2 Black-Bodies, White Suns

2.1 Introduction

A black-body is simply a body or object that is a perfect absorber of light and therefore, by a fundamental reciprocal relation, a perfect emitter. Although the perfect black-body represents a mathematical ideal, physical objects and devices can approach black-body properties reasonably closely. Even though defined so simply, black-bodies have played a surprisingly large role in the evolution of physics. Attempts to understand the spectral distribution of light emitted by heated black-bodies led directly to the development of quantum mechanics (Duck 2000). One particular approximation to a black-body, the paradoxically white and bright sun, has had an even more significant impact on human culture, being indispensable to human life itself.

Black-bodies feature prominently in the theory of the limiting performance of solar cells. Not only is the radiation emitted by the sun a good approximation to that from a very hot black-body, but an ideal solar cell would be expected to be a good absorber of light and hence be related to a black-body in some way. The details of this relationship forms the focus of many of the following chapters. An additional reason for the interest in black-bodies in solar cell theory is that, given their prominence in the development of physics, their thermodynamics have been thoroughly explored. This is particularly helpful when seeking to evaluate limiting solar cell performance.

2.2 Black-Body Radiation

A simple way to make a black-body is to put a small hole in an otherwise fully enclosed cavity (Fig. 2.1). Any ray entering the cavity via this hole will have great difficulty escaping, particularly if the cavity walls have moderately good absorption properties. The cavity therefore acts as an excellent absorber of radiation entering the hole. As a result, the area of the hole acts like an almost ideal black-body radiator for the inverse emission process.

From experimental data for such cavities, Max Planck first guessed the correct expression for the spectral variation of black-body radiation in 1900 and then attempted to derive this expression theoretically (Duck 2000; Bailyn 1994). Simply stated, he had to stretch both statistics and physics to achieve his goal! In

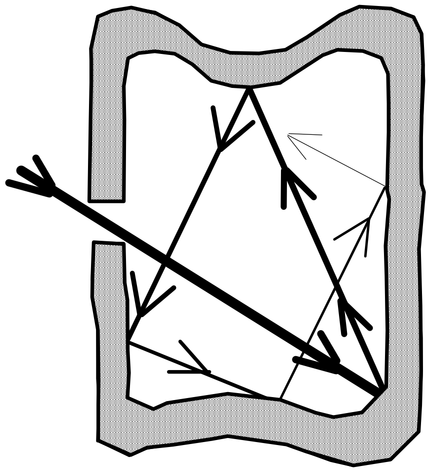

Fig. 2.1: A cavity with a small hole in it acts as an almost perfect absorber of light and therefore emits light from the hole with almost ideal black-body properties.

the latter area, he attempted to understand how his expression arose from the interaction of light with molecules of the black-body, modelled as oscillators. By working backwards to get the correct result, he was forced to hypothesise that these oscillators possessed energy only in multiples of a fundamental quantum.

Albert Einstein first saw the full ramifications of the quantum idea. In 1905, he suggested that light itself must have an intrinsic particle aspect (Duck 2000; Bailyn 1994). By assuming that an electromagnetic wave was not continuous but lumped into particles, he could explain existing experimental results from the photoelectric effect very simply (ejection of electrons from illuminated metals). This also allowed a different approach from Planck's to calculating the properties of a black-body (Bose 1924). As summarised by Richard Feynman (Feynman et al. 1965), the two approaches are equivalent. From one point of view, the radiation emitted by a black-body can be regarded as being in equilibrium with a large number of oscillators, one for each frequency component of light, with each oscillator in different excited states. From the other, the same property can be analysed in terms of Einstein's particles. The number of particles having a particular energy correspond to the state of excitation of the corresponding oscillator.

Satyendra Nath Bose was first to derive the black-body formula from the quantum particle point of view in 1924 (Feynman et al. 1965). Bose showed that the quantum particle hypothesis combined with the statistics of these particles were enough in themselves to give Planck's formula (Bailyn 1994). The particle hypothesis is that light consists of quanta of energy $E = hf$ and momentum $k = hf/c$, giving the following expression for the spatial components of this momentum:

$$k_x^2 + k_y^2 + k_z^2 = h^2 f^2 / c^2 \tag{2.1}$$

Using a forerunner of Heisenberg's Uncertainty Principle of 1926 $(\Delta x \Delta k_x \geq h)$, Bose imagined a 6-dimensional $xyzk_xk_yk_z$ phase space with the phase space volume divided into a mosaic of cells each corresponding to a separate state and each of volume:

