JOHN-JULES CH. MEYER

MODAL EPISTEMIC AND DOXASTIC LOGIC

1 INTRODUCTION

Knowledge has always been a topic central to philosophy (cf. e.g. [Glymour, 1992]). Since ancient times philosophers have been interested in the way knowledge comes to us and in what way it relates to reality, the world in which we live. As is the case with so many things, during this century also the topic of knowledge has become the subject of formal investigations. Questions arose such as what the logical properties of knowledge are, and in order to come up with answers to these, logics have been devised to study these questions in a formal setting. These logics are now generally called *'epistemic logics'*, i.e., logics pertaining to 'knowledge'. Mostly, also the notion of 'belief', which is sometimes thought of as a weaker form of knowledge (but this is debated among philosophers (cf. [Gettier, 1963; Pollock, 1986; Voorbraak, 1993]) is considered, and we will also incorporate this notion in our treatment. Sometimes logics of belief are referred to with the special term *'doxastic logics'*, but we will just use the term 'epistemic logic(s)' for logics of knowledge and belief. Jaakko Hintikka [1962] was the first who proposed a modal logic approach to knowledge and belief. We follow the tradition of most recent treatments of modal logics, including those of knowledge and belief, by adopting possible world semantics in the style of Kripke [1963].

Recently, that is, in the last decade or two, these logics have attracted new attention from computer scientists and researchers of artificial Intelligence (AI), since it appeared that the notions of knowledge and belief played an important role in understanding advanced systems of software. It was appreciated that the use of notions of knowledge and belief enabled one to describe systems that are complicated both in physical and in logical respect: for both so-called distributed systems as systems in which information is manipulated in a non-trivial way, such as knowledge-based systems the concept of knowledge is important, and to give these systems a sound basis it appeared to be worth-while to use epistemic logic [Fagin *et al.*, 1995; Fagin and Vardi, 1986; Fischer and Immerman, 1986; Halpern, 1986; Halpern, 1987; Halpern and Fagin, 1989; Halpern and Moses, 1984a; Halpern and Moses, 1985; Halpern and Moses, 1990; Halpern and Moses, 1992; Moses, 1988; Moses, 1992; Voorbraak, 1988].

In this chapter we will give an overview of the development of epistemic logic in recent years as well as its application in computer science and AI.

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 10, 1–38.
© 2003, Kluwer Academic Publishers. Printed in the Netherlands.

2 THE 'STANDARD' MODAL APPROACH TO KNOWLEDGE AND BELIEF

2.1 The Modal Language of Knowledge and Belief

The core (propositional) language that we will use consists of a fixed set of propositional atoms closed under the usual propositional connectives and modal operators K, M, B and N. (For ease of presentation we will treat both knowledge and belief in the same model. Of course, one might leave out one of the two, and then also simplify the models accordingly.)

Formally, this is given by the following definition:

DEFINITION 1. Let $\mathcal{P}$ be a denumerable set of (propositional) atoms. The language $\mathcal{L}_{core}$ consists of

1. $p \in \mathcal{L}_{core}$ for all $p \in \mathcal{P}$;
2. if $\varphi, \psi \in \mathcal{L}_{core}$ then also $\neg\varphi, \varphi \wedge \psi$ and $\varphi \vee \psi \in \mathcal{L}_{core}$;
3. if $\varphi \in \mathcal{L}_{core}$ then also $K\varphi, B\varphi \in \mathcal{L}_{core}$.

We will furthermore use the abbreviations $\varphi \to \psi$ for $\neg\varphi \vee \psi$, $\varphi \leftrightarrow \psi$ for $(\varphi \to \psi) \wedge (\psi \to \varphi)$, $\bot$ for $p \wedge \neg p$ for some fixed $p \in \mathcal{P}$, $\top$ for $\neg\bot$, $M\varphi$ for $\neg K\neg\varphi$ and $N\varphi$ for $\neg B\neg\varphi$.

