Preface

The subject of *antenna design*, primarily a discipline within electrical engineering, is devoted to the manipulation of structural elements of and/or the electrical currents present on a physical object capable of supporting such a current. Almost as soon as one begins to look at the subject, it becomes clear that there are interesting *mathematical* problems which need to be addressed, in the first instance, simply for the accurate modelling of the electromagnetic fields produced by an antenna. The description of the electromagnetic fields depends on the physical structure and the background environment in which the device is to operate.

It is the coincidence of a class of practical engineering applications and the application of some interesting mathematical optimization techniques that is the motivation for the present book. For this reason, we have thought it worthwhile to collect some of the problems that have inspired our research in applied mathematics, and to present them in such a way that they may appeal to two different audiences: *mathematicians* who are experts in the theory of mathematical optimization and who are interested in a less familiar and important area of application, and *engineers* who, confronted with problems of increasing sophistication, are interested in seeing a systematic mathematical approach to problems of interest to them. We hope that we have found the right balance to be of interest to both audiences. It is a difficult task.

Our ability to produce these devices at all, most designed for a particular purpose, leads quite soon to a desire to *optimize* the design in various ways. The mathematical problems associated with attempts to optimize performance can become quite sophisticated even for simple physical structures. For example, the goal of choosing antenna feedings, or surface currents, which produce an antenna pattern that matches a desired pattern (the so-called *synthesis problem*) leads to mathematical problems which are *ill-posed* in the sense of Hadamard. The fact that this important problem is not well-posed causes very concrete difficulties for the design engineer.

Moreover, most practitioners know quite well that in any given design problem one is confronted with not only a single measure of antenna performance, but with several, often conflicting, measures in terms of which the designer would like to optimize performance. From the mathematical point of view, such problems lead to the question of multi-criteria optimization whose techniques are not as well known as those associated with the optimization of a single cost functional.

Sooner or later, the question of the efficacy of mathematical analysis, in particular of the optimization problems that we treat in this book, must be addressed. It is our point of view that the results of this analysis is *normative*; that the analysis leads to a description of the theoretically optimal behavior against which the radiative properties of a particular realized design may be measured and in terms of which decisions can be made as to whether that realization is adequate or not.

From the mathematical side, the theory of mathematical optimization, a field whose antecedents pre-date the differential and integral calculus itself, has historically been inspired by practical applications beginning with the apocryphal isoperimetric problem of Dido, continuing with Newton's problem of finding the surface of revolution of minimal drag, and in our days with problems of mathematical programming and of optimal control. And, while the theory of optimization in finite dimensional settings is part of the usual set of mathematical tools available to every engineer, that part of the theory set in infinite dimensional vector spaces, most particularly, those optimization problems whose state equations are partial differential equations, is perhaps not so familiar.

For each of these audiences it may be helpful to cite two recent books in order to place the present one amongst them. It is our view that our monograph fits somewhere between that of Balanis [16] and the recent book of Cessenat [23], our text being more mathematically rigorous than the former and less mathematically intensive than the latter. On the other hand, while our particular collection of examples is not as wide-ranging as in [16], it is significantly more extensive than in [23]. We also mention the book of Stutzman and Thiele [132] which specifically treats antenna design problems exclusively, but not in the same systematic way as we do here. Moreover, to our knowledge the material in our final chapter does not appear outside of the research literature. The recent publications of the IEEE, [35] and [84], while not devoted to the problems of antenna design, are written at a level similar to that found in our book.

While this list of previously published books does not pretend to be complete, we should finally mention the classic work of D.S. Jones [59]. Part of that text discusses antenna problems, including the synthesis problem. The author discusses the approach to the description of radiated fields for wire antennas, and dielectric cylinders, and the integral equation approach to more arbitrarily shaped structures, with an emphasis on methods for the computation of the fields. But while Jones does formulate some of the optimization problems we consider, his treatment is somewhat brief.