$$h^3 = dx\, dy\, dz\, dk_x\, dk_y\, dk_z \tag{2.2}$$

At fixed x, y, and z, the three dimensional k_x k_y k_z sub-space is of interest. A spherical cell in the k_x k_y k_z sub-space between radius k and $k + dk$ contains photons with approximately the same frequency, $f = ck/h$. The total phase volume $d\Omega$ associated with these photons is:

$$d\Omega = 4\pi k^2 dk \iiint dxdydz = 4\pi V(hf/c)^2 d(hf/c) \tag{2.3}$$

where V is the physical volume of the cavity being considered. The number of states enclosed is:

$$\rho_f df = (g/h^3) d\Omega = g(4\pi V/c^3) f^2 df \tag{2.4}$$

where the degeneracy factor, g equals 2. This is because light, considered as a planar transverse wave, needs two parameters to specify how it is orientated (i.e., has two possible polarisation states for each value of f) or has two possible spins, if considered as a particle.

Most readers, as for the author, probably find it easier to think in 3-dimensional rather than 6-dimensional space. Some feeling for the previous mathematics can be gained by thinking in the 3-dimensional xk_xk_y space of Fig. 2.2, as might be important for fixed y, z and k_z. The k_xk_y plane is the analogue of the 3-dimensional $k_xk_yk_z$ space, in this example. The analogue of the spherical shell is the annular ring shown for three such planes in Fig. 2.2. The volume corresponding to k_x and k_y values within this ring within the 3-dimensional space is just the area of the ring multiplied by the extent of the region in the x-direction, for the fixed values of y and z. This corresponds to the tubular volume shown. The calculation is simplified since the geometry in the k_xk_y plane is independent of x-value. For the 6-dimensional case, the volume of the shell in the $k_xk_yk_z$ sub-space is independent of x,y and z co-ordinates. The volume in the 6-dimensional space is found for an incremental physical volume around any particular x, y, z co-ordinate and integrated over the entire physical volume. The latter converts to a multiplication due to the non-dependence upon spatial co-ordinates.

Bose then calculated the most probable distribution of photons amongst his mosaic of cells within phase space. He was able to show that the average number of particles is given by the function that now bears his name (together with

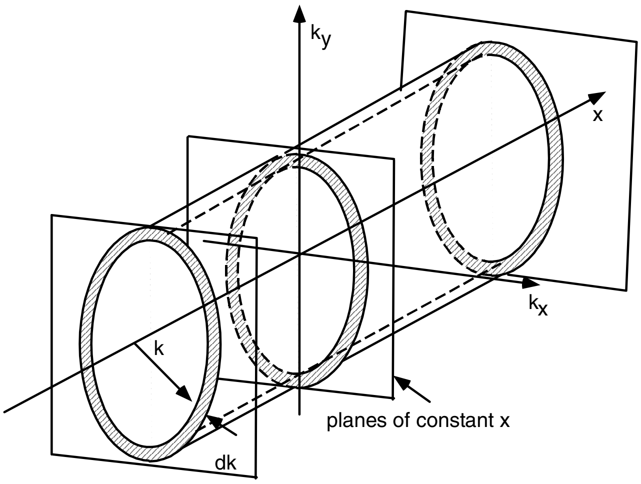

Fig. 2.2: 3-dimensional xk_xk_y space analogue of the calculation conducted in 6-dimensional $xyzk_xk_yk_z$ space in text.

Einstein's who was quick to realise the significance of Bose's work and to develop it). The Bose-Einstein distribution function is given by:

$$f_{BE} = 1/(e^{hf/kT} - 1) \tag{2.5}$$

The total photon energy per unit volume in the cavity in the frequency range df is therefore given by:

$$u_V df = \frac{8\pi(hf^3/c^3)df}{e^{hf/kT} - 1} \tag{2.6}$$

2.3 Black-Body in a Cavity

If a black-body is inserted in the cavity of Fig. 2.1, as in Fig. 2.3, several properties of the black-body can be deduced (Siegal and Howell 1992).

A black-body must be a perfect emitter of light as well as a perfect absorber. This follows since the black-body absorbs all incident radiation from the cavity. After a period of time, the black-body and the cavity will reach a common equilibrium temperature. Since there can be no net energy transfer when at a common temperature, the black-body must be emitting the maximum amount of radiation. This must follow because anything less than a perfect absorber would have to emit less to remain in equilibrium.

