

Contents

1	General Properties of Lattices	1
1.1	Lattices in Real Vector Spaces	1
1.2	Lattices in Euclidean Spaces	4
1.3	Duality	7
1.4	Automorphism Groups	12
1.5	Bilinear and Quadratic Forms	17
1.6	Quadratic Forms	18
1.7	The Dictionary Relating Lattices and Quadratic Forms	20
1.8	Packings	24
1.9	More on Integral Lattices	26
1.10	Tensor Product and Exterior Powers	30
1.11	Notes on Chapter 1	33
2	Geometric Inequalities	37
2.1	The Hadamard Inequality	37
2.2	The Hermite Inequality	39
2.3	The Mordell Inequality	41
2.4	Mahler's Compactness Theorem	43
2.5	Lattice Constants	45
2.6	Extreme Lattices for an Open Star Body	47
2.7	The Lattice Constant for a Convex Star Body	51
2.8	Generalizations of the Hermite Invariant	55
2.9	The HKZ Reduction	59
2.10	Exercises for Chapter 2	62
2.11	Notes on Chapter 2	63
3	Perfection and Eutaxy	67
3.1	Symmetric Endomorphisms	67
3.2	Linear Forms on Spaces of Endomorphisms	73
3.3	Linear Inequalities	77
3.4	A Characterization of Extreme Lattices	78
3.5	Perfect Configurations	81
3.6	Eutactic Configurations and Extreme Lattices	84
3.7	The Lamination Process	92

3.8	Dual-Extreme Lattices	94
3.9	Exercises for Chapter 3	100
3.10	Notes on Chapter 3.	105
4	Root Lattices	109
4.1	The \mathbb{Z}^n Lattice	110
4.2	The \mathbb{A}_n Lattice	110
4.3	The \mathbb{D}_n Lattice	112
4.4	The \mathbb{D}_n^+ Packing and the \mathbb{E}_8 Lattice	114
4.5	The Lattices \mathbb{E}_7 and \mathbb{E}_6	117
4.6	Graphs and Inclusions Between Root Lattices	120
4.7	Perfection and Eutaxy	124
4.8	Some Other Constructions for Root Lattices	125
4.9	Residual Quadratic Forms	129
4.10	Root Systems	131
4.11	Exercises for Chapter 4	138
4.12	Notes on Chapter 4	145
5	Lattices Related to Root Lattices	147
5.1	The Coxeter–Barnes Lattices \mathbb{A}_n^r	147
5.2	The Coxeter Lattices \mathbb{A}_n^r	153
5.3	Barnes’s Lattices P_n	157
5.4	Craig’s Difference Lattices	163
5.5	Lattices Related to the \mathbb{D}_n Lattice	171
5.6	Unimodular Lattices	174
5.7	Around the Leech Lattice	177
5.8	Exercises for Chapter 5	182
5.9	Notes on Chapter 5	187
6	Low-Dimensional Perfect Lattices	189
6.1	A Combinatorial Characterization of the \mathbb{A}_n Lattices	190
6.2	Perfect Lattices up to Dimension 4	194
6.3	Dual-Extreme Lattices up to Dimension 4	196
6.4	Perfect Lattices in Dimension 5	200
6.5	Perfect Lattices in Dimensions 6 and 7	208
6.6	Some Indications About 8-Dimensional Perfect Lattices	212
6.7	Exercises for Chapter 6	219
6.8	Notes on Chapter 6	223
7	The Voronoi Algorithm	227
7.1	Voronoi Domains	227
7.2	Contiguity	234
7.3	Finiteness Results	237
7.4	The Voronoi Graphs	238
7.5	Lattices Contiguous to \mathbb{A}_n	241

