
Contents

1	Introduction	1
1.1	Introduction	1
1.2	The Catenary and Brachystochrone Problems	3
1.2.1	The Catenary	3
1.2.2	Brachystochrones	7
1.3	Hamilton's Principle	10
1.4	Some Variational Problems from Geometry	14
1.4.1	Dido's Problem	14
1.4.2	Geodesics	16
1.4.3	Minimal Surfaces	20
1.5	Optimal Harvest Strategy	21
2	The First Variation	23
2.1	The Finite-Dimensional Case	23
2.1.1	Functions of One Variable	23
2.1.2	Functions of Several Variables	26
2.2	The Euler-Lagrange Equation	28
2.3	Some Special Cases	36
2.3.1	Case I: No Explicit y Dependence	36
2.3.2	Case II: No Explicit x Dependence	38
2.4	A Degenerate Case	42
2.5	Invariance of the Euler-Lagrange Equation	44
2.6	Existence of Solutions to the Boundary-Value Problem*	49
3	Some Generalizations	55
3.1	Functionals Containing Higher-Order Derivatives	55
3.2	Several Dependent Variables	60
3.3	Two Independent Variables*	65
3.4	The Inverse Problem*	70

4	Isoperimetric Problems	73
4.1	The Finite-Dimensional Case and Lagrange Multipliers	73
4.1.1	Single Constraint	73
4.1.2	Multiple Constraints	77
4.1.3	Abnormal Problems	79
4.2	The Isoperimetric Problem	83
4.3	Some Generalizations on the Isoperimetric Problem	94
4.3.1	Problems Containing Higher-Order Derivatives	95
4.3.2	Multiple Isoperimetric Constraints	96
4.3.3	Several Dependent Variables	99
5	Applications to Eigenvalue Problems*	103
5.1	The Sturm-Liouville Problem	103
5.2	The First Eigenvalue	109
5.3	Higher Eigenvalues	115
6	Holonomic and Nonholonomic Constraints	119
6.1	Holonomic Constraints	119
6.2	Nonholonomic Constraints	125
6.3	Nonholonomic Constraints in Mechanics*	131
7	Problems with Variable Endpoints	135
7.1	Natural Boundary Conditions	135
7.2	The General Case	144
7.3	Transversality Conditions	150
8	The Hamiltonian Formulation	159
8.1	The Legendre Transformation	160
8.2	Hamilton's Equations	164
8.3	Symplectic Maps	171
8.4	The Hamilton-Jacobi Equation	175
8.4.1	The General Problem	175
8.4.2	Conservative Systems	181
8.5	Separation of Variables	184
8.5.1	The Method of Additive Separation	185
8.5.2	Conditions for Separable Solutions*	190
9	Noether's Theorem	201
9.1	Conservation Laws	201
9.2	Variational Symmetries	202
9.3	Noether's Theorem	207
9.4	Finding Variational Symmetries	213

10 The Second Variation	221
10.1 The Finite-Dimensional Case	221
10.2 The Second Variation	224
10.3 The Legendre Condition	227
10.4 The Jacobi Necessary Condition	232
10.4.1 A Reformulation of the Second Variation	232
10.4.2 The Jacobi Accessory Equation	234
10.4.3 The Jacobi Necessary Condition	237
10.5 A Sufficient Condition	241
10.6 More on Conjugate Points	244
10.6.1 Finding Conjugate Points	245
10.6.2 A Geometrical Interpretation	249
10.6.3 Saddle Points*	254
10.7 Convex Integrands	257
A Analysis and Differential Equations	261
A.1 Taylor's Theorem	261
A.2 The Implicit Function Theorem	265
A.3 Theory of Ordinary Differential Equations	268
B Function Spaces	273
B.1 Normed Spaces	273
B.2 Banach and Hilbert Spaces	278
References	283
Index	287



<http://www.springer.com/978-0-387-40247-5>

The Calculus of Variations

van Brunt, B.

2004, XIV, 292 p., Hardcover

ISBN: 978-0-387-40247-5