

Contents

1. Introduction	1
2. Fractals	5
2.1 Fractal Structures	5
2.2 Fractal Dimensions	7
2.3 Methods for Obtaining Fractal Dimensions	10
2.4 Fractal Dimension of Aerogels	13
2.5 Brownian Motion and its Fractal Nature	15
3. Percolating Networks as Random Fractals	19
3.1 Critical Exponents and Scaling Relations	19
3.2 Fractal Dimension	27
3.3 Finite-Size Scaling and Scaling Relations	29
3.4 Nodes–Links–Blobs Model	32
4. Multifractals	35
4.1 Hierarchical Resistor Network Model	35
4.2 Mass Exponent and Generalized Dimension	40
4.3 Multifractal Spectrum	44
4.4 Relation between $\tau(q)$ and $f(\alpha)$	45
4.5 Direct Determination of $f(\alpha)$	47
4.6 Correlations between Box Measures	48
4.7 Profiles of $\tau(q)$, D_q , $f(\alpha)$, and $z(q)$	50
4.8 Parabolic Approximation and Distribution Functions of Measures	54
4.9 Growth Probability of DLA	56
5. Anomalous Diffusion on Fractal Networks	59
5.1 Anomalous Diffusion	59
5.2 Spectral Dimension	62
5.3 Spectral Density of States of Fractal Networks	64
5.4 Scaling Argument for Spectral Density of States	65
5.5 Localization of Excitations on Fractal Networks	66
5.6 Phonons and Fractons in Percolating Networks	68

6. Atomic Vibrations of Percolating Networks	71
6.1 Spectral Density of States and Spectral Dimension	71
6.2 Missing Modes at Low Frequencies	75
6.3 The Hump at High Frequencies	76
6.4 Localized Nature of Fracton Excitations	79
6.5 Networks with Vector Elasticity: Scaling Arguments	81
6.6 Simulation Results for Vector Elasticity	86
7. Scaling Arguments for Dynamic Structure Factors	89
7.1 Dynamic Structure Factors: Inelastic Neutron Scattering	89
7.2 Single-Length Scaling Arguments	91
7.3 Numerical Simulations of $S(q, \omega)$	94
7.4 Inelastic Light Scattering in Percolating Systems	96
8. Spin Waves in Diluted Heisenberg Antiferromagnets	101
8.1 Spin Waves in Percolating Antiferromagnets	101
8.2 Antiferromagnetic Spectral Dimension and the Upper Bound of \tilde{d}_{AF}	104
8.3 Numerical Simulations of Antiferromagnetic Fractons	107
8.4 Scaling Theory of $S(q, \omega)$ for Percolating Antiferromagnets	109
8.5 Large-Scale Simulations for $S(q, \omega)$	110
9. Anderson Transition	115
9.1 Coherent Transport of Electrons	115
9.2 Anderson Localization	117
9.3 Scaling Theory of the Anderson Transition	120
9.4 Universality Classes	125
9.5 Numerical Studies	129
9.6 Dynamical Properties at the Anderson Transition	134
9.6.1 Scaling Form of Dynamic Quantities at Criticality	134
9.6.2 Diffusion of Wave Packets	135
9.6.3 Two-Particle Correlation Function	142
9.6.4 Diffusion Length	144
10. Multifractals in the Anderson Transition	149
10.1 Multifractality of a Critical Wavefunction	149
10.2 Multifractality of Spectral Measures	155
10.3 Relations between Multifractality and Dynamical Properties at the Transition Point	160
10.4 Conformal Invariance	162
10.5 Universality at Higher Landau Levels	168
10.6 Finite-Size Scaling for the q th Moment	172

Appendices	177
A. Multifractality of the HRN Model	177
B. Spectral Dimensions for Deterministic Fractals	182
B.1 Sierpinski Gasket	182
B.2 Mandelbrot–Given Fractal	184
C. Diffusion and Dynamics on Networks	186
C.1 Atomic Vibrations	187
C.2 Spin Waves in Diluted Ferro- and Antiferromagnets	191
C.3 Superconducting Networks	192
D. Wigner Distributions	194
References	197
Subject Index	203

Fractal Concepts in Condensed Matter Physics

Nakayama, T.; Yakubo, K.

2003, XIII, 210 p., Hardcover

ISBN: 978-3-540-05044-5