

Warfare Can Be Calculated

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Combat Modelling (CM) is calculating of combat outcomes based on mathematical models. During the cold war a lot of research was spent to develop CM, but still CM can hardly be considered a scientific undertaking. CM is rather a kind of practical tool for support to military decision makers. CM may be based on different types of mathematical models, traditionally deterministic models of the Lanchester type. These models are often used without due regard to underlying assumptions, and the outcomes may differ from the “true” mean outcomes usually wanted. CM based on stochastic models, especially time continuous and state discrete Markovian processes is in principle better than deterministic models. Stochastic CM has an innate potential for determining the “true” mean outcome usually wanted, but unfortunately stochastic CM often leads to very complex models.

The Danish Defence Research Establishment (DDRE) has developed a stochastic combat model, Defence Dynamics (DD) intended to describe tri-service combat. DD overcomes many of the weaknesses of the Lanchester type of models. DD is based on a universal combat model for a duel between two arbitrary military units. This model is a stochastic model able to describe almost any kind of duel. By use of a new concept: “A representative pair” the model has been generalized to arbitrary homogeneous and heterogeneous combat situations. Unfortunately the generalization requires an approximation, but it seems to be tolerable. DD determines outcomes much closer to the “true” mean outcomes than is possible with traditional CM. DD has supported practical problem solving both for the Danish defence and for NATO.

1 Introduction

Combat modelling is a kind of mathematical modelling intended to calculate combat outcomes. During the cold war combat modelling became a major tool to support NATO military decision makers. In those times very few people wondered if warfare might be calculated. The important problem was how warfare should be calculated. Generally speaking it was agreed that combat modelling is of no practical value with regard to reliable prediction of real life combat outcomes. The uncertainty related to this kind of prediction was considered too high. This conclusion is in no way surprising, because real life combat is influenced by a huge number of both qualitative and quantitative factors, e.g.,

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- Personnel
- Leadership
- Moral
- Training and education
- Weapon- and sensor systems
- Command, control and information systems
- Strategies and tactics
- Terrain
- Weather
- Light conditions

which may only be known to a limited and uncertain extent.

Nevertheless combat modelling was during the cold war still considered a useful tool, when military decision makers had to deal with problems like:

- Long term planning with regard to structuring of military forces
- Acquisition of weapon- and sensor systems
- Acquisition of CCIS systems
- Development of new military concepts and doctrines
- Optimisation of logistic systems

In relation to this kind of problems it was generally believed that it would be sufficient to consider mean combat outcomes. It was furthermore believed that combat modelling based on the Lanchester tradition would be sufficient for providing combat outcomes close to or equal to the “true” mean combat outcomes wanted.

But despite all the research and despite all the resources spent, combat modelling can hardly be considered a fully scientific undertaking. The reason for this statement is that combat models can hardly be exposed to empirical testing. Certainly it is not physically impossible to carry out adequate empirical testing on combat models, but in practice it will usually be considered unacceptable both for economic and moral reasons. Some scattered pieces of empirical evidence may be obtained from studies of war history, but this will hardly be sufficient to establish combat modelling as scientific.

Nevertheless combat modelling is generally accepted as a practical tool for military decision makers. Combat modelling may be useful for military decision makers to support summarizing, processing and reaching conclusions from relevant and available information. This information may be quite overwhelming and almost impossible to treat in any other way. Combat modelling may be considered a kind of thinking device enabling the military decision maker to a better understanding and solution of problems.

2 Possible Types of Combat Models

In fact there is a broad spectrum of possible types of mathematical models, which might provide the theoretical basis for combat modelling. Underlying each of these types of models is a set of specific assumptions, limitations and possibilities. Different mathematical criteria with regard to classification of combat models may be established. Based on a classification it might be easier to discuss, criticize and select a type of combat model relevant for a specific application.

The following set of mathematical criteria is considered sufficient for the present purpose:

- Descriptive models / optimisation models
- Deterministic models / stochastic models
- Time continuous models / time discrete models
- State continuous models / state discrete models.

In fact most military problems suited for combat modelling are decision problems. Consequently it should be obvious to select *optimisation models* (i.e., mathematical models, which include both a decision function and an objective function) as a basis for combat modelling. But unfortunately optimisation models will in most cases become overwhelming in mathematical complexity when applied to traditional military problems, so they will not be considered any further at this occasion. The only possibility left is *descriptive models*, i.e., mathematical models, which do not include any explicit decision function.

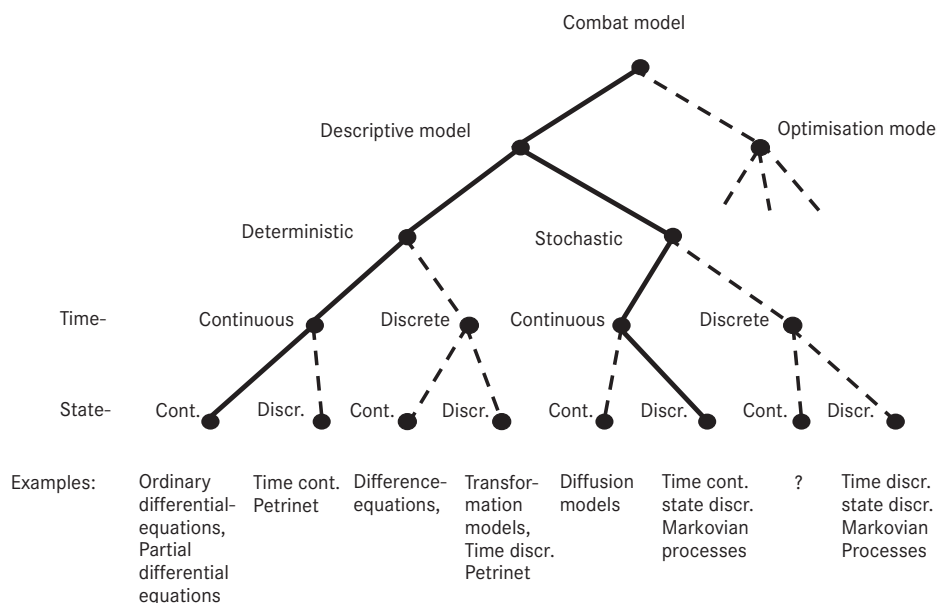


Figure 1. Possible mathematical types of combat models.

