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LEIBNIZ'S LOGICAL SYSTEMS: A RECONSTRUCTION

1. INTRODUCTORY REMARKS

The system of logic plays a central rôle in Leibniz's methodology of science. According to his basic ideas, any scientific question should be posed in the form "Is S a P ?" and its answer might be found either in *Encyclopædia Universalis* of the human knowledge (when the fact is synthetic) or in the corresponding relation between *Numeri Characterici* of S and P (when the fact is analytic). Leibniz believed that "a few selected men", as he wrote, could systematize (in a *quinquenum*!) all data and establish a correspondence between the elementary notions and their characteristic numbers. As we see, the logical system determines both the formal translation of the scientific sentences and the mechanism of checking their truthfulness.

In the style of his epoch, Leibniz has identified the whole logical reasoning with syllogistic. His efforts to turn human reasoning into some kind of mathematics have been in two directions. The first one may be called *algebraic*. It is presented in various manuscripts during all his life. The main goal has been the abstract representation of the syllogistic laws and in such a way, obtaining a special algebra of notions. The second direction of investigations (all of them concentrated in April 1679) may be called *arithmetical*. The central idea has been embedding that logical algebra into arithmetic. In other words, to establish an isomorphism between the logical algebra of terms and a suitable algebra of integers. Therefore, Leibniz's "Calculemus!" was meant to construct an adequate model of the basic syllogistic relations and operations into the arithmetic of integers.

All Leibniz's trials to find some natural model of syllogistic can be retraced from the first publications of his logical papers (Couturat, *Logique* and *Opuscules*) as well as from the new academic edition of his philosophical manuscripts (Leibniz, *Sämtliche Schriften*). Unfortunately, those trials were unsuccessful. Finally Leibniz has radically changed the arithmetical interpretation: in the place of simple relations between integers based on divisibility, relations between *pairs* of (relatively prime) numbers appeared. In 1946, J. Ślupecki (*Syllogistic*) proved that the last model was adequate; see also the celebrated book of J. Łukasiewicz (*Aristotle's Syllogistic*). However, the model of pairs possesses two big disadvantages. The first one is that the initial naturalness has died. Actually, it is not fatal from the mathematical point of view. However, the second disadvantage is principal: it is not clear how the

model of pairs of integers can be extended to involve the generalizations of the Aristotelian syllogistic containing, e.g., term negation or term conjunction.

It was shown (Sotirov, *Arithmetizations*) that Leibniz's primary plan to explore divisibility of integers had been vital and two kinds of arithmetical models adequate to many systems of syllogistic were built up. Later on the arithmetical interpretation was extended to cover the pure monadic predicate calculus, i.e., the entire logic of properties (Sotirov, *Monadic*). Finally, it was shown that the Leibniz style arithmetization could be extended to the monadic predicate calculus containing equality as well (Sotirov, *Equality*). All papers cited here are available from <http://www.math.bas.bg/~vlsot/>; the detailed proofs can also be found there.

The following methodological notice may be added here: any variant of syllogistic is decidable while the whole arithmetic is not. In such a way, speaking *in abstracto*, the calculation is not able to increase the "algorithmicity" of any disputation according to Leibniz's dreams. Nevertheless, that fragment of arithmetic in which all variants of syllogistic were imbedded is decidable because it contains *multiplication* only (Skolem, *Satzfunktionen*; Boolos and Jeffrey, *Computability*, § 25). The same fragment was used for the monadic predicate calculus (with or without equality).

The present paper is neither historical, philosophical nor archival. Its goal is not to expose the development of Leibniz's ideas, their philosophical underground or their exact places in his manuscripts. Our aim is to reconstruct the variety of logical systems sketched by Leibniz and in such a way, to obtain a working mechanism: probably the mechanism that Leibniz would describe if he was alive today.

2. ARITHMETIZATIONS OF THE TRADITIONAL SYLLOGISTIC

The Aristotelian syllogistic will be treated in the style that became canonical (Łukasiewicz, *Aristotle's Syllogistic*). For this purpose the language of the classical propositional calculus is extended by *term variables* or, for short, *terms* $t_1, t_2, \dots$ together with two binary *term relations*: A and I. Syllogistic *atoms* are all formulae of the kind sAp or sIp with s and p being terms. A *sylogism* is any propositional formula with all propositional letters replaced by syllogistic atoms. The formal connectives used are $\neg, \wedge, \vee, \Rightarrow, \&, \rightarrow$ and $\leftrightarrow$ are used as informal abbreviations.

