

# Contents

<b>Preface</b> .....	v
<b>Preliminaries</b> .....	xi
<b>1. Simplicial Complexes</b> .....	1
1.1 Topological Spaces .....	1
1.2 Homotopy Equivalence and Homotopy .....	4
1.3 Geometric Simplicial Complexes .....	7
1.4 Triangulations .....	10
1.5 Abstract Simplicial Complexes .....	13
1.6 Dimension of Geometric Realizations .....	16
1.7 Simplicial Complexes and Posets .....	17
<b>2. The Borsuk–Ulam Theorem</b> .....	21
2.1 The Borsuk–Ulam Theorem in Various Guises .....	22
2.2 A Geometric Proof .....	30
2.3 A Discrete Version: Tucker’s Lemma .....	35
2.4 Another Proof of Tucker’s Lemma .....	42
<b>3. Direct Applications of Borsuk–Ulam</b> .....	47
3.1 The Ham Sandwich Theorem .....	47
3.2 On Multicolored Partitions and Necklaces .....	53
3.3 Kneser’s Conjecture .....	57
3.4 More General Kneser Graphs: Dol’nikov’s Theorem .....	61
3.5 Gale’s Lemma and Schrijver’s Theorem .....	64
<b>4. A Topological Interlude</b> .....	69
4.1 Quotient Spaces .....	69
4.2 Joins (and Products) .....	73
4.3 $k$ -Connectedness .....	78
4.4 Recipes for Showing $k$ -Connectedness .....	80
4.5 Cell Complexes .....	82

<b>5. <math>\mathbb{Z}_2</math>-Maps and Nonembeddability</b>	87
5.1 Nonembeddability Theorems: An Introduction	88
5.2 $\mathbb{Z}_2$ -Spaces and $\mathbb{Z}_2$ -Maps	92
5.3 The $\mathbb{Z}_2$ -Index	95
5.4 Deleted Products Good ...	108
5.5 ... Deleted Joins Better	112
5.6 Bier Spheres and the Van Kampen–Flores Theorem	116
5.7 Sarkaria’s Inequality	121
5.8 Nonembeddability and Kneser Colorings	124
5.9 A General Lower Bound for the Chromatic Number	128
<b>6. Multiple Points of Coincidence</b>	145
6.1 $G$ -Spaces	145
6.2 $E_n G$ Spaces and the $G$ -Index	149
6.3 Deleted Joins and Deleted Products	157
6.4 The Topological Tverberg Theorem	161
6.5 Many Tverberg Partitions	165
6.6 Necklace for Many Thieves	167
6.7 $\mathbb{Z}_p$ -Index, Kneser Colorings, and $p$ -Fold Points	170
6.8 The Colored Tverberg Theorem	174
<b>A Quick Summary</b>	179
<b>Hints to Selected Exercises</b>	185
<b>References</b>	187
<b>Index</b>	203

Using the Borsuk-Ulam Theorem  
Lectures on Topological Methods in Combinatorics and  
Geometry

Matousek, J.

2003, XII, 214 p., Softcover

ISBN: 978-3-540-00362-5