To keep it simple only two options are open to determine a combat outcome, especially a mean combat outcome by use of descriptive models: Deterministic and stochastic descriptive models. A *deterministic model* can usually only be justified if it is able to produce a combat outcome close to or equal to some “true” mean outcome. But this is meaningless because a mean outcome usually is not even defined in relation to a deterministic model. With a *stochastic model* it will be meaningful and in principle possible to determine a mean combat outcome, which is “true” at least within the framework specified by the stochastic model.

Of course time is a major factor for combat modelling and it has to be portrayed within the model. Again only two simple options need to be considered: Time discrete – and time continuous models. It is a basic fact of war that events relevant for combat modelling may occur at any time, so for that reason it should be obvious to apply *time continuous models*. Nevertheless time discrete models are quite frequently used in practice because they are especially well suited for discrete numerical computations. But in principle use of time discrete models introduces an error, because the events are forced within the model to occur only at the discrete points of time allowed by the model. This error will be reduced if the time steps chosen are diminished, but then the amount of work for solving the model and the cumulated numerical errors will increase. Time discrete models exclude the use of strong analytical tools such as differentiation and integration.

In combat modelling the number of operational military units at an arbitrary point of time is usually considered the state of the relevant military system. This state factor may also be portrayed in two different ways: Discrete or continuous. A *state discrete model* is the natural choice, because the real life number of units is usually considered an integer. But unfortunately a state discrete model may be too demanding to operate if the number of units on each side is high. With a *state continuous model* the real life number of units will be considered a real number and this may be a gross approximation, especially when the number of units are small, especially smaller than one.

Based on this kind of considerations it should in principle be obvious to choose *descriptive, stochastic, time continuous and state discrete types of models* as the best-suited type of mathematical model with regard to combat modelling. It might for instance be a time continuous and state discrete Markovian process model. Unfortunately this type of combat model has only been implemented a few times, because it has a tendency to become unacceptably complex.

In common practice combat modelling is quite often based on *descriptive, deterministic, time continuous and state continuous types of models*. Usually it is a set of ordinary differential equations of the Lanchester type. The argument for using differential equations is that they are sufficient to determine the mean combat outcome. But this argument is doubtful, when the mean outcome is not defined within a deterministic model.

3 NATO Combat Modelling

Today combat modelling within NATO is considered to be a fairly well established and almost concluded research area. But this assessment is highly doubtful. Most combat modelling within NATO is still to a high extent based on deterministic descriptive models, i.e., ordinary differential or difference equations of the Lanchester type often used without due respect to underlying assumptions and limitations.

To exemplify this criticism a few combat models of the Lanchester type based on ordinary differential equations are considered. The simplicity of the examples chosen is necessary for purposes of presentation and computation, but the criticism will also be valid with regard to more advanced models of the Lanchester type.

3.1 Lanchester square model

The Lanchester square model is the best known of the classical Lanchester models. Lanchester considered this model especially relevant for modern warfare (1917):

$$\frac{db(t)}{dt} = -\kappa_r \cdot r(t) \quad b(0) = B$$

$$\frac{dr(t)}{dt} = -\kappa_b \cdot b(t) \quad r(0) = R$$

B initial number of blue units

$b(t)$ number of operational blue units at time t

κ_b kill rate for one operational blue unit against operational red units

R initial number of red units

$r(t)$ number of operational red units at time t

κ_r kill rate for one operational red unit against operational blue units

This model describes combat between two homogeneous forces (a homogeneous force is a force including only one type of military units, e.g., tanks), both fighting under the assumption about full tactical information.

Full tactical information means that an arbitrary operational unit is at any time able to detect at least that many hostile operational units as it is capable of killing. Furthermore it is assumed that all operational units on each side are able to fully share their information and coordinate their firepower among the operational hostile units. In fact firepower is the only limiting factor. The capabilities for detection and coordination are assumed sufficient for firing.

This assumption may be quite unrealistic, because it is difficult to identify many modern combat situations, where the assumption is fulfilled on both sides. For example very few army operations are carried out with full tactical information

on both sides. Within NATO the square model is often used without sufficient consideration to this assumption.

The square model is simple and it may be solved by analytical means. From the analytical solution it is possible to derive the following criterion for victory:

$$R^2 \cdot \kappa_r > B^2 \cdot \kappa_b \Rightarrow \text{red victory}$$

$$R^2 \cdot \kappa_r < B^2 \cdot \kappa_b \Rightarrow \text{blue victory}$$

$$R^2 \cdot \kappa_r = B^2 \cdot \kappa_b \Rightarrow \text{mutual destruction}$$

The saying, that “Quantity is more important than quality”, presumably originated from this criterion.

Numerical example:

$$B = 200, R = 400$$

$$\kappa_b = 0.40, \kappa_r = 0.15$$

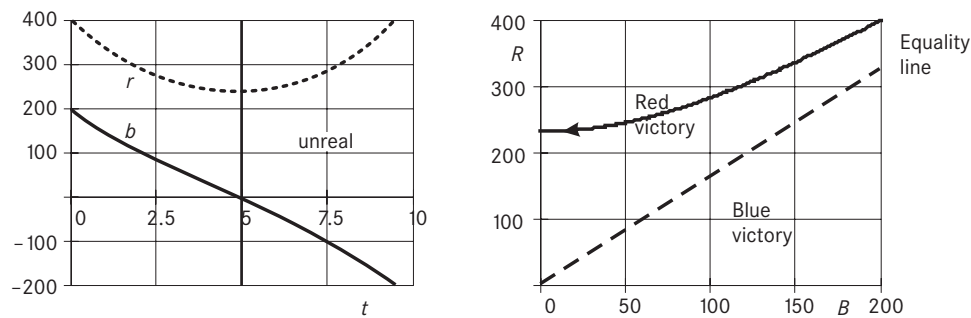


Figure 2. Combat outcomes derived from the Lanchester square model.

The Lanchester square model is a time and state continuous model in practice often assumed to be a kind of mean value model. The logic behind the model deteriorates with diminishing numbers of operational units and it certainly collapses when the number of operational units becomes less than one or even negative. The model is a pair of ordinary non-linear differential equations and it exhibits chaotic behaviour in combat situations close to the equality line (equality leads to mutual destruction).

