Preface

Overview and Goals

Combinatorial design theory is one of the most beautiful areas of mathematics. Design theory has its roots in recreational mathematics, but it evolved in the twentieth century into a full-fledged mathematical discipline with diverse applications in statistics and computer science. The fundamental problems in design theory are simple enough that they can be explained to non-mathematicians, yet the solutions of those problems have involved the development of innovative new combinatorial techniques as well as ingenious applications of methods from other areas of mathematics such as algebra and number theory. Many classical problems remain unsolved to this day as well.

This book is intended primarily to be a textbook for study at the senior undergraduate or beginning graduate level. Courses in mathematics or computer science can be based on this book. Regardless of the audience, however, it requires a certain amount of "mathematical maturity" to study design theory. The main technical prerequisites are some familiarity with basic abstract algebra (group theory, in particular), linear algebra (matrices and vector spaces), and some number-theoretic fundamentals (e.g., modular arithmetic and congruences).

Topic Coverage and Organization

The first seven chapters of this book provide a thorough treatment of the classical core of the subject of combinatorial designs. These chapters concern symmetric BIBDs, difference sets, Hadamard matrices, resolvable BIBDs, Latin squares, and pairwise balanced designs. A one-semester course can cover most of this material. For example, when I have taught courses on designs, I have based my lectures on material selected from the following chapters and sections:

- Chapter 1: Sections 1.1–1.3, Section 1.4 (optional), Sections 1.5–1.6
- Chapter 2: Sections 2.1–2.4
- Chapter 3: Sections 3.1–3.4
- Chapter 4: Sections 4.1–4.4, Section 4.5 (optional), Section 4.6
- Chapter 5: Sections 5.1–5.2, Section 5.3 (optional)
- Chapter 6: Sections 6.1, Section 6.2 (optional), Sections 6.3–6.8
- Chapter 7: Sections 7.1–7.3

There are many variations possible, of course. Typically, I would provide a complete proof of the Bruck-Ryser-Chowla Theorem or the Multiplier Theorem, but not both. It is possible to omit Wilson's Construction for MOLS in order to spend more time on pairwise balanced designs. Another option is to include the optional Section 6.2 and omit some of the material in Chapter 7. Yet another possibility is to present an introduction to t-designs (incorporating some material from Chapter 9, Sections 9.1 and 9.2) and delete some of the optional sections listed above.

More advanced or specialized material is covered in the last four chapters as well as in some later sections of the first seven chapters. The main topics in the last four chapters are minimal pairwise balanced designs, t-designs, orthogonal arrays and codes, and four selected applications of designs (in the last chapter).

Key Features

There are several features of this book that will make it useful as a textbook. Complete, carefully written proofs of most major results are given. There are many examples provided throughout in order to illustrate the definitions, concepts, and theorems. Numerous and varied exercises are provided at the end of each chapter. As well, certain mathematical threads flow through this book:

- The linear algebraic method of proving Fisher's Inequality reappears several times.
- The theme of Boolean functions is introduced in the study of bent functions and revisited in the discussion of Reed-Muller codes and a brief treatment of resilient functions.
- The use of permutation groups as a construction technique is pervasive.
- Elegant combinatorial arguments are used in many places in preference to alternative proofs that employ heavier mathematical machinery.
- Finite fields are used throughout the book. For this reason, some background material on finite fields is summarized in an Appendix. However, another option for an instructor is to specialize constructions utilizing finite fields $\mathbb{F}_q$ to the more familiar fields $\mathbb{Z}_p$, where p is a prime.

As mentioned earlier, there are a variety of advanced or specialized topics that are discussed in the book. Highlights include the following:

- regular Hadamard matrices and excess of Hadamard matrices;
- bent functions;
- bounds and constructions for minimal pairwise balanced designs;
- the Ryser-Woodall Theorem;
- constructions and bounds for t-wise balanced designs, including a proof of the Kramer Conjecture;
- a survey of the combinatorial connections between orthogonal arrays, codes, and designs;
- constructions and bounds for various classes of optimal codes and orthogonal arrays;
- Reed-Muller codes;
- resilient functions;
- four selected applications of designs: authentication codes, threshold schemes, group testing, and two-point sampling.

It must be recognized that design theory is an enormous subject, and any choice of optional material in a 300 page book is dependent on the whim of the author! Thus there are many interesting or important areas of design theory that are not discussed in the book. I hope, however, that readers of the book will find a fascinating mix of topics that serve to illustrate the breadth and beauty of design theory.

Audience

As mentioned above, this book is primarily intended to be a textbook. In addition, all of the material in this book is suitable for self-study by graduate students, who will find it provides helpful background information concerning research topics in design theory. Researchers may also find that some of the sections on advanced topics provide a useful reference for material that is not easily accessible in textbook form.

Acknowledgments

I have benefitted from the suggestions, comments and encouragement of many people while this book was being written. In particular, I would like to thank Charlie Colbourn, Don Kreher, and Brett Stevens. Special thanks goes to Dameng Deng for his help with proofreading. Also, I appreciate the assistance and advice of Wayne Yuhasz and Wayne Wheeler from Springer during this project.

Waterloo, Ontario, Canada *Douglas R. Stinson*
July, 2003

http://www.springer.com/978-0-387-95487-5

Combinatorial Designs
Constructions and Analysis
Stinson, D.
2004, XVI, 300 p., Hardcover
ISBN: 978-0-387-95487-5