7.6	The Voronoi Algorithm in Dimension 4	242
7.7	The Facets of \mathbb{D}_n and the 5-Dimensional Perfect Lattices	244
7.8	Determination of the Contiguous Form	253
7.9	Perfect Forms in Dimensions 6 and 7	254
7.10	Exercises for Chapter 7	258
7.11	Notes on Chapter 7	260
8	Hermitian Lattices	263
8.1	Complex and Quaternionic Structures	263
8.2	Hurwitz Lattices: Enlargements of \mathbb{D}_n	268
8.3	Hurwitz Lattices: Around Dimension 16	274
8.4	Eisenstein Lattices: A Construction of Barnes	280
8.5	Eisenstein Lattices: The Coxeter–Todd Lattice	284
8.6	A General Construction of Hermitian Lattices	292
8.7	Quadratic Hermitian Structures	298
8.8	Beyond Dimension 24	306
8.9	Exercises for Chapter 8	310
8.10	Notes on Chapter 8	316
9	The Configurations of Minimal Vectors	321
9.1	Minimal-Equivalent Lattices	321
9.2	Classes of Dimension $n \leq 3$	329
9.3	Classification in Dimension 4	333
9.4	Weakly Eutactic Lattices in a Minimal Class	339
9.5	The Classification of Eutactic Lattices	343
9.6	Perfect Pairs of Lattices	348
9.7	Complements	354
9.8	Exercises for Chapter 9	357
9.9	Notes on Chapter 9	361
10	Extremal Properties of Families of Lattices	363
10.1	Some Elementary Results on Lie Groups	364
10.2	Perfection and Eutaxy	366
10.3	Extremality	368
10.4	Minimal Classes	373
10.5	Dual-Extreme Lattices	374
10.6	The Rankin Invariants	376
10.7	Exercises for Chapter 10	379
10.8	Notes on Chapter 10	380
11	Group Actions	383
11.1	Rational and Integral Representations	383
11.2	G -Lattices	385
11.3	G -extreme Lattices	388
11.4	Cyclotomic Lattices	393

11.5	Isodual Lattices, Modular Lattices, and Normal Lattices	400
11.6	Normal Lattices	403
11.7	Extreme Symmetric and Symplectic Lattices	407
11.8	Isodual Lattices: Examples and Classification Results	414
11.9	Rationality and Finiteness Questions	417
11.10	Exercises for Chapter 11	421
11.11	Notes on Chapter 11	425
12	Cross-Sections	427
12.1	Embedding a Lattice in a Larger One	427
12.2	X-Rays of Lattices	430
12.3	Lattices with a Fixed Cross-Section	433
12.4	A Characterization of Relatively Extreme Lattices	437
12.5	Patchwork Lattices	438
12.6	Exercises for Chapter 12	439
12.7	Notes on Chapter 12	440
13	Extensions of the Voronoi Algorithm	443
13.1	Contiguity Relative to a Space of Symmetric Matrices	444
13.2	The Voronoi Algorithm Relative to a Space of Symmetric Matrices	448
13.3	Perfect G -Lattices	451
13.4	Two-Dimensional Centralizers	454
13.5	Cyclotomic Lattices	458
13.6	Lattices with a Fixed Section and Patchwork Lattices	460
13.7	Examples	463
13.8	Exercises for Chapter 13	464
13.9	Notes on Chapter 13	465
14	Numerical Data	467
14.1	Low-Dimensional Perfect Lattices	467
14.2	Root Lattices	468
14.3	Eutactic Lattices up to Dimension 4	469
14.4	The Hermite Constant	472
14.5	Invariants Related to Duality	474
14.6	The Kissing Number	476
15	Appendix 1: Semi-Simple Algebras and Quaternions	479
15.1	Semi-Simple Algebras	479
15.2	Quaternion Algebras	481
15.3	Algebraic Lattices over Dedekind Domains	482
15.4	Arithmetic in Separable Algebras	484
15.5	Number Fields	485
15.6	Quaternions Again	486
15.7	Ideal Class Set	487

16 Appendix 2: Strongly Perfect Lattices	489
16.1 Spherical Designs	489
16.2 Strong Perfection	491
16.3 An Infinite Series	492
16.4 Modular Lattices	493
16.5 Group Theory	494
16.6 Designs on Grassmannian Varieties	495
References	497
List of Symbols	511
Index	517



<http://www.springer.com/978-3-540-44236-3>

Perfect Lattices in Euclidean Spaces

Martinet, J.

2003, XXII, 526 p., Hardcover

ISBN: 978-3-540-44236-3