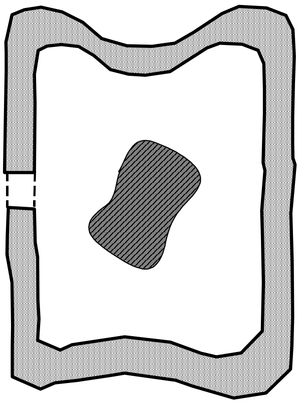

Fig. 2.3: Black-body in a cavity.

The radiation within the cavity must be isotropic. If the black-body, after equilibriating, were rotated or moved to another position in the cavity, it must remain at the same temperature since there are no heat inputs to the system. Consequently, it would emit the same radiation as before with the reasonable assumption that the emission rate is a function of temperature only. It follows that the black-body must be absorbing the same amount of radiation. For arbitrary geometries, this could only occur if the radiation were isotropic. By developing this argument, it can be shown that a black-body must be a perfect emitter in each direction and at each wavelength (Siegal and Howell 1992).

2.4 Angular Dependence of Emitted Radiation

Figure 2.4 shows the segment, dA, of a black-body surface emitting radiation in a direction defined by the angles θ and φ. Also shown is the projected area, dA_p of this element normal to the (θ,φ) direction, with an area of $dA\cos\theta$.

It is straightforward to show that the radiation flux emitted per unit solid angle and per unit projected area by a black-body must be constant. (This can be proved by imagining that the hemispherical shell shown in Fig. 2.4 represents part of the surface of a spherical black-body cavity exchanging radiation with the central element). As opposed to this, the radiation emitted per unit area of emitting surface must be proportional to $\cos\theta$. Surfaces with this characteristic are known as Lambertian, and feature prominently in solar cell light trapping theory (Green 1995). Figures 2.5(a) and (b) show this difference in the two emission characteristics, diagramatically.

The amount of radiation emitted into a finite range of angles (finite solid angle) also can be calculated using the construction of Fig. 2.4. A solid angle is defined as the area intercepted by the section of space that encompasses the solid

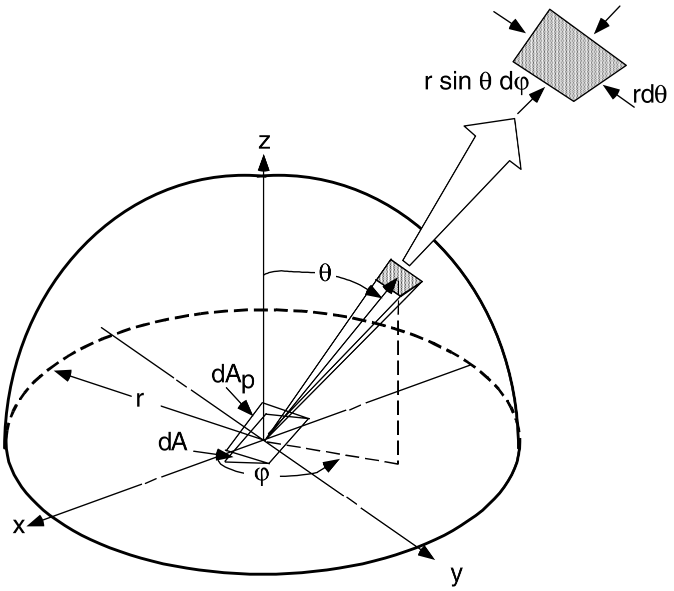

Fig. 2.4: Construction for examining angular dependence of emission from a small area, *dA*, of the surface of a black-body (after Siegal and Howell 1992).

angle, on an arbitrarily sized hemisphere centred at the angle's vertex, divided by radius of this hemisphere squared. Hence, the solid angle corresponding to the segment on the surface of the hemisphere of Fig. 2.4 defined by small variations $d\theta$ and $d\varphi$ equals $sin\theta\ d\theta d\varphi$. The radiation emitted per unit solid angle and per unit projected area is *R*, independent of θ and φ for a black-body (regardless of whether "radiation" refers to energy, entropy or photon fluxes, or to one wavelength or to the radiation integrated over all).