3.2 Guerrilla model

Another quite popular combat model of the Lanchester type is a model that might be called “the guerrilla model”:

$$\frac{dr(t)}{dt} = -\delta_b \cdot r(t) \cdot b(t) ; \quad r(0) = R$$

$$\frac{db(t)}{dt} = -\delta_r \cdot r(t) \cdot b(t) ; \quad b(0) = B$$

R initial number of red units

$r(t)$ number of operational red units at time t

δ_r detection rate for one operational red unit against an arbitrary operational blue unit

B initial number of blue units

$b(t)$ number of operational blue units at time t

δ_b detection rate for one operational blue unit against an arbitrary operational red unit

This model also describes combat between two homogeneous forces. One possible interpretation of the model is that each force is able to kill all hostile operational units that have been detected. In fact this model describes a combat situation in which the saying “If you are seen, you are dead” will be true. This might be valid for combat between two guerrilla forces searching for each other in a wilderness. The detection rate is the only limiting factor. Firepower and the capability for coordination are sufficient for killing all detected hostile units. This assumption may also be unrealistic, because it is difficult to identify many regular combat situations, where detection is the only crucial factor on both sides.

The guerrilla model is simple and it may be solved by analytical means. From the analytical solution it is possible to derive the following criterion for victory:

$$R \cdot \delta_r > B \cdot \delta_b \Rightarrow \text{red victory}$$

$$R \cdot \delta_r < B \cdot \delta_b \Rightarrow \text{blue victory}$$

$$R \cdot \delta_r = B \cdot \delta_b \Rightarrow \text{mutual destruction}$$

With the assumption valid for “the guerrilla model” quality and quantity obviously are of equal importance.

This type of model has also been used to describe outcomes resulting from indirect fire combat, i.e., outcomes relevant when each of two artillery forces at random shoot against the territory occupied by the other.

Numerical example:

$$B = 200, R = 400$$

$$\delta_b = 0.40, \delta_r = 0.15$$

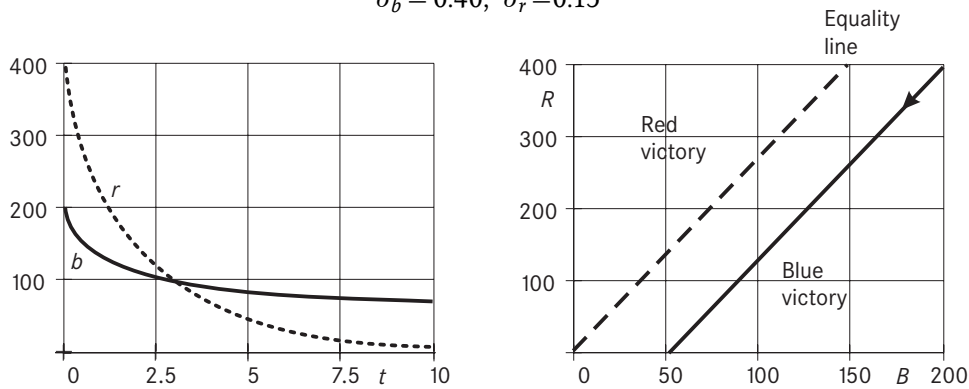


Figure 3. Combat outcomes derived from the Guerilla model.

“The guerrilla model” also is a time and state continuous model in practice often assumed to be a kind of mean value model. The logic behind the model deteriorates with diminishing numbers of operational units and it certainly collapses when the number of operational units becomes less than one. With this model the number of operational units cannot become negative.

This model is a pair of ordinary non-linear differential equations, but it does not exhibit any chaotic behaviour.

3.3 Mixed combat model

The last example of a combat model of the Lanchester type treated here is a mixed combat model (a mixture of the square model and “the guerrilla model”):

$$\frac{dr(t)}{dt} = -\kappa_b \cdot b(t) \quad r(0) = R$$

$$\frac{db(t)}{dt} = -\delta_r \cdot b(t) \cdot r(t) \quad b(0) = B$$

R initial number of red units

$r(t)$ number of operational red units at time t

δ_r detection rate for one operational red unit against an arbitrary operational blue unit

B initial number of blue units

$b(t)$ number of operational blue units at time t

κ_b kill rate for one operational blue unit against operational red units

The model describes combat between two homogeneous forces. The interpretation might be that red is advancing over an open terrain looking for and fighting against blue. Blue is fighting back from concealed and camouflaged positions. The detection rate is the only limiting factor for red. The firepower is the only limiting factor for blue.

The model may be solved by analytical means. From the analytical solution it is possible to derive the following criterion for victory:

$$R^2 \cdot \delta_r > 2 \cdot B \cdot \kappa_b \Rightarrow \text{red victory}$$

$$R^2 \cdot \delta_r < 2 \cdot B \cdot \kappa_b \Rightarrow \text{blue victory}$$

$$R^2 \cdot \delta_r = 2 \cdot B \cdot \kappa_b \Rightarrow \text{mutual destruction}$$

With the assumptions valid for the mixed model quality is more important than quantity for red, while quality and quantity are equally important to blue.

Numerical example:

$$B = 200, R = 400$$

$$\kappa_b = 0.40, \delta_r = 0.15$$

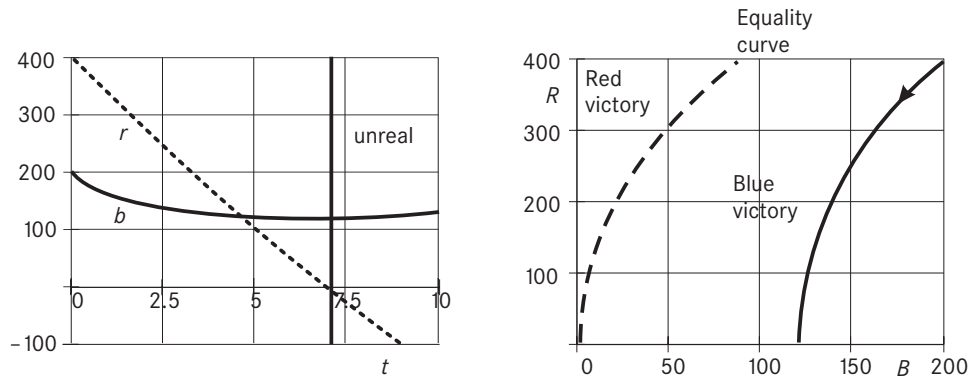


Figure 4. Combat outcomes derived from the mixed combat model.

