Chapter 3
Types of Comprising Entities

I - Collective Classes and Distributive Classes

An important step in the development of a theory of what I have called 'controversial' comprising entities (entities designated by terms such as 'class', 'manifold', 'collection', 'aggregate', etc.) is the clear distinction between talk of the many comprised entities, on the one hand, and talk of the entity which comprises them on the other hand. It seems natural to consider the latter as simply a peculiar mode of the former, one in which the many are 'spoken of as one'. Indeed, it may turn out that the comprising entity is identical to the many comprised entities. However, the question whether this is so can only be considered with clarity if our thought and talk of the former is clearly distinguished from our thought and talk of the latter.

In the 19th Century gradually more attention was being paid to comprising entities, due to the emerging importance of their role in connection with the logical analysis of language and with work on the foundations of mathematics. This led to a recognition (for example, in the works of Cantor, Schröder and Frege) of the need for a distinction between talk of a comprising entity and talk of the many entities it comprises.

Once it is clear that talk of a comprising entity cannot simply be assumed to be a peculiar kind of talk of the many comprised entities, it is also clear that existence of the comprising entity cannot without justification be supposed to be merely an aspect of the existence of the many comprised entities. It is then realized that, granted the existence of the many comprised entities, an *additional* assumption is required to guarantee the existence of the comprising entity. The general assumption playing this role in Cantorian set-theory, for example, is the Comprehension Principle, according to which, as we have seen in section IV of Chapter 1, whenever there exist entities *x*s, there exists a class to which the *x*s belong as members.[1]

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[1] See also Machover 1996, 12.

In addition to laying down assumptions regarding conditions under which a comprising entity exists, the development of a theory of comprising entities involves the introduction of assumptions regarding the relations between comprised entities and entities which comprise them.

A particularly noteworthy development in this respect was the distinction, due principally to Frege, between two different types of comprising entity, corresponding to two different types of relations between comprised and comprising entities. Frege criticised Schröder for failing to distinguish between two notions of a class:[2] The first is the notion of a class as what Frege calls a 'collective whole'. In specifying such a notion of classes, we 'do not bring them into connection with concepts'.[3] To see what is meant by 'bringing them into connection with concepts' it is perhaps best to contrast this notion of a class with the other one discussed by Frege.

The second notion is that of a class as the *extension* of a concept. The extension of the concept *man* is an entity which is internally articulated in accordance with the concept, in a way that the collective whole is not. A collective whole is an unarticulated occupant of a specified spatial domain. The difference between the two notions is highlighted by the fact that there are two ways in which an entity can belong to a class-as-extension: It can belong to it as a member, or as a subclass.[4] By contrast, there is only one way in which an entity can belong to a class-as-collective-whole: It can only belong to it as a part.

The relation *is a subclass of* is transitive, as is the relation *is a part of*. The relation *is a member of*, by contrast, is not transitive. Furthermore, both *is a subclass of* and *is a part of* may be assumed to be reflexive,[5] but not so the relation *is a member of*. In as far as a theory of classes is meant to serve as a tool in the formalisation of logic, it is classes-as-extensions that are required, for it is only such classes that allow us to avoid conflating, for example, the logical form of 'Socrates is a human-being' with that of 'the Greeks are human beings'.

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[2] See Frege 1895.

[3] Frege 1895, 87.

[4] As Frege notes (Frege 1895, 92), Husserl similarly observes that there are two ways in which an entity can belong to a class-as-extension, in his 1891 review of the same work of Schröder's work.

[5] To take them as reflexive relations is to assume that an "improper part" is a part, and that an "improper subclass" is a subclass.

Frege's distinction can be thought of as marking a point of bifurcation, at which modern theorizing on classes became separate from modern theorizing on wholes and parts (in a sense which involves the provision of a systematic theory of the relation of parthood).

His classes-as-collective-wholes are predecessors of contemporary classical and neoclassical mereological sums, and his classes-as-extensions are predecessors of contemporary classes and sets.Classes-as-extensions were soon to meet grave difficulties centring on Russell's well-known Paradox, which led to considerations limiting the application of the Comprehension Principle. One way (adopted by Russell) to effect such limitations was by means of introducing a theory of types. Another (adopted by Zermelo and his followers) was to construct an *axiomatic* set theory, where only a limited range of instances of the Comprehension Principle were assumed.[6]

On the other hand, reflection on these difficulties led Stanisław Leśniewski to develop a theory which did away with classes-as-extensions, or *distributive* classes, as he called them, and only made use of Frege's other type of class, which he called *collective* classes.[7] He named his theory of collective classes *mereology*, the etymology of the term expressing the theory's interpretation as a theory of parts and wholes (as already noted, '*meros*' is the Greek for 'part'). In what follows, I shall adopt Leśniewski's terminology, speaking of Frege's distinction as the distinction between distributive and collective classes.

From the perspective of the common origin of modern theories of classes (i.e. theories of distributive classes) and of parts and wholes (i.e. theories of collective classes), it is understandable that Leśniewski did not consider mereology to be any more ontologically presumptuous than the set or class theories of Cantor, Frege, and Russell and Whitehead. Where the latter theories assume the existence of an entity (i.e. a class) which has certain individuals as members, mereology assumes the existence of an entity (i.e. a mereological sum) which has those individuals as parts. That is, both theories assume the existence of a single comprising entity, and they only differ with respect to the formal characteristics of the relations between the comprising entity and the individuals it comprises.

Moreover, the fact that class theory could only be safeguarded against contradiction by apparently placing limitations to the Comprehension

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[6] See Machover 1996, 13-14.

[7] See Küng 1963, 105-115; Simons 1987, 101-104.

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