Erratum

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Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems

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Dear Reader,

As a result of technical problems with the data processing, some figures in the printed version of this book have not been correctly reproduced. Here are the pdf files with the correct Figs. 6.17 (p. 116), 6.18 (p. 118), 8.12 (p. 167) and B.1 (p. 202).

Please accept our apologies for the inconvenience this may have caused you.

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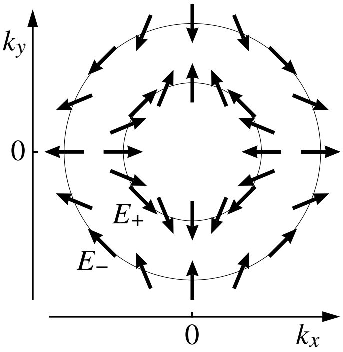

Fig. 6.17. Lowest-order spin orientation $\langle \boldsymbol{\sigma} \rangle$ of the eigenstates $|\psi_\pm^{\mathrm{BIA}}(\boldsymbol{k}_\|)\rangle$ in the presence of BIA. The *inner* and *outer* circles show $\langle \boldsymbol{\sigma} \rangle$ along contours of constant energy for the upper and lower branches E_+ and E_-, respectively, of the spin-split dispersion

The spin orientation (6.63) of the eigenfunctions (6.62) as a function of the direction of the in-plane wave vector is indicated by arrows in Fig. 6.17. For the Rashba spin splitting, we see in Fig. 6.2 that if we move clockwise on a contour of constant energy $E(\boldsymbol{k}_\|)$, the spin vector rotates in the same direction, consistent with the axial symmetry of the Rashba term. On the other hand, (6.63) and Fig. 6.17 show that in the presence of BIA, the spin vector rotates counterclockwise for a clockwise motion in $\boldsymbol{k}_\|$ space. The different symmetries of the Hamiltonians for BIA and SIA, which become visible in the quantity $\langle \boldsymbol{\sigma} \rangle$, is also the reason for the anisotropy of the $B = 0$ spin splitting even in the leading order of $\boldsymbol{k}_\|$ that was obtained in Sect. 6.4.1 for the case where both BIA and SIA are present.

In the above discussion we have assumed that the wave functions are two-component spinors. When the eigenstates $|\psi\rangle$ are multicomponent envelope functions (4.1), we must evaluate the expectation value of $\boldsymbol{S} = \boldsymbol{\sigma} \otimes \mathbb{1}_{\mathrm{orb}}$ where the identity operator $\mathbb{1}_{\mathrm{orb}}$ refers to the orbital part of $|\psi\rangle$. For the transformed Hamiltonian $\mathcal{H}'$ in (5.1), the spin operator $\boldsymbol{S}'$ has the block form

$$S'_x = \begin{pmatrix} 0 & \mathbb{1}_{\mathrm{orb}} \\ \mathbb{1}_{\mathrm{orb}} & 0 \end{pmatrix}, \quad S'_y = \begin{pmatrix} 0 & -\mathrm{i}\mathbb{1}_{\mathrm{orb}} \\ \mathrm{i}\mathbb{1}_{\mathrm{orb}} & 0 \end{pmatrix}, \quad S'_z = \begin{pmatrix} \mathbb{1}_{\mathrm{orb}} & 0 \\ 0 & -\mathbb{1}_{\mathrm{orb}} \end{pmatrix}. \tag{6.64}$$

The inverse unitary transformation that relates $\mathcal{H}'$ to $\mathcal{H}$ gives the spin operator in the unprimed basis. For the extended Kane model, we obtain

$$S_i = \begin{pmatrix} \frac{2}{3}J_i & -2U_i & 0 & 0 & 0 \\ -2U_i^\dagger & -\frac{1}{3}\sigma_i & 0 & 0 & 0 \\ 0 & 0 & \sigma_i & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3}J_i & -2U_i \\ 0 & 0 & 0 & -2U_i^\dagger & -\frac{1}{3}\sigma_i \end{pmatrix}, \qquad i = x, y, z \tag{6.65}$$

where the matrices J_i, U_i, and σ_i are defined in Table C.2. Once again, the expectation value $\langle\psi|\boldsymbol{S}|\psi\rangle$ is a three-component vector that can be identified with the spin orientation of the multicomponent wave function $|\psi\rangle$. We remark that while the vector $\langle \boldsymbol{\sigma} \rangle$ of a spin-1/2 system is always strictly

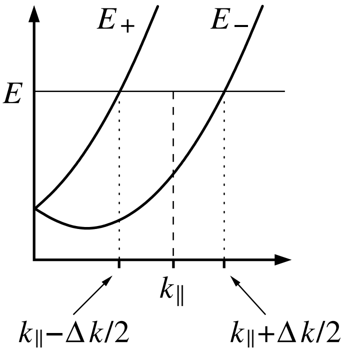

Fig. 6.18. For a given energy E and a fixed direction of the in-plane wave vector $\boldsymbol{k}_\|$, we determine $\boldsymbol{k}_\| \mp \Delta\boldsymbol{k}/2$ such that $E = E_+(\boldsymbol{k}_\| - \Delta\boldsymbol{k}/2) = E_-(\boldsymbol{k}_\| + \Delta\boldsymbol{k}/2)$. Here E_+ and E_- denotes the upper and lower branches, respectively, of the spin-split dispersion

provided we can ignore anisotropic contributions to $B = 0$ spin splitting. However, see also Chap. 9.