This is a time and state continuous model, in practice often assumed to be a kind of mean value model. The logic behind the model deteriorates with diminishing numbers of operational units and it certainly collapses when the number of operational units becomes less than one or even negative.

The model is a pair of ordinary non-linear differential equations, but it does not exhibit any chaotic behaviour.

The three simple combat models presented and discussed exemplify deterministic combat models of the Lanchester type, which are frequently used within NATO. The examples have demonstrated that the models are based on certain cru-

cial assumptions embedded within the models. Quite often these types of models are used without sufficient awareness of and regard to the assumptions.

3.4 Comparisons of deterministic and stochastic combat models

Another point of criticism against deterministic combat models of the Lanchester type is that the concept of a mean combat outcome is not even defined within these models. To define and determine a mean combat outcome it is necessary to use a stochastic model.

To illustrate this point the three deterministic combat models already considered are compared to three corresponding continuous and state discrete Markovian process models. The stochastic models are based on the same assumptions and parameters as the deterministic models. For each of the three relevant situations a numerical comparison between the combat outcome determined with the deterministic model of the Lanchester type and the “true” mean outcome determined by the corresponding Markovian model has been carried out. To keep it simple and manageable the initial numbers of the two military forces considered have been kept low: 1 blue and 2 red military units. (Note that in the figure below “EB” and “ER” signify the “true” mean outcomes determined by Markovian models for blue and red respectively. “B” and “R” are the outcomes determined by deterministic models.)

Numerical example:

Comparison between the outcome determined by the square Lanchester model and the “true” mean outcome determined by a corresponding time continuous and state discrete Markovian process combat model.

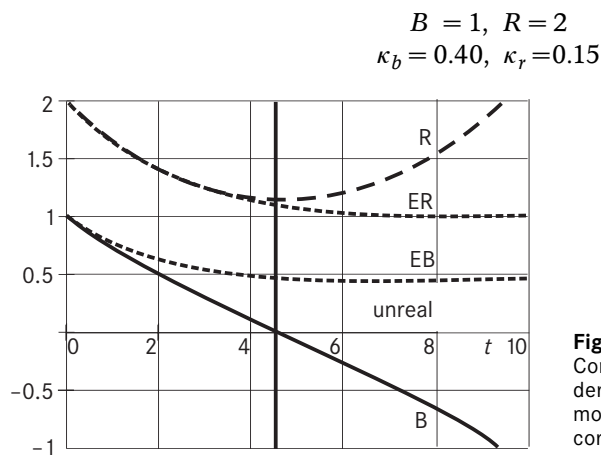


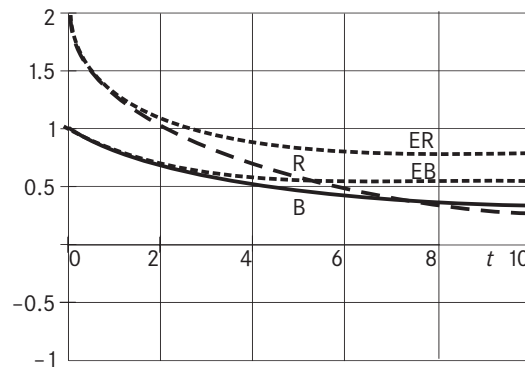
Figure 5. Comparison between combat outcomes derived from the Lanchester square model and mean outcomes derived from a corresponding stochastic model.

Numerical example:

Comparison between the outcome determined by “the Guerrilla model” and a “true” mean outcome determined by a corresponding time continuous and state discrete Markovian process combat model.

$$B = 1, R = 2$$

$$\delta_b = 0.40, \delta_r = 0.15$$

**Figure 6.**

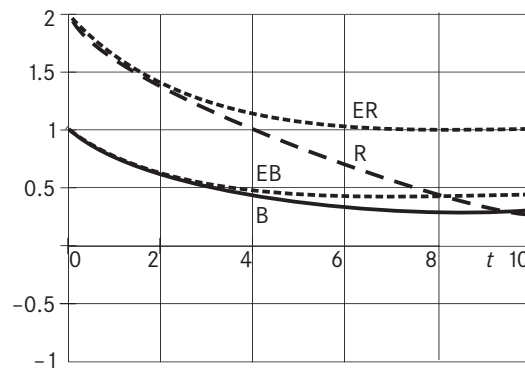
Comparison between combat outcomes derived from the Guerilla model and mean outcomes derived from a corresponding stochastic model.

Numerical example:

Comparison between the outcome determined by the mixed model and a “true” mean outcome determined by a corresponding time continuous and state discrete Markovian process combat model.

$$B = 1, R = 2$$

$$\kappa_b = 0.40, \delta_r = 0.15$$

**Figure 7.**

Comparison between combat outcomes derived from the mixed model and mean outcomes derived from a corresponding stochastic model.

The three comparisons reveal considerable differences between the combat outcome determined by a deterministic model of the Lanchester type and the “true” mean combat outcome determined by a corresponding Markovian process model. The differences will not be large for high numbers of units, but they will increase with diminishing numbers of units left. So with regard to determining the “true” mean outcome of a combat a stochastic model obviously is preferable to a deterministic model especially for small numbers of units. In fact the mean combat outcome is only defined in relation to a stochastic model.

Furthermore this stochastic combat model does have other advantages. A state discrete Markovian model only operates with integer numbers of operational units. The integer number can never become negative and the “smaller than one” problem has completely disappeared.

For all these reasons stochastic models are more attractive than corresponding deterministic models. But unfortunately stochastic models also suffer from serious drawbacks compared to the deterministic models. These drawbacks may be most prohibitive in practice:

- The mathematical complexity has increased dramatically.
- The number of equations blows up with factors of maybe millions.
- The time for solution of the models becomes unacceptably lengthy.

4 Defence Dynamics

In 1980 The Danish Defence Command asked the Danish Defence Establishment (DDRE) to develop a tri-service combat model as a tool for long term defence planning. Later this model became known as *Defence Dynamics* (DD). The idea was that DD should be able to predict a mean combat outcome if a possible Danish tri-service defence structure was attacked by WAPA. The Danish Defence Command had set up a list with 10 - 20 possible Danish defence structures and 5 - 10 possible WAPA attack structures against Denmark.