The part of the language in which only the K- (and M-)modalities are used, will be denoted $\mathcal{L}_K$, while the language with only B- (and N-) modalities will be denoted $\mathcal{L}_B$. Formulas without any occurrence of a modal operator will be called *objective*, and for a set $\Phi \subseteq \mathcal{L}_{core}$ the subset of objective formulas will be referred to as $Obj(\Phi)$. For any modal operator $\bigcirc$ (viz. K, M, B, N, but also for any other modal operator that we shall introduce in the sequel), we use the convention that $\bigcirc\Phi = \{\bigcirc\varphi \mid \varphi \in \Phi\}$. For ease of notation, we will also use this convention for $\bigcirc = \neg$.

The operator K will denote 'it is known that' and B will denote 'it is believed that', while the dual operators M and N will denote possibility on the basis of knowledge and belief, respectively.

This will be the core language that we will use for the moment. Later on, we will extend this language to accommodate for further epistemic modalities.

2.2 Kripke-style Semantics

The core language $\mathcal{L}_{core}$ is given an interpretation on the basis of Kripke-models, as is usual for a modal language. We will use *tt* and *ff* for the truth values.

DEFINITION 2. A Kripke model is a structure $\mathcal{M}$ of the form $\langle S, \pi, R, T\rangle$, where

- S is a non-empty set (the set of *states* or *possible worlds*);
- $\pi : S \to (\mathcal{P} \to \{tt, ff\})$ is a truth assignment function to the atoms per state;
- $R \subseteq S \times S$ is the knowledge accessibility relation;
- $T \subseteq S \times S$ is the belief accessibility relation.

The intuition behind the accessibility relation is the following: if $R(s,t)$ holds, then the state t is considered possible on the basis of the available knowledge in state s. In some way, having a number of available alternatives $t_1, t_2, t_3, \ldots$ for which $R(s,t_i)$ holds, expresses the knower's uncertainty about the true nature of the state s/he is in. Something similar holds for the relation T with respect to belief.

The interpretation of the language is now given by the following.

DEFINITION 3. Let $\mathcal{M} = \langle S, \pi, R, T \rangle$ be a given model and $s \in S$. Then:

- $\mathcal{M}, s \models p$ iff $\pi(s)(p) = tt$, for $p \in \mathcal{P}$;
- $\mathcal{M}, s \models \neg\varphi$ iff not $\mathcal{M}, s \models \varphi$;
- $\mathcal{M}, s \models \varphi \wedge \psi$ iff $\mathcal{M}, s \models \varphi$ and $\mathcal{M}, s \models \psi$;
- $\mathcal{M}, s \models \varphi \vee \psi$ iff $\mathcal{M}, s \models \varphi$ or $\mathcal{M}, s \models \psi$;
- $\mathcal{M}, s \models K\varphi$ iff $\mathcal{M}, t \models \varphi$ for all t with $R(s,t)$;
- $\mathcal{M}, s \models B\varphi$ iff $\mathcal{M}, t \models \varphi$ for all t with $T(s,t)$

A formula φ is *valid in a model* $\mathcal{M} = \langle S, \pi, R, T \rangle$, denoted $\mathcal{M} \models \varphi$, if $\mathcal{M}, s \models \varphi$ for every $s \in S$. A formula φ is *valid* with respect to a set MOD of models, denoted $MOD \models \varphi$, if $\mathcal{M} \models \varphi$ for every model $\mathcal{M} \in MOD$. If MOD is the set of *all* Kripke models of the above form, we generally write $\models \varphi$ instead of $MOD \models \varphi$.

One may now verify that the following holds:

PROPOSITION 4.

1. $\models K(\varphi \to \psi) \to (K\varphi \to K\psi)$, *and likewise for* B;
2. *If* $\models \varphi$ *then* $\models K\varphi$ *and* $\models B\varphi$.

This proposition says that by modelling knowledge and belief by a (normal) modal logic, the notions knowledge and belief are closed under logical implication. Furthermore, validities are always known and believed. Although perhaps defensible as properties of idealized notions of knowledge and belief, these properties are not always desirable: they give rise to the problem of logical omniscience, to which we will return in a later section.



http://www.springer.com/978-1-4020-1644-8

Handbook of Philosophical Logic
Volume 10
Gabbay, D.; Guenthner, F. (Eds.)
2003, XIII, 348 p., Hardcover
ISBN: 978-1-4020-1644-8