The definition (6.66) presupposes that the spin expectation values $\langle \boldsymbol{S} \rangle_+$ and $\langle \boldsymbol{S} \rangle_-$ are strictly antiparallel to each other. In (6.61) we saw that for the Rashba Hamiltonian, this condition is fulfilled exactly. This is closely related to the fact that for the Rashba Hamiltonian, the spin subband eigenstates $|\psi_+^{\mathrm{SIA}}(\boldsymbol{k}_\|)\rangle$ and $|\psi_-^{\mathrm{SIA}}(\boldsymbol{k}'_\|)\rangle$ are orthogonal, independent of the magnitude of $\boldsymbol{k}_\|$ and $\boldsymbol{k}'_\|$ as long as the wave vectors $\boldsymbol{k}_\|$ and $\boldsymbol{k}'_\|$ are parallel to each other.[18] In general, $|\psi_+(\boldsymbol{k}_\| - \Delta\boldsymbol{k}/2)\rangle$ and $|\psi_-(\boldsymbol{k}_\| + \Delta\boldsymbol{k}/2)\rangle$ are only approximately orthogonal, so that $\langle \boldsymbol{S} \rangle_+$ and $\langle \boldsymbol{S} \rangle_-$ are only approximately antiparallel. However, we find that the angle between the vectors $\langle \boldsymbol{S} \rangle_+$ and $\langle \boldsymbol{S} \rangle_-$ is always very close to 180°, with an error $\lesssim 1°$, so we neglect this point in the remaining discussion.

Even though we can evaluate the spin expectation value $\langle \boldsymbol{S} \rangle$ for each spin subband separately, we do not attempt to define an effective magnetic field $\boldsymbol{\mathcal{B}}$ for each spin subband. This is due to the fact that $\boldsymbol{\mathcal{B}}$ is commonly used to discuss phenomena such as spin precession [108] and spin relaxation [107], which cannot be analyzed for each spin subband individually.

The allowed directions of the effective magnetic field $\boldsymbol{\mathcal{B}}$ can be deduced from the symmetry of the QW (Table 3.4). The spin-split states for a fixed wave vector $\boldsymbol{k}_\|$ are orthogonal to each other, i.e. the spin vectors of these states are antiparallel. The spin orientation of eigenstates for different wave vectors in the star of $\boldsymbol{k}_\|$ are connected by the symmetry operations of the system [77]. Accordingly, only those spin orientations of the spin-split eigenstates are permissible for which every symmetry operation maps orthogonal states onto orthogonal states. In a QW grown in the crystallographic direction [001], the effective field $\boldsymbol{\mathcal{B}}$ is parallel to the plane of the quasi-2D system. Indeed, the field $\boldsymbol{\mathcal{B}}$ due to SIA is always in the plane of the well. For growth directions

[18] From a group-theoretical point of view, this can be traced back to the fact that $|\psi_+(\boldsymbol{k}_\|)\rangle$ and $|\psi_-(\boldsymbol{k}_\|)\rangle$ transform according to different irreducible representations of the group of the wave vector $\boldsymbol{k}_\|$.

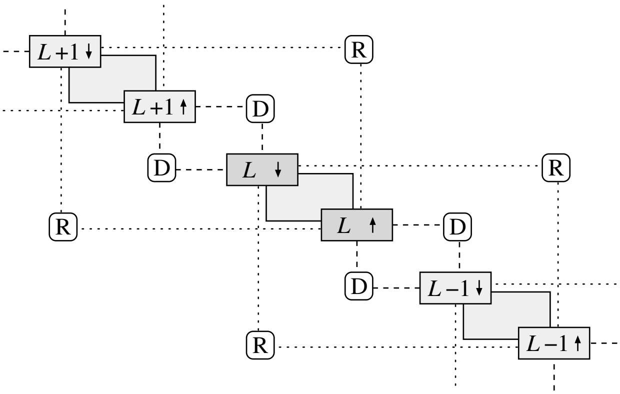

Fig. 8.12. Off-diagonal couplings between spin-split Landau harmonic oscillators $|L\rangle \otimes |\sigma\rangle$ ($\sigma = \uparrow, \downarrow$) in an inversion-asymmetric electron system. Matrix elements due to the Dresselhaus and Rashba terms are marked by the letters D and R, respectively