After prolonged and difficult discussions concerning combat modelling DDRE decided to base DD on a so-called “*Universal combat model*”. This model was intended to predict stochastic outcomes for a duel between two military units independent of their types and services. Furthermore this model was intended to be most flexible with regard to different kinds of relevant assumptions concerning the actual combat situation. It was decided to apply time continuous and state discrete Markovian processes as the relevant mathematical framework. This type of mathematical model was chosen to ensure that it would be possible to determine the “true” mean combat outcome defined within the model framework. As exemplified earlier this type of mathematical model would avoid both the “smaller than zero” and “smaller than one” problem, which otherwise might be most frustrating in combat situations involving only a few military units. The universal combat model was intended to serve as a basis for development of a full-scale tri-service model.

DDRE succeeded in developing the so-called “universal combat model”, which with very few exceptions is applicable to duel situations in general. One exception is a duel between a sea mine and a landing ship. For the few exceptional cases it was necessary to develop specific models.

4.1 One-on-one combat

The “Universal Combat Model” was originally based on a quite limited number of basic factors, which it was considered important to take into consideration:

– *Tactical manoeuvres*

During combat a military unit can usually change between a passive and an active state. Within the passive state the unit is not able to detect and fight the hostile unit and the hostile unit is not able to detect and fight the unit. Within the active state the unit may be able to detect and fight the hostile unit and the hostile unit may be able to detect and fight the unit.

– *Stand off advantage*

If the unit has a stand off advantage due to long-range sensor- and weapon systems it is able to detect and fire upon the hostile unit without any risk of retaliation.

– *Detection*

Detection of the hostile unit depends on the unit’s sensor systems and the signature of the hostile unit compared to its background. Detection may be highly influenced by weather, light conditions, decoys and means of concealment and camouflage.

– *Firing*

Conditioned by detection of the hostile unit the unit is able to fire upon it. Firing depends on the actual weapon systems and types of ammunition available to the unit.

– *Kill*

Conditioned by detection of and firing the unit may be able to kill the hostile unit. Kill depends on the actual weapon systems, the types of ammunition available to the unit and the physical protection of the hostile unit.

Later on further features have been added to “the universal combat model”, for example:

- coordination among units within a group,
- decoy effects due to killed hostile units,
- limited ammunition supplies,
- limited sensor- and weapon ranges (front sections),

- initial detection, when the unit first appears from the passive state,
- Suppression.

To keep the presentation simple these further features will not be considered here.

The basic factors are reflected within the universal combat model partly by the following possible states for a military unit:

- s** The unit is passive.
- k** The unit has a stand off advantage compared to the hostile unit.
- n** The unit is active, but it has not been detected by the hostile unit.
- d** The unit is active and it has been detected by the hostile unit.
- e** The unit is active (and the hostile unit is passive). Note “ $e = n + d$ ”.
- u** The unit is killed.

Of course the hostile unit may independently of the unit occupy the same kind of states with a single exception; it is not possible for both units simultaneously to have a stand off advantage (state k).

Taken together a pair of two opposing units may consequently occupy and change between the following 16 combined states: ss, se, su, ks, kn, kd, ku, es, nn, nd, dn, dd, eu, us, ue and uu. Note the combined states where the hostile unit is in state k have been omitted. The 16 combined states are considered to be the possible states for the universal combat model, i.e., a time continuous and state discrete Markovian process.

To make this mathematical model operational the possible transitions between the states are characterized by the following (transition) rates:

Rates	Blue	Red
Activation	η_b	η_r
Passivation	σ_b	σ_r
Loss of stand off advantage	ε_b	ε_r
Detection	δ_b	δ_r
Loss of detection	γ_b	γ_r
Kill	κ_b	κ_r

Table 1.
Transition rates relevant for one-on-one combat.

The information necessary for setting up a time continuous and state discrete Markovian process may be summarized within the transition rate matrix characteristic for this type of stochastic model (Table 2).

The diagonal elements within the matrix are, as a consequence of mathematical technicalities, defined as the negative sum of all other elements within the relevant row. Note furthermore the state uu will be impossible with this type of model, because it does not allow two events (two killings) to occur simultaneously.

The corresponding Chapman-Kolmogorov equation for determination of the time dependent probability distribution for the actual combined state of the two units is described by 14 first order linear differential equations, which may

A	ss	se	su	ks	kn	kd	ku	es	nn	nd	dn	dd	eu	us	ue	uu
ss	-	σ_r		σ_b				σ_b								
se	η_r	-		σ_b	σ_b			σ_b	σ_b	σ_b	σ_b					
su			-			σ_b						σ_b				
ks	η_b			-	σ_r	σ_r										
kn		η_b		η_r	-	γ_b										
kd				δ_b	-											
ku			η_b		κ_b	-										
es				ε_b				-	σ_r	σ_r	σ_r	σ_r				
nn				ε_b				η_r	-	γ_b	γ_r					
nd					ε_b			δ_b	-		γ_r					
dn						δ_r		δ_r	-	γ_b						
dd						δ_r	δ_b	δ_r	δ_b	-						
eu							κ_b	κ_b	κ_b		-					
us														-	σ_r	
ue														η_r	-	
uu									κ_r	κ_r						-

Table 2.
Transition rate matrix
relevant for one-on-one
combat.

be solved relatively easily by use of either analytical or numerical methods. The information included in the transition rate matrix may also be demonstrated in a more conspicuous graphical way by use of a transition diagram. The combined states are represented by nodes and the possible transitions between the states are represented by arrows connecting the relevant nodes.