The obvious difficulty in attempting to write for a dual audience lies in the necessity to include the information necessary for both groups to understand the basic material. There are few mathematicians who understand the fundamental facts about antennas, or even what is meant by an antenna pattern; it is not unknown but still unusual for engineers to know about ordered vector spaces or even weak-star convergence in Banach spaces.

It is impossible to make this single volume self-contained. Our choice is to present introductory material about antennas, together with some elementary examples in the introductory chapter. That discussion may then serve as a motivation for a more wide-ranging analysis. On the other hand, in order to continue with the flow of ideas, we have chosen to place a summary of the mathematical tools that we will use in the Appendix. That background material may be consulted from time to time as the reader may find necessary and convenient.

The chapter which follows gives some basic information about Maxwell's equations and the asymptotic behavior of solutions which is then used in Chapter 3. There we formulate a general class of optimization problems with radiated fields generated by bounded sources. Most importantly, we give several different measures of antenna performance related to the desired behavior of the radiated fields far from the antenna itself. These cost functionals are related to various properties of this far field and we discuss, in particular, their continuity properties which are of central importance to the problems of optimization.

In the fourth chapter, we concentrate on one particular problem, the synthesis problem mentioned earlier, and on its resolution. Since the problem is ill-posed, we give there a brief discussion of the mathematical nature of this class of problems.

The following two chapters then discuss, respectively, the boundary value problems for the two-dimensional Helmholtz equation, particularly important for treating TE and TM modes, and for the three-dimensional time-harmonic Maxwell equations. Our discussion, in both instances, includes some background in the numerical treatment of those boundary value problems.

Chapter 7, which together with Chapter 8 forms the central part of our presentation, contains the analysis of various optimization problems for specific examples based on the general framework that we constructed in Chapter 3. It is our belief that, while the traditional antenna literature analyzes the various concrete antenna structures somewhat independently, emphasizing the specific properties of each, a more over-arching approach can guide our understanding of the entire class of problems. In any specific application it is inevitable that there will come a time when the very particular details of the physical nature of the antenna will need to addressed in order to complete the design. That being said, the general analytical techniques we study here are applicable to antennas whether they take the form of a planar array of patches or of a line source on the curvilinear surface of the wing of an aircraft. For some of the standard (and simplest) examples, we include a numerical treatment which, quite naturally, will depend on the specifics of the antenna; a curvilinear line source will demand numerical treatment different from an array of radiating dipoles.

In the final chapter, Chapter 8, we address optimization problems arising when (as is most often the case) there is a need to optimize antenna performance with respect to two or more, often conflicting, measures. To give a simple example, there is often a desire to produce both a focused main beam and to minimize the electromagnetic energy trapped close to the antenna itself e.g, to maximize both directivity and gain simultaneously. In such a situation, the end result of such an analysis is a "design curve" which concretely represents the trade-offs that a design engineer must make if the design is to be in some sense optimal.

These problems fall within the general area of multi-criteria optimization **which was initially** investigated in the field of mathematical economics. More recently, such techniques have been applied to structural engineering problems, as for example the problem of the design of a beam with maximal rigidity and minimal mass, and even more recently, in the field of electromagnetics. While there is now an extensive mathematical literature available, the numerical treatment of such problems is most often, but not exclusively, confined to the "bi-criteria" case of two cost functionals. Our numerical illustrations are confined to this simplest case.

We make no pretense that our presentation is complete. Experts in antenna engineering will find many interesting situations have not been discussed. Likewise, experts in mathematical optimization will see that there are many techniques that have not been applied. We will consider our project a success if we can persuade even a few scientists that this general area, lying as it does on the boundary of applied mathematics and engineering, is both an interesting one and a source of fruitful problems for future research.

Finally, we come to the most pleasant of the tasks to face those who write a monograph, namely that of thanking those who have supported and encouraged us while we have been engaged in this task. There are so many!

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