

Preface

This book is about three seemingly independent areas of mathematics: combinatorial group theory, the theory of Lie algebras and affine algebraic geometry. Indeed, for many years these areas were being developed fairly independently. Combinatorial group theory, the oldest of the three, was born in the beginning of the 20th century as a branch of low-dimensional topology. Very soon, it became an important area of mathematics with its own powerful techniques.

In the 1950s, combinatorial group theory started to influence, rather substantially, the theory of Lie algebras; thus combinatorial theory of Lie algebras was shaped, although the origins of the theory can be traced back to the 1930s.

In the 1960s, B. Buchberger introduced what is now known as Gröbner bases. This marked the beginning of a new, “combinatorial”, era in commutative algebra. It is not very likely that Buchberger was directly influenced by ideas from combinatorial group theory, but his famous algorithm bears resemblance to Nielsen’s method, although in a more sophisticated form. More recently, in the 1990s, ideas from combinatorial group theory started to have a more direct and significant impact on the areas of commutative algebra and affine algebraic geometry through the work of several mathematicians including the authors of this book. It is remarkable that, also in the 1990s, ideas from algebraic geometry found their way into combinatorial group theory through the work of G. Baumslag, A. G. Myasnikov, V. Remeslennikov, O. Kharlampovich and B. I. Plotkin, thus creating a “two-way traffic” of ideas that allows algebraic geometry to shed light on group theory and vice versa.

The main purpose of this book is to show how ideas from combinatorial group theory have spread to the other two areas, with the main focus on the area of commutative algebra where the influence of these ideas has been especially spectacular.

We would like to emphasize that we only consider here purely combinatorial methods and results; in particular, we leave out important interactions of group theory with topology and geometry that were recently used with great success in solving several difficult problems about free groups and their automorphisms. We also leave out geometric methods in affine algebraic geometry and concentrate on combinatorial ones, in particular on those that come from combinatorial group theory.

The book is targeted at research mathematicians as well as graduate students with an interest in the general area of algebra.

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