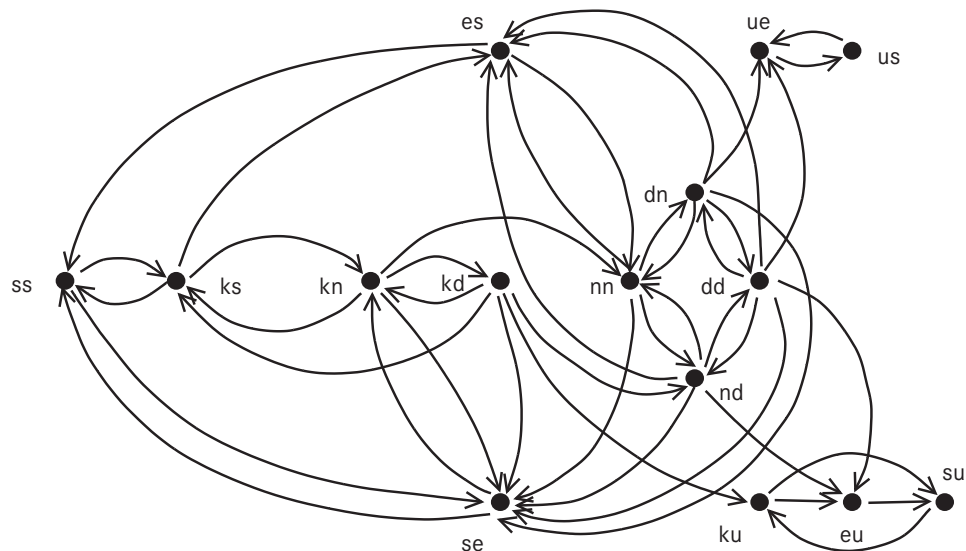


Figure 8. Transition diagram for one-on-one combat.

4.2 Many-on-many homogeneous combat

To generalize one-on-one combat to many-on-many homogeneous combat it is necessary to introduce further military units on each side:

- B Initial number of blue homogeneous units
- R Initial number of red homogeneous units.

But first and foremost it is necessary to have an idea that makes it possible to avoid the overwhelming number of combined states due to possible combinations of blue and red operational units. Note, that already 200 blue units and 400 red units immediately imply $(200 + 1) \cdot (400 + 1) = 80\,601$ possible combined states, when the only states allowed for a unit is operational or killed. This might be considered an immediate threat of a set of 80 600 ordinary differential equations.

To escape from this major complication the idea of a *representative pair of opposing units* has been introduced:

In principle all units on each side fight under identical conditions. For that reason it should be sufficient for modelling purposes to select an arbitrary pair of opposing representative units fighting a one-on-one combat and then superimpose the influences from all the other non-representative units.

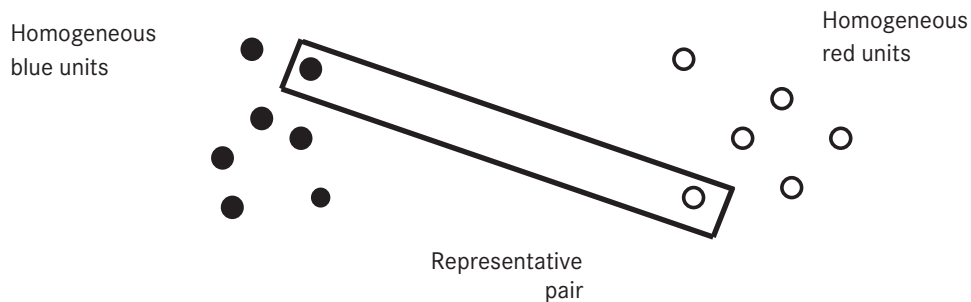


Figure 9. The concept of a representative pair.

Based on this idea it might be sufficient to elaborate on the Markovian model for one-on-one combat, hoping to avoid a dramatic increase in the number of possible states. Of course a number of conditions related to the original one-on-one universal combat model has to be reconsidered to develop a model for a representative pair within a many-on-many homogeneous combat situation.

States/transitions related to detection/non-detection of a representative unit have to be subjective, i.e., only relevant to the opposing unit within the representative pair. Other opposing units may have a different point of view with regard to detection/non-detection of the representative unit. It means for instance that a unit may be detected by the opposing unit within the representative pair, but not by any other opposing unit and vice versa.

States/transitions related to everything else are (in most cases) considered objective, i.e., all opposing units have the same point of view concerning the representative unit. It means for instance that if a unit is killed, it is killed from the point of view of all opposing units.

If the representative pair is in *state kd*, it means that the representative blue unit has a stand off advantage (state k) with regard to all opposing units. Furthermore the representative blue unit has detected the representative red unit (state d) with certainty and an arbitrary non-representative red unit with a probability p_k :

$$p_k = \frac{p_{kd}}{p_k} ; \quad p_{k\bullet} = p_{ks} + p_{kn} + p_{kd} + p_{ku}$$

The expected kill rate from the representative blue unit in state k against an arbitrary detected red unit becomes:

$$\alpha_{bk} = E \left\{ \frac{\kappa_b}{1+X} \right\} = \sum_{x=0}^{R-1} \frac{\kappa_b}{1+x} \binom{R-1}{x} p_k^x (1-p_k)^{R-1-x} = \frac{\kappa_b}{R p_k} [1 - (1-p_k)^R]$$

X is the stochastic number of non-representative red units detected by the representative blue unit in state k. X is binomially distributed with the parameters $R - 1$ and p_k .

Something quite similar is valid for the representative pair in *state ed*. The representative blue unit is active and exposed (state e) with regard to opposing units. Furthermore the representative blue unit has detected the representative red unit (state d) with certainty and an arbitrary non-representative red unit with a probability p_e .

$$p_e = \frac{p_{ed}}{p_e} ; \quad p_{e\bullet} = p_{es} + p_{em} + p_{ed} + p_{en} + p_{ee} + p_{eu}$$

The expected kill rate from the representative blue unit in state e against an arbitrary detected red unit becomes:

$$\alpha_{be} = E \left\{ \frac{\kappa_b}{1+X} \right\} = \sum_{x=0}^{R-1} \frac{\kappa_b}{1+x} \binom{R-1}{x} p_e^x (1-p_e)^{R-1-x} = \frac{\kappa_b}{R p_e} [1 - (1-p_e)^R]$$

Here X is the stochastic number of non-representative red units detected by the representative blue unit in state e. X is binomially distributed with the parameters $R - 1$ and p_e .

Now the total expected kill rate from all non-representative blue units against the representative red unit in state e may be determined by:

$$K_b = (B-1) \left(\frac{p_{kd}}{p_e} \alpha_{bk} + \frac{p_{ed}}{p_e} \alpha_{be} \right)$$

α_{bk} , α_{be} and K_b is state dependent kill rates, which shall be introduced into the transition rate matrix for one-on-one combat to describe the full threat of a kill on the representative red unit due to all blue units.