The standard and the most intuitive semantics of the Aristotelian syllogistic is that in the theory of sets: if S and P are arbitrary *non-empty* sets, sAp ("Every s is a p ") is translated into $S \subseteq P$, sIp ("Some s is a p ") into $S \cap P \neq \emptyset$, and the formal propositional connectives are replaced by the informal ones. Thus any sylogism is translated into a sentence about non-empty sets. If this sentence is true, i.e., if the expression so obtained is a set-theoretical tautology, the sylogism is said to be *true*. It is *true in a given (non-empty) set U* when any replacement of its terms with (non-empty) subsets of U gives a true sentence. This semantics will be called *Scholastic* according to Leibniz's own use of this word.

Another semantics in the theory of sets is possible; it will be named *Leibnizian*, being (partially) accepted by him. When a non-empty set U is given, term variables are evaluated by subsets of U *different from U*. If S and P are such sets, sAp is

interpreted as $S \supseteq P$, sIp as $S \cup P \neq U$, and the formal propositional connectives are replaced by informal ones. A syllogism is said to be *true in U* when the sentence obtained after any replacement of all term variables with subsets of U (different from U) is true. The syllogism is *true* when it is true in any set U .

On the base of both set-theoretical intuitions, two translations of the traditional syllogistic into arithmetic are possible. The first one will be named *Scholastic*. Let $a_1, a_2, \dots$ denote arbitrary integers *greater than 1*. Given a syllogism, replace $t_i A t_j$ with " a_i is a divisor of a_j ", $t_i I t_j$ with " a_i and a_j have a common divisor greater than 1", or: $\text{g.c.d.}(a_i, a_j) > 1$, and the formal propositional connectives with informal ones. Call the syllogism *arithmetically true* (in the *Scholastic sense*) if the sentence so obtained is an arithmetical truth.

Fact 1 (Adequacy of the Scholastic arithmetical interpretation): *A traditional syllogism is true if and only if it is arithmetically true (in the Scholastic sense).*

If the empty set is admitted to evaluate terms in the Scholastic semantics, the arithmetical interpretation can be modified: 1 has to be added to the list of divisors, being the number corresponding to empty terms.

Further, for the arithmetical interpretation named *Leibnizian*, let u (the Universe number) be an arbitrary integer greater than 1, and let $a_1, a_2, \dots$ be its arbitrary proper divisors, i.e., $a_i < u$ for any i (however, $a_i = 1$ is permitted). Replace $t_i A t_j$ with a relation " a_i is divisible by a_j ", and $t_i I t_j$ with "there is a *prime* divisor of u dividing neither a_i nor a_j ", or: $\text{l.c.m.}(a_i, a_j) < u$. Finally, formal propositional connectives are replaced with their informal analogues. The syllogism is said to be *arithmetically true* (in the *Leibnizian sense*) *with respect to u* if the sentence so obtained is an arithmetical truth. The syllogism is *arithmetically true* (in the *Leibnizian sense*) if it is arithmetically true, in the same sense, with respect to any $u > 1$.

Fact 2 (Adequacy of the Leibnizian arithmetical interpretation): *A traditional syllogism is true if and only if it is arithmetically true in the Leibnizian sense.*

In this semantics, if empty terms are admitted, they have to be evaluated by the Universe number u .

3. ARITHMETIZATIONS OF SYLLOGISTIC WITH NEGATIVE TERMS

Expand the language of syllogistic by adding an operation of term negation $-$; then, if t is a term, $-t$ ("non- t ") is a term, too. The definition of atoms is modified by permitting s and p to be arbitrary terms in sAp and in sIp as well. In both set-theoretical semantics, a *universal set* U is introduced. According to the tradition, terms are evaluated by subsets of U different from $\emptyset$ and U . If t is evaluated by a set T , the value of $-t$ is the complement of T to U . The rest of the definition of a true syllogism remains the same. In both arithmetical interpretations, a Universe number $u > 1$ *without multiple factors* is introduced together with the following rules: 1) all evaluating integers are divisors of u different from 1 and u ; 2) if term t is evaluated by an integer a then the term $-t$ is evaluated by the fraction u/a .

Fact 3 (Adequacy of both Scholastic and Leibnizian arithmetical interpretations of syllogistic with term negation): *A syllogism (possibly with*



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