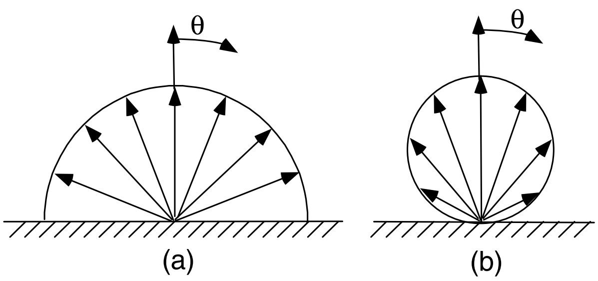

Fig. 2.5: Directional emission characteristics of a black-body (or Lambertian) surface: (a) Emitted flux per unit solid angle and per unit projected area normal to the emission direction; (b) per unit surface area.

Hence, the radiation passing through the element on the hemisphere's surface per unit emitting surface area, dA, equals $R cos\theta \ sin\theta d\theta d\varphi$. Integrating over a range of θ and φ gives:

$$\begin{aligned}\int_{\varphi_1}^{\varphi_2}\int_{\theta_1}^{\theta_2} R\,cos\,\theta sin\,\theta\;d\theta\;d\varphi &= R\int_{\varphi_1}^{\varphi 2}\int_{sin\;\theta_1}^{sin\,\theta_2} sin\,\theta\;d(sin\,\theta\,)d\varphi \\ &= R\left(\frac{sin^2\theta_2 - sin^2\theta_1}{2}\right)(\,\varphi_2 - \varphi_1\,)\end{aligned} \tag{2.7}$$

If $\varphi_1 = \theta_1 = 0$, $\theta_2 = \pi/2$ and $\varphi_2 = 2\pi$, the total radiation emitted per unit surface area passing through the hemisphere is calculated as πR.

Figure 2.4 can also be used to find the total light emitted by an elemental black-body surface area on Earth intercepted by the sun. Consider the case when the sun is directly overhead. In this case, $sin\theta_2 = r_s/d_{es}$ and $\theta_2 = 2\pi$ where r_s is the radius of the sun (695,990 km) and d_{es} is the distance from the elemental area to the sun [calculable as the mean sun-earth distance, a mere 149,597,871 km, minus a small distance approximately equal to up to the Earth's radius, 6,378 km, divided by the air mass (Green 1982) corresponding to the sun's position]. Hence, $\theta_2 = 0.26657(1)°$ and the light intercepted equals $f_\omega \pi$ where f_ω equals $(r_s/d_{es})^2$, or $2.1646(1) \times 10^{-5}$. The number in brackets represents the uncertainty in the last digit of the preceding number due to changes in the position on Earth relative to the sun. Greater variability is introduced by the eccentricity of the Earth's orbit (1.67%), which introduces a similar percentage variability into θ_2 and about double this percentage into f.

If the element and sun were at the same temperature, this flow would have to be balanced by an equal and opposite flow from the sun. Hence, the same expression gives the total amount of the radiation emitted over the entire sun's surface that strikes an elemental area on Earth, where R is the corresponding radiation emitted at the sun's temperature. This result could be derived more directly by considering a huge sphere centred at the sun, but intersecting the Earth at the elemental area's position.

The fact that direct sunlight arrives over such a small range of angles can be used to advantage in photovoltaics. This compactness allows the direct component of sunlight to be concentrated, which increases cell efficiency by increasing cell voltage outputs. Alternatively, the small angular spread can allow the net rate of recombination within the cell to be suppressed by restricting the cell's angular acceptance of light and hence its total light emission (Green 1995; Araujo 1990). Both approaches give rise to the same limiting efficiency.

With free space between sun and the cell, the best that can be done for concentrated sunlight is to design an optical system that steers all the black-body radiation emitted by the incremental earth-based element onto the sun. In this case, all light emitted by the element into the hemisphere would reach the sun, i.e., an amount equal to πR (i.e., $f = 1$). By the same argument as before, a corresponding amount from the sun's surface would reach the incremental area with such optics. Hence, the maximum possible concentration level possible equals $(d_{es}/r_s)^2$ or 46,198(2), if the intervening medium is vacuum. (Other values for this quantity in the literature stem from either using rounded values for the different parameters involved or a less accurate solid-angle argument to deduce the amount of sunlight intercepted). At this concentration level, the intensity at the sun's surface is reproduced at the cell surface, a frightening thought but something that is close to achievable.