Corresponding α_{re} and K_r are state dependent transition rates, which shall be introduced into the transition rate matrix for one-on-one combat to describe the full threat of a kill on the representative blue unit due to all red units. But unfortunately this derivation required an *approximation*. The kill rates α_{bk} , α_{be} and α_{re} for a representative unit against an arbitrary opposing unit are determined by averaging over all non-representative opposing units instead of using the actual (but unknown) number of detected non-representative opposing units.

The great advantage with this approximation is that it reduces the number of possible states within the relevant Markovian process from maybe millions to 16, but never the less it still is an approximation. A number of sensitivity analyses have been carried out for modest combat situations. The mean number of operational units determined based on this approximation deviated in no case more than 9% from the “true” mean number of operational units determined with a correct Markovian process model.

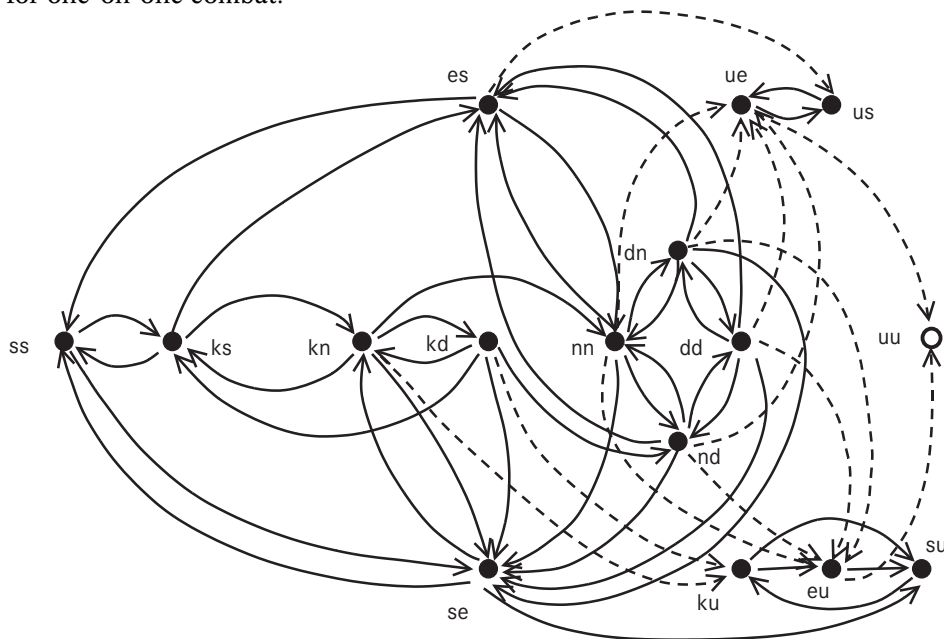
Based on this approximation the transition rate matrix valid for the representative pair within a many-on-many homogeneous combat becomes:

A	ss	se	su	ks	kn	kd	ku	es	nn	nd	dn	dd	eu	us	ue	uu
ss	-	σ_r		σ_b				σ_b								
se	η_r	-		σ_b	σ_b			σ_b	σ_b		σ_b	σ_b				
su		K_b	-				σ_b						σ_b			
ks	η_b			-	σ_r	σ_r										
kn		η_b		η_r	-	γ_b										
kd					δ_b	-										
ku			η_b		K_b	$\alpha_{bk}+K_b$	-									
es				ε_b				-	σ_r	σ_r	σ_r	σ_r				
nn				ε_b				η_r	-	γ_b	γ_r					
nd					ε_b			δ_b	-		γ_r					
dn								δ_r		-	γ_b					
dd									δ_r	δ_b	-					
eu							ε_b	K_b	$\alpha_{be}+K_b$	K_b	$\alpha_{be}+K_b$	-				
us								K_r						-	σ_r	
ue								K_r	K_r		$\alpha_{re}+K_r$	$\alpha_{re}+K_r$		η_r	-	
uu													K_r		K_b	-

Table 3. Transition rate matrix relevant for one-on-many homogeneous combat.

So in this way many-on-many homogeneous combat is still represented by a time continuous and state discrete Markovian process with only 16 possible states. It means that the corresponding Chapman-Kolmogorov equation for determination of the probability distribution for the actual state of the representative pair is described by 15 first order non-linear differential equations, which may be solved relatively easily by numerical methods of integration.

The information included in the transition rate matrix for many-on-many homogeneous combat may be demonstrated in a more conspicuous graphical way by use of a corresponding transition diagram. Note, the dashed arrows indicate rates, which have been modified or added compared to the transition rate matrix for one-on-one combat.



4.3 Many-on-many heterogeneous combat

It will be necessary to define a representative pair of opposing units for each relevant combination of homogeneous forces fighting each other within each heterogeneous sub battle.

The principles and the approximation necessary for this generalization are quite similar to those applied for generalization from one-on-one combat to many-on-many homogeneous combat. For each relevant representative pair of opposing units it is now necessary to superimpose not only the influence from all non-representative units within their own homogeneous force. Now it is also necessary to superimpose the influence from all units within all the other homogeneous forces taking part in the heterogeneous sub battle.

When DD was first applied to support long term planning for the Danish Defence Command a few thousands of representative pairs were defined. The maximum number of non-linear differential equations, which were set up and integrated simultaneously, amounted to approximately 20.000, when a heterogeneous sub battle going on in Schleswig-Holstein was really intense.

Within the heterogeneous battle each sub heterogeneous battle is described with a many - on - many heterogeneous combat model, which determines the time dependent probability distributions for each relevant representative pair. From these probability distributions the mean number of blue and red units surviving the sub battle may easily be determined

Due to the averaging approximation it has not up till now been possible to determine the “true” outcome of a heterogeneous battle exactly. But the battle outcome determined by Defence Dynamics is still considered to be a magnitude of size closer to the “true” battle outcome than outcomes generated with deterministic combat models of the Lanchester type.

For the time being DDRE hopes to avoid introducing the basic approximation into Defence Dynamics by use of a special blending of numerical and Monte Carlo simulation methods for solving the model. In this way it should be possible to generate combat outcomes very close to or identical to “true” mean combat outcomes. But this still requires a lot of research to be seen.

