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SYNTHETIC GEOMETRY AND *AUFBAU*

1. INTRODUCTION

From antiquity to the beginnings of the 20th century philosophers took geometry as a paradigmatic example of science. Geometry defined what was to be considered as scientific knowledge. “More geometrico” was a sign of quality for philosophical and scientific argumentation. Philosophy and science had to make a great conceptual effort to get rid of this overwhelming and sometimes depressing epistemic ideal of geometry. Today this aim has been achieved to a large extent. Geometry as a philosophical topic is of secondary importance at most. Certainly, it does not occupy centre stage in the contemporary discussion of epistemology and philosophy of science. Not even in philosophy of mathematics, geometry is considered as a hot topic. For instance, in Tymoczko’s (already somewhat dated) anthology *New Directions in the Philosophy of Mathematics* (Tymoczko, 1985) no contribution deals with geometry in its classical or modern form. *A priori* there is no need to deplore this situation, geometry may belong to those topics that for good reasons are no longer on the agenda of contemporary philosophy. Be this as it may, this situation is markedly different from that of the beginning of the 20th century. For philosophers such as Russell, Cassirer or Carnap, to name but a few, the philosophical problems posed by geometry played a central role in their investigations. It may be sufficient to mention just a few bio-bibliographical facts: (i) Russell started his philosophical career in 1897 with the dissertation *The Foundations of Geometry*. Somewhat later he published *The Foundations of Mathematics* in which he treated geometry at great length; (ii) In *Substanzbegriff und Funktionsbegriff* Cassirer dedicated a central chapter to the topic of concept formation in geometry, considered by him as a paradigmatic case for concept formation in science in general (Cassirer, 1985 (1910)). Throughout his life, he considered *Klein’s Erlanger Programm* as a guideline for his “critical idealism”; (iii) Carnap started his philosophical career with the dissertation *Der Raum, Ein Betrag zur Wissenschaftslehre* (Carnap, 1922). Although this work was largely ignored by Carnap-scholars, it is an important early work that contains the germs of many of the ideas to be unfolded and elaborated later.

Der Raum is a rather eclectic work dealing with a lot of topics, often only touching the problems and not giving them an in-depth treatment. In a first

approximation, it may be characterized as a Neo-Kantian work although the influences of Husserlian phenomenology and Russellian logical constructivism cannot be overlooked. In this paper, I do not aim at the probably rather difficult task to give a comprehensive interpretation of it. Rather, I'd like to concentrate on one specific topic which was to play a crucial role in Carnap's philosophical thought as a whole. This topic is concerned with the role of geometry for what later was to become Carnap's theory of constitutional systems. More precisely, I'd like to put forward the following thesis:

Geometry, as synthetic geometry, was an important source of inspiration of Carnap's philosophical thought. The constitutional theory of *Der Logische Aufbau der Welt* is inspired, to a large extent, by the relational systems of synthetic geometry treated in *Der Raum*. Moreover, since the *Aufbau* program was decisive for Carnap's philosophy in general, geometry had a substantial influence on his philosophy in general.

This thesis may be generalized: Geometry was of utmost importance not only for Carnap, but for the epistemology and philosophy of science of the early 20th century in general. Not only Logical Empiricism, but also rival philosophical currents such as the Marburg Neokantianism, or Husserl's phenomenology were deeply engaged in wrestling with the problems geometry presented to philosophy.

At that time, philosophers used to have a much richer idea of geometry than today. The philosophy of geometry was not restricted to the worn out topic of Euclidean vs. Non-Euclidean geometry. For whatever reasons, since then the philosophical community has suffered a sort of amnesia with respect to geometry. If, as recently has been claimed by various authors, a more profound understanding of the philosophy of the beginnings of the 20th century is crucial for contemporary epistemology and philosophy of science, and if geometry is an essential ingredient in the philosophy of that period, then this amnesia is a serious obstacle for contemporary philosophy. Philosophers will have to make some efforts to regain the lost territory of geometry for the philosophical discourse of the 21st century.

Now, let us be more specific and concentrate on the role geometry played in Carnap's philosophy, particularly in the *Aufbau*. With respect to this topic I contend that the constitutional theory of the *Aufbau* follows the patterns of the construction (or constitution) of geometric systems in synthetic geometry. The fact that essential motives for the constitutional theory of the *Aufbau* can be found in synthetic geometry is not only of interest for the history of philosophy. A better understanding of the geometric backdrop of the *Aufbau* may help defuse some of the criticisms of the feasibility of the *Aufbau* program. Many of them are based on misunderstandings or neglect the geometric background of the *Aufbau*. Two examples are the influential objections of Goodman (Goodman, 1963, 1977) and Quine (Quine, 1951) against the *Aufbau* program.

The outline of this article is as follows: To set the stage, in the next section *Synthetic Geometry as a Theory of Ordnungsgefüge* the basics of synthetic

geometry are recalled in an informal way. In section 3 *Conceptual Geometry in Der Raum* it is argued that the roots of what was to become in the *Aufbau* the theory of constitutional systems can already be found in *Der Raum* under the heading of a geometrical theory of “conceptual geometries.” “Conceptual geometries” may be considered as the earliest (and somewhat primitive) forerunners of what was to be called constitutional systems and, later, linguistic or ontological frameworks. At this early stage the geometric origins of these systems are still quite visible while later they tend to be obscured. The subsequent development of conceptual geometries is manifest in a number of unpublished manuscripts written in the interim period between *Der Raum* and *Aufbau*, in particular in *Quasizerlegung* – which is dealt with in section 4. In this section it is shown how Carnap’s famous – or notorious – method of quasianalysis fits into the framework of synthetic geometry. In section 5 *From Synthetic Geometry to Constitutional Theory* it is argued that some of the objections against the feasibility of the *Aufbau* program are unfounded since they are based on inadequate understanding of the geometrical background of this work.

2. SYNTHETIC GEOMETRY AS A GENERAL THEORY OF *ORDNUNGSGEFÜGE*

Having lamented about the amnesia contemporary philosophy suffers with respect to geometry and its relevance for philosophy it would be expedient to go on with a refresher course “Synthetic geometry for philosophers.” For obvious reasons, this is not possible. Instead, I’d like to explicate one leitmotif of synthetic geometry from which two characteristics of the synthetic-geometric thought emerged that became crucial for Carnap’s “constitutional theory in the spirit of synthetic geometry.”

The leitmotif of synthetic geometry is order. As Carnap put it, geometry is a general theory of *Ordnungsgefüge*. The expression *Ordnungsgefüge* is not a terminus technicus in mathematics. It seems that Carnap understood the term in a semi-technical sense intended to mean something like “relational structure” or “structured set.” By conceiving a domain as an *Ordnungsgefüge* one imposes an order, or structure on it. This is achieved by certain stipulations of order (“*Ordnungssetzungen*”).

Characterized as a general theory of *Ordnungsgefüge* synthetic geometry is conceived of as having a strong applicative dimension. That is to say, the *Ordnungsgefüge* synthetic geometry is dealing with are to be applied to many different cases. In *Der Raum* Carnap explains this fact for projective geometries at great length. Synthetic geometry as a general theory of *Ordnungsgefüge* offers an arsenal of possible conceptual schemes or perspectives applicable to many domains. This applicative dimension renders it deeply pluralistic: It goes without saying that no single structure can cope with all applications. Rather, the point of geometry is to study the variety of different geometries.

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