If the receiving element is immersed in a medium of refractive index, n, maximally achievable concentration is increased to $(d_{es}/r_s)^2 n^2$ (Smestad et al. 1990). Concentrators approaching this limiting performance actually have been built. The present record appears to be a concentration ratio of 84,000 corresponding to an output power of 7.2 kW/cm^2 (Cooke at al. 1990). Figure 2.6 shows the experimental set-up. Note that the refractive index of air under standard conditions is a little higher than unity (1.000278) but will be taken as unity throughout this text.

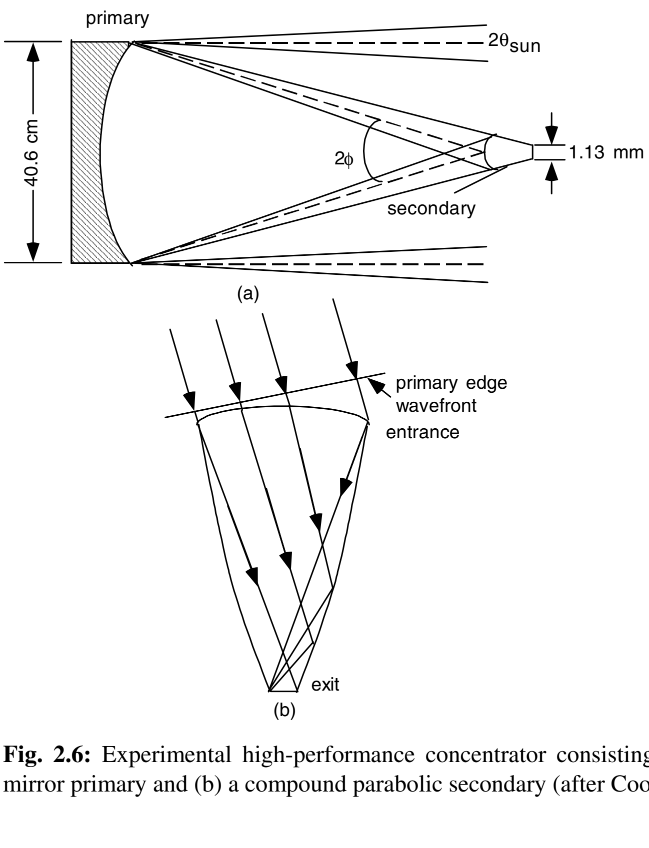

Fig. 2.6: Experimental high-performance concentrator consisting of (a) a parabolic mirror primary and (b) a compound parabolic secondary (after Cooke et al. 1990).

2.5 Direct and Diffuse Efficiencies

As opposed to the directional radiation in space, the Earth's atmosphere scatters incoming sunlight so that some is incident on terrestrial cells from all accessible directions in the sky. A terrestrial system that converts only the direct component of sunlight can therefore waste a lot of energy, even if it converts the direct very efficiently. Usually, the quoted efficiency for concentrating photovoltaic and solar thermal electric systems is based on only the direct component of incident sunlight, rather than the combined direct plus diffuse. Tracking of the sun is also essential once a reasonable concentration level is reached.

A standard non-concentrating photovoltaic system converts both direct and diffuse sunlight. Since sun tracking is not essential, such tracking is usually regarded as a bonus when implemented, giving up to 40% more energy compared to a stationary system of the same peak rating.

The overall efficiency of conversion, η, could therefore be regarded as a composite of a direct and diffuse component:

$$\eta = f_{dir}\,\eta_{dir} + (1 - f_{dir})\,\eta_{diff} \tag{2.8}$$

where f_{dir} is the fraction of available light that is direct, η_{dir} is the conversion efficiency for this light while η_{diff} is that for diffuse light. For a normal concentrator system, $\eta_{diff} = 0$, while for a normal non-concentrating system, $\eta \approx \eta_{diff}$. Fundamentally, $\eta_{diff} < \eta_{dir}$ due to the performance gains possible by taking advantage of light collimation. In both common cases, the overall η is appreciably less than that possible if all light were direct.

It is possible to improve on these norms. For example, the system shown in Fig. 2.7 could be designed to convert the direct component of sunlight with high efficiency while also efficiently converting the diffuse (Goetzberger 1990). The approach also accommodates losses due to non-idealities in the concentrating optics. (The diffuse converters would have to be very inexpensive to make such a scheme attractive in practice, however).

In the following chapters, both η_{dir} and η_{diff} will be calculated. The rationale for calculating the former is that it is the relevant efficiency, in principle, for space systems and for the conversion of the direct component of terrestrial sunlight. It also represents the highest value obtainable for any given conversion approach, allowing unambiguous comparison between different approaches and comparison also with thermodynamic limits.