4.4 Practical considerations to use of Defence Dynamics

DD is considered a tool for summarizing, processing and reaching conclusions with relevant and available information. This tool is intended to provide support to decision makers operating within a broad spectrum of military problems. In any case use of DD requires the following type of input information:

- A blue force structure, including types and numbers of weapon systems, ammunition and sensor systems
- A red force structure, including types and numbers of weapon systems, ammunition and sensor systems
- Strategies and concepts for both blue and red force
- CCIS, Command, Control and Information Systems
- The weather situation
- Light conditions
- Topographical information
- A technological database concerning weapon- and sensor systems and parameters for all relevant one-on-one combat models.

It certainly is a most demanding job to provide and maintain all this information necessary for using DD.

Based on the input information DD generates a combat outcome. This combat outcome is an approximation to the “true” mean outcome. Based on this outcome the decision maker may find support to make one or another decision.

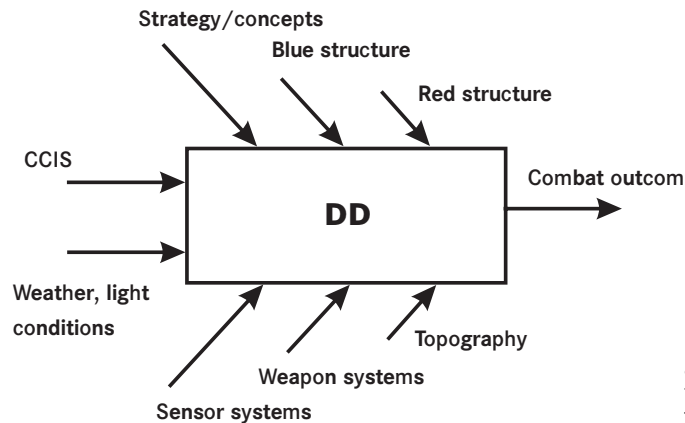


Figure 11. Defence Dynamics generates a combat outcome based on the type of information specified.

The practical procedure for using DD may be illustrated by the following flow chart:

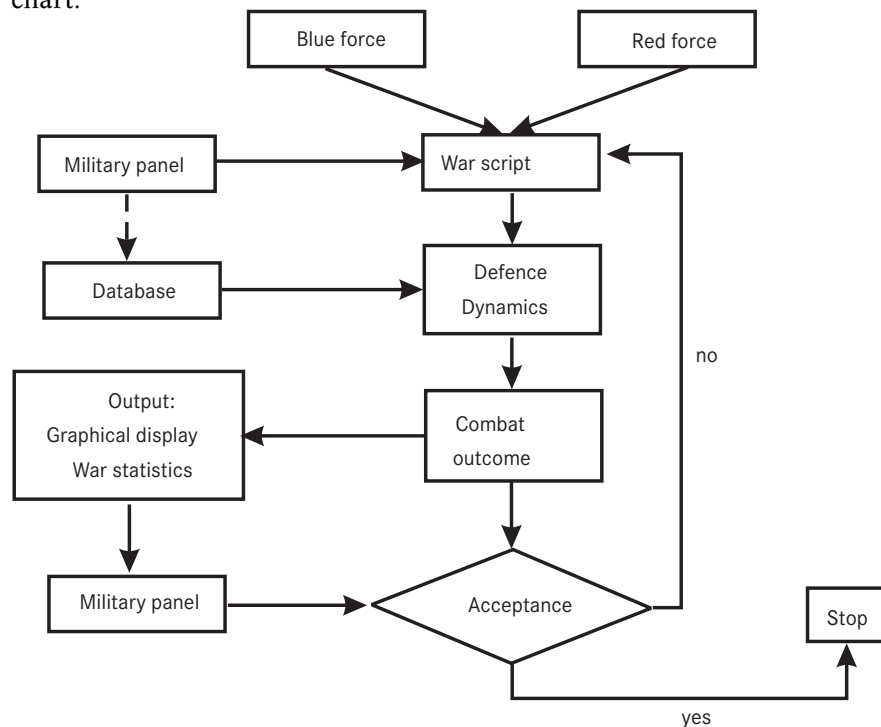


Figure 12. Practical procedure for using Defence Dynamics.

A military panel defines the relevant blue and red force. Afterwards the military panel works out a so-called war script for the heterogeneous battle situation considered. The war script specifies all movements and engagements for blue and red forces based on actual strategies and concepts. DD exploits the war script to control the setting up and removal of the relevant many-on-many heterogeneous combat models necessary to describe parallel and serial sub battle situations. A team of military and analytical people has prepared the database with information concerning all relevant weapon-sensor systems. This team furthermore has to analyze all relevant representative one-on-one combat situations to provide the parameters necessary. DD draws the relevant parameters directly from the database when it establishes the combat models. Based on all this input information an approximated mean battle outcome is determined. This outcome is presented to the military panel by use of graphical displays and war statistics. The panel decides if the outcome is in agreement with the strategies and concepts intended. If so the outcome is accepted, otherwise the war script is revised and a new computer run is implemented. This will continue until the panel finally accepts the combat outcome. From the concluding outcome the conclusions relevant to the actual practical application will be drawn.

The total process is time consuming and difficult to carry out. Furthermore the process will quite often provoke disagreements among the members of the military panel. Such disagreements may delay the process further.

Despite the problems and the heavy amount of work involved DD has been used on a number of occasions. The most important applications have been:

- 1982 "The October War". The Danish Long Term Planning Group. The Danish Defence Command.
- 1989 Evaluation of possible new Air Defence Systems. The Danish Advisory and Analysis Group. The Danish Defence Ministry.
- 1989 Division Jutland defensive combat in Schleswig-Holstein. The Danish Advisory and Analysis Group. The Danish Ministry of Defence.
- 1989 Comparison of the TOW system and Leopard tanks used for defensive purposes. (European disarmament negotiations.) ACAG/NATO.
- 1992 Development of a new doctrine for the Danish army mobile combat. The Danish Defence College.
- 1994 Analyses intended to investigate the concept of Stable Defence. Defence Research Group, NATO.
- 1994 Development of an optimal structure and size for a Danish armoured battalion. The Danish Advisory and Analysis Group. The Danish Ministry of Defence.

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