

# Table of Contents

<b>1</b>	<b>Introduction</b> . . . . .	1
1.1	Background . . . . .	1
1.2	Notation and Mathematical Preliminaries . . . . .	4
1.3	Physical Derivation of the Heat Equation . . . . .	7
1.4	Problems . . . . .	12
<b>2</b>	<b>A Two-Point Boundary Value Problem</b> . . . . .	15
2.1	The Maximum Principle . . . . .	15
2.2	Green's Function . . . . .	18
2.3	Variational Formulation . . . . .	20
2.4	Problems . . . . .	23
<b>3</b>	<b>Elliptic Equations</b> . . . . .	25
3.1	Preliminaries . . . . .	25
3.2	A Maximum Principle . . . . .	26
3.3	Dirichlet's Problem for a Disc. Poisson's Integral . . . . .	28
3.4	Fundamental Solutions. Green's Function . . . . .	30
3.5	Variational Formulation of the Dirichlet Problem . . . . .	32
3.6	A Neumann Problem . . . . .	35
3.7	Regularity . . . . .	37
3.8	Problems . . . . .	38
<b>4</b>	<b>Finite Difference Methods for Elliptic Equations</b> . . . . .	43
4.1	A Two-Point Boundary Value Problem . . . . .	43
4.2	Poisson's Equation . . . . .	46
4.3	Problems . . . . .	49
<b>5</b>	<b>Finite Element Methods for Elliptic Equations</b> . . . . .	51
5.1	A Two-Point Boundary Value Problem . . . . .	51
5.2	A Model Problem in the Plane . . . . .	57
5.3	Some Facts from Approximation Theory . . . . .	60
5.4	Error Estimates . . . . .	63
5.5	An A Posteriori Error Estimate . . . . .	66
5.6	Numerical Integration . . . . .	67
5.7	A Mixed Finite Element Method . . . . .	71
5.8	Problems . . . . .	73

<b>6</b>	<b>The Elliptic Eigenvalue Problem</b> .....	77
6.1	Eigenfunction Expansions .....	77
6.2	Numerical Solution of the Eigenvalue Problem .....	87
6.3	Problems .....	93
<b>7</b>	<b>Initial-Value Problems for ODEs</b> .....	95
7.1	The Initial Value Problem for a Linear System .....	95
7.2	Numerical Solution of ODEs .....	101
7.3	Problems .....	106
<b>8</b>	<b>Parabolic Equations</b> .....	109
8.1	The Pure Initial Value Problem .....	109
8.2	Solution by Eigenfunction Expansion .....	114
8.3	Variational Formulation. Energy Estimates .....	120
8.4	A Maximum Principle .....	122
8.5	Problems .....	124
<b>9</b>	<b>Finite Difference Methods for Parabolic Problems</b> .....	129
9.1	The Pure Initial Value Problem .....	129
9.2	The Mixed Initial-Boundary Value Problem .....	138
9.3	Problems .....	146
<b>10</b>	<b>The Finite Element Method for a Parabolic Problem</b> .....	149
10.1	The Semidiscrete Galerkin Finite Element Method .....	149
10.2	Some Completely Discrete Schemes .....	156
10.3	Problems .....	159
<b>11</b>	<b>Hyperbolic Equations</b> .....	163
11.1	Characteristic Directions and Surfaces .....	163
11.2	The Wave Equation .....	166
11.3	First Order Scalar Equations .....	169
11.4	Symmetric Hyperbolic Systems .....	173
11.5	Problems .....	181
<b>12</b>	<b>Finite Difference Methods for Hyperbolic Equations</b> .....	185
12.1	First Order Scalar Equations .....	185
12.2	Symmetric Hyperbolic Systems .....	192
12.3	The Wendroff Box Scheme .....	196
12.4	Problems .....	198
<b>13</b>	<b>The Finite Element Method for Hyperbolic Equations</b> .....	201
13.1	The Wave Equation .....	201
13.2	First Order Hyperbolic Equations .....	205
13.3	Problems .....	216

<b>14</b>	<b>Some Other Classes of Numerical Methods</b>	217
14.1	Collocation Methods	217
14.2	Spectral Methods	218
14.3	Finite Volume Methods	219
14.4	Boundary Element Methods	221
14.5	Problems	223
<b>A</b>	<b>Some Tools from Mathematical Analysis</b>	225
A.1	Abstract Linear Spaces	225
A.2	Function Spaces	231
A.3	The Fourier Transform	238
A.4	Problems	240
<b>B</b>	<b>Orientation on Numerical Linear Algebra</b>	245
B.1	Direct Methods	245
B.2	Iterative Methods. Relaxation, Overrelaxation, and Acceleration	246
B.3	Alternating Direction Methods	248
B.4	Preconditioned Conjugate Gradient Methods	249
B.5	Multigrid and Domain Decomposition Methods	250
	<b>Bibliography</b>	253
	<b>Index</b>	257

Partial Differential Equations with Numerical Methods

Larsson, S.; Thomee, V.

2003, X, 262 p., Hardcover

ISBN: 978-3-540-01772-1