Optimal η_{dir} is obtained when all light emitted by the cell strikes the sun. This can be ensured, in principle, by appropriate concentrating optics which produces f values of unity for both radiation emitted by the sun and by the cell. Alternatively, the cell can be designed, in principle, so that it absorbs only light from the small range of incident angles involved (Green 1995; Araujo 1990) and hence emits light only into this range (angularly selective black-body properties). In this case, f

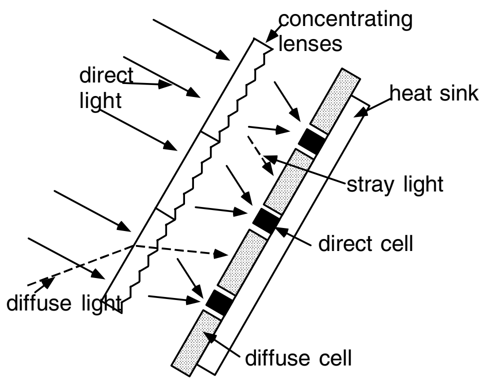

Fig. 2.7: A combined direct-diffuse light converter capable of higher conversion efficiencies based on the total incident light intensity.

has the much lower value of f_ω (2.1646 x 10^{-5}) for both sun and cell. Similar mathematics apply to both cases, although either approach is difficult to implement in practice. Concentration levels of 1,000 normally are regarded as very high for experimental photovoltaic systems, while there is not yet any demonstration of performance gain by the second approach. Intermediate options are possible whereby the sun is concentrated to less than the maximally possible concentration level with the cell then designed to accept only the resulting angular spread (Green 1995).

η_{diff} is calculated by applying the lower value of f for light received from the sun (f_ω), while applying the higher value of unity to that emitted by the cell.

2.6 Black-Body Emission Properties

From Sect. 2.2, expressions for the emitted black-body flux can be deduced regardless of whether particle, energy or entropy fluxes are being discussed. The flux of an extensive quantity carried by particles (an example of an extensive quantity is the total photon energy in the cavity), in the general case, can be related to its volume density by multiplying by a quarter of the mean particle velocity. In the present case, this means that the cavity quantities of Sect. 2.2 need to be multiplied by $c/4$ to convert to fluxes (see box below).

The particle, energy and entropy fluxes emitted by a black-body per unit surface area into a hemisphere over the energy range, E_1 to E_2 are given by:

$$\dot{N}(E_1, E_2) = \frac{2\pi}{h^3c^2}\int_{E_1}^{E_2}\frac{E^2 dE}{e^{E/kT}-1} \tag{2.9}$$

$$\dot{E}(E_1, E_2) = \frac{2\pi}{h^3c^2}\int_{E_1}^{E_2}\frac{E^3 dE}{e^{E/kT}-1} \tag{2.10}$$

$$\dot{S}(E_1, E_2) = \frac{\dot{E}}{T} - \frac{2\pi k}{h^3c^2}\int_{E_1}^{E_2} E^2 \, ln(1-e^{-E/kT})dE \tag{2.11}$$

$$= \frac{4\dot{E}}{3T} - \frac{2\pi k}{3h^3c^2}\left| E^3 \, ln(1-e^{-E/kT}) \right|_{E_1}^{E_2} \tag{2.12}$$

Relationship Between Extensive Variables and Fluxes

To convert from quantities in a cavity such as energy per unit volume to fluxes (energy per unit area per second), it is apparent from dimensional analysis that the former have to be multiplied by a velocity. But what velocity?

If all light were moving in a direction perpendicular to the area dA of interest, the appropriate velocity would be $c/2$. This is because half the light in a volume cdA would go in each direction in unit time. However, within the black-body cavity, the radiation is isotropic and moves with velocity c in all directions. A fraction $d\Omega/4\pi$ can be considered going in any direction specified to within an element of solid angle $d\Omega$.

Consider a hole of area dA in the cavity as in Fig. 2.4 but, this time, assume the hemisphere penetrates into the cavity and that it represents a shell of finite thickness, dr. The area dA subtends a solid angle $dA\cos\theta/r^2$ from the point of view of the small volume $r^2 \sin\theta d\theta d\varphi dr$. A fraction of the energy in this volume determined by this solid angle reaches the element dA in a time interval between t and $t + dt$, where $dt = dr/c$.

