Preface

This text is an elementary introduction to differential geometry. Although it was written for a graduate-level audience, the only requisite is a solid background in calculus, linear algebra, and basic point-set topology.

The first chapter covers the fundamentals of differentiable manifolds that are the bread and butter of differential geometry. All the usual topics are covered, culminating in Stokes' theorem together with some applications. The students' first contact with the subject can be overwhelming because of the wealth of abstract definitions involved, so examples have been stressed throughout. One concept, for instance, that students often find confusing is the definition of tangent vectors. They are first told that these are derivations on certain equivalence classes of functions, but later that the tangent space of $\mathbb{R}^n$ is "the same" as $\mathbb{R}^n$. We have tried to keep these spaces separate and to carefully explain how a vector space E is canonically isomorphic to its tangent space at a point. This subtle distinction becomes essential when later discussing the vertical bundle of a given vector bundle.

The following two chapters are devoted to fiber bundles and homotopy theory of fibrations. Vector bundles have been emphasized, although principal bundles are also discussed in detail. Special attention has been given to bundles over spheres because the sphere is the simplest base space for nontrivial bundles, and the latter can be explicitly classified. The tangent bundle of the sphere, in particular, provides a clear and concrete illustration of the relation between the principal frame bundle and the associated vector bundle, and a short section has been specifically devoted to it.

Chapter 4 studies bundles from the point of view of differential geometry, by introducing connections, holonomy, and curvature. Here again, the emphasis is on vector bundles. The last section discusses connections on principal bundles, and examines the relation between a connection on the frame bundle and that on the associated vector bundle.

Chapter 5 introduces Euclidean bundles and Riemannian connections, and then embarks on a brief excursion into the realm of Riemannian geometry. The basic tools, such as Levi-Civita connections, isometric immersions, Riemannian submersions, the Hopf-Rinow theorem, etc., are introduced, and should prepare the reader for more advanced texts on the subject. The relation between curvature and topology is illustrated by the classical theorems of Hadamard-Cartan and Bonnet-Myers.

Chapter 6 concludes with Chern-Weil theory, introducing the Pontrjagin, Euler, and Chern characteristic classes of a vector bundle. In order to illustrate these concepts, vector bundles over spheres of dimension ≤ 4 are reinterpreted in terms of their characteristic classes. The generalized Gauss-Bonnet theorem is also discussed here.

This book grew out of a series of graduate courses taught over the years at the University of Oklahoma. Although there were many outstanding texts available that collectively contained the sequence of topics I wished to present, none did this on its own, with the possible exception of Spivak's monumental treatise. In the end, I often found myself during a course following one author on a particular topic, another on a second one, and so on. As a result, the approach here at times closely parallels that of other texts, most notably Gromoll-Klingenberg-Meyer [**15**], Poor [**32**], Steenrod [**35**], Spivak [**34**], and Warner [**36**].

There are several options for using the material as the textbook for a course, depending on the instructor's inclination and the pace she/he wants to set. A leisurely paced one-semester course on manifolds could cover the first chapter. Similarly, a one-semester course on bundles could be based on Chapters 2 and 3, assuming the students are already familiar with the concept of manifolds. I have also used Chapter 1, parts of Chapter 4, and Chapter 5 for a two-semester course in differential geometry.

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