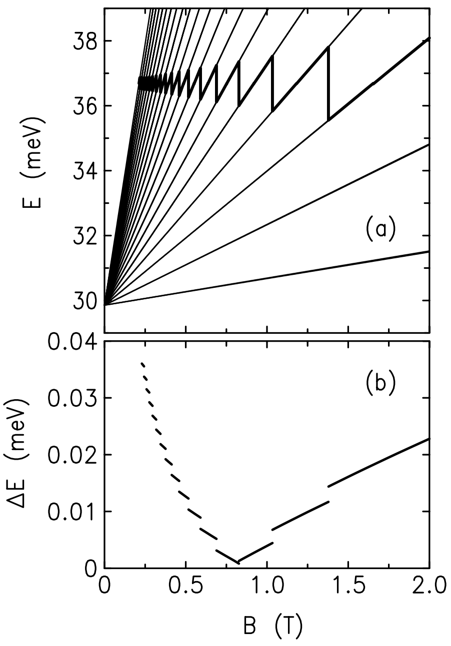

Fig. 8.13. (**a**) Landau levels $E_{\mathcal{N}\pm}$ and (**b**) positive energy difference $\Delta E = |E_{\mathcal{N}+} - E_{\mathcal{N}-}|$ between the Landau levels at the Fermi energy in a 150 Å wide GaAs–$Al_{0.3}Ga_{0.7}As$ QW with $N_s = 2 \times 10^{11}\ \mathrm{cm}^{-2}$ in the presence of an external electric field $\mathcal{E}_z = 50$ kV/cm, calculated by means of an 8×8 Hamiltonian. In (**a**), the highest filled Landau level is marked by a *bold line*

For this calculation, we used an 8×8 Hamiltonian that takes BIA and SIA into account fully. We see that the spin splitting vanishes at about $B = 0.8$ T, consistent with the results in Sect. 6.4.1, where it was shown that in GaAs QWs the spin splitting is usually dominated by the Dresselhaus term.

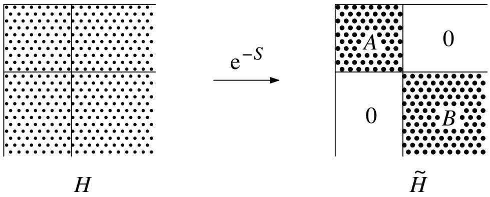

Fig. B.1. Removal of off-diagonal elements of H

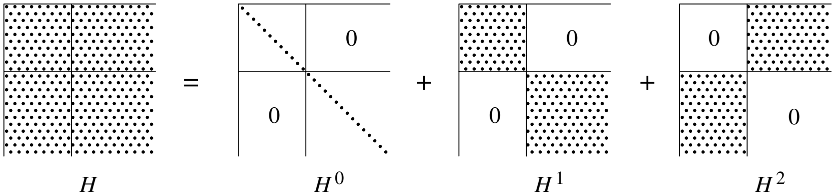

Fig. B.2. Representation of H as $H^0 + H^1 + H^2$

S such that the transformation (B.2) converts H^2 into a block-diagonal form similar to H^1 while keeping the desired block-diagonal form of $H^0 + H^1$. In order to determine the operator S, we expand e^S in a series

$$\mathrm{e}^S = 1 + S + \frac{1}{2!}S^2 + \frac{1}{3!}S^3 \ldots \tag{B.4}$$

and construct S by successive approximations. Substituting (B.4) into (B.2), and noting that the operator S must be anti-Hermitian, i.e. $S^\dagger = -S$, we obtain

$$\tilde{H} = \sum_{j=0}^{\infty} \frac{1}{j!}\left[H, S\right]^{(j)} = \sum_{j=0}^{\infty} \frac{1}{j!}\left[H^0 + H^1, S\right]^{(j)} + \sum_{j=0}^{\infty} \frac{1}{j!}\left[H^2, S\right]^{(j)} , \tag{B.5}$$

where the commutators $\left[A, B\right]^{(j)}$ are defined by

$$\left[A, B\right]^{(j)} = [\ldots[[A \underbrace{, B], B], \ldots B]}_{j \text{ times}} . \tag{B.6}$$

Since S must be non-block-diagonal like H^2, the block-diagonal part $\tilde{H}_\mathrm{d}$ of $\tilde{H}$ contains the terms $\left[H^0 + H^1, S\right]^{(j)}$ with even j and $\left[H^2, S\right]^{(j)}$ with odd j:

$$\tilde{H}_\mathrm{d} = \sum_{j=0}^{\infty} \frac{1}{(2j)!}\left[H^0 + H^1, S\right]^{(2j)} + \sum_{j=0}^{\infty} \frac{1}{(2j+1)!}\left[H^2, S\right]^{(2j+1)} . \tag{B.7}$$

Conversely, the non-block-diagonal part $\tilde{H}_\mathrm{n}$ of $\tilde{H}$ must contain the terms $\left[H^0 + H^1, S\right]^{(j)}$ with odd j and $\left[H^2, S\right]^{(j)}$ with even j:

$$\tilde{H}_\mathrm{n} = \sum_{j=0}^{\infty} \frac{1}{(2j+1)!}\left[H^0 + H^1, S\right]^{(2j+1)} + \sum_{j=0}^{\infty} \frac{1}{(2j)!}\left[H^2, S\right]^{(2j)} . \tag{B.8}$$

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