The amount of energy flowing through this hole in the time dt from this small volume equals $u_\nu c \cos\theta \sin\theta d\theta d\varphi dt dA$. The total amount arriving per unit area per unit time is given by:

$$\int_0^{\pi/2\pi}\int_0^{2\pi} u_\nu \, c\cos\theta \, \sin\theta \, \frac{d\theta d\varphi}{4\pi} = \frac{u_\nu c}{4}$$

This is the total emitted flux per unit time per unit area. For particles other than photons, a similar calculation shows that the volume density of an extensive variable is converted to a flux by multiplying by $\bar{v}/4$, where $\bar{v}$ is the average particle velocity.

Most readers may not be as familiar with entropy fluxes as with particle and energy fluxes. Chapter 3 will give a fuller discussion of the entropy flux and its importance in determining limiting efficiency. An interesting feature of the entropy expression is noted in passing. If we regarded the energy flux radiated by the black-body as being supplied from a heat reservoir at very close to the same temperature (similar to the way that the hydrogen fusion reaction at the sun's core supplies the energy emitted by the sun), the entropy flow from this reservoir to the area involved is given by the energy flux divided by T, different from the entropy flux emitted.

The above implies there is entropy production at the surface emitting the light (Planck 1959; De Vos and Pauwels 1983) to provide the entropy balance [the production rate is equal to the second term on the right of Eq. (2.11)]. Similarly, entropy generation is associated with light absorption. When calculated separately, the latter can give a negative contribution, although the overall balance is always be positive, becoming zero only when the emitted and absorbed light is identical.

As shown in Appendix C, the integrals involved in the above equations can be evaluated in terms of the standard Bose - Einstein integrals:

$$\beta_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{\varepsilon^j d\varepsilon}{e^{\varepsilon-\eta}-1} \tag{2.13}$$

where $\Gamma(j+1)$ equals the Gamma function given by $j!$ for positive integers as arguments. For the case where $E_1 = 0$ and $E_2 = \infty$, the integral involved can be expressed in terms of $\beta_j(0)$ which equals the Reiman zeta function given by:

$$\xi(j+1) = \sum_{n=1}^{\infty} (1/n)^{j+1} \tag{2.14}$$

This sum can be expressed in terms of π for even integral arguments:

$$\xi(2) = \pi^2/6, \ \xi(4) = \pi^4/90, \quad \xi(6) = \pi^6/945$$

Using these results gives:

$$\dot{N}(0,\infty) = \frac{2\pi(kT)^3\Gamma(3)\xi(3)}{h^3c^2} = \dot{E}(0,\infty)/2.70117kT \tag{2.15}$$

$$\dot{E}(0,\infty) = \frac{2\pi(kT)^4}{h^3c^2}\Gamma(4)\xi(4) = \sigma T^4 \tag{2.16}$$

$$\dot{S}(0,\infty) = \frac{4\dot{E}(0,\infty)}{3T} = \frac{4\sigma T^3}{3} \tag{2.17}$$

where σ is the Stefan - Boltzmann constant given by:

$$\sigma = 2\pi^5k^4/(15h^3c^3) = 5.670400(40)x10^{-12}W/cm^2/K^4 \tag{2.18}$$

Note that the average photon carries the energy 2.70117kT [equal to $3\xi(4)/\xi(3)$ times kT].

If properties are less than ideal, these quantities can be multiplied by appropriate averaged emissivities (equal to the corresponding absorptivities). Wavelength dependent emissivities could be included within the integrals of Eqs. (2.9) to (2.11). Angularly dependent ones would need to be incorporated prior to this stage of development.

Exercise

2.1 Assuming the sun can be modelled as an ideal black-body at 6000 K, calculate the Earth's surface temperature, assuming its temperature is uniform over its entire surface and that the sun is the only source providing energy to it. Assume the following models for the Earth's radiative properties:
(a) Ideal black-body properties for the Earth;
(b) Grey-body properties with an absorptance of 0.7;
(c) Spectrally sensitive absorptance such that the absorptance averaged over the sun's energy spectrum is 0.7. The reduced emissivity at longer wavelengths, due to greenhouse gases, is modelled as a decreased absorptance of 0.6 averaged over wavelengths corresponding to the Earth's radiative emission.
(d) How much change in the latter absorptance is required to heat the Earth's temperature by 2°C?

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