1
Introduction to Structures

examples, definition, and properties

A vibration is a motion
that can't make up its mind
which way it wants to go.
—From Science Exam

Flexible structures in motion have specific features that are not a secret to a structural engineer. One of them is resonance—strong amplification of the motion at a specific frequency, called natural frequency. There are several frequencies that structures resonate at. A structure movement at these frequencies is harmonic, or sinusoidal, and remains at the same pattern of deformation. This pattern is called a mode shape, or mode. The modes are not coupled, and being independent they can be excited separately. More interesting, the total structural response is a sum of responses of individual modes. Another feature—structural poles—are complex conjugate. Their real parts (representing modal damping) are typically small, and their distance from the origin is the natural frequency of a structure.

1.1 Examples

In this book we investigate several examples of flexible structures. This includes a simple structure, composed of three lumped masses, a two-dimensional (2D) truss and a three-dimensional (3D) truss, a beam, the Deep Space Network antenna, and the International Space Station structure. They represent different levels of complexity.

1.1.1 A Simple Structure

A three-mass system—a simple structure—is used mainly for illustration purposes, and to make examples easy to follow. Its simplicity allows for easy analysis, and for straightforward interpretation. Also, solution properties and numerical data can be displayed in a compact form.

The system is shown in Fig. 1.1. In this figure m_1, m_2, and m_3 represent system masses, k_1, k_2, k_3, and k_4, are stiffness coefficients, while d_1, d_2, d_3, and d_4, are damping coefficients. This structure has six states, or three degrees of freedom.

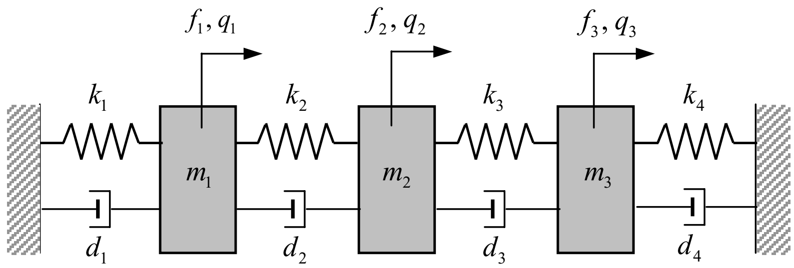

Figure 1.1. A simple structure.

1.1.2 A 2D Truss

The truss structure in Fig. 1.2 is a more complex example of a structure, which can still easily be simulated by the reader, if necessary. For this structure, l_1=15 cm, l_2=20 cm are dimensions of truss components. Each truss has a cross-sectional area of 1 cm^2, elastic modulus of 2.0×10^7 N/cm^2, and mass density of 0.00786 kg/cm^3. This structure has 32 states (or 16 degrees of freedom). Its stiffness and mass matrices are given in Appendix C.1.

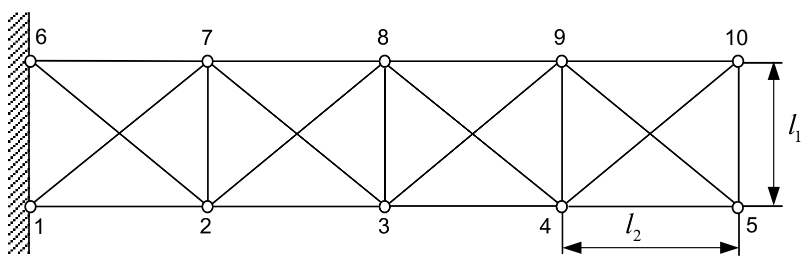

Figure 1.2. A 2D truss structure.

1.1.3 A 3D Truss

A 3D truss is shown in Fig. 1.3. For this truss, the length is 60 cm, the width 8 cm, the height 10 cm, the elastic modulus is 2.1×10^7 N/cm^2, and the mass density is 0.00392 kg/cm^3. The truss has 72 degrees of freedom (or 144 states).

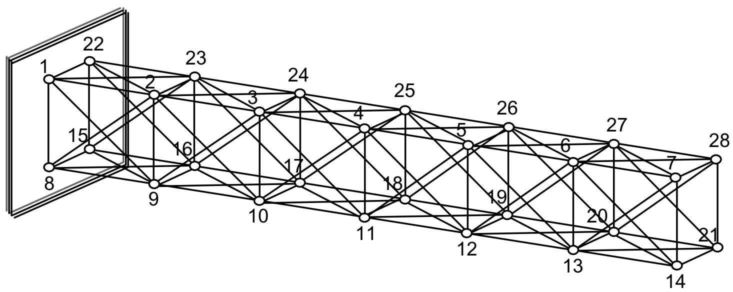

Figure 1.3. A 3D truss structure.

1.1.4 A Beam

A clamped beam is shown in Fig. 1.4. It is divided into n elements, with n–1 nodes, and two fixed nodes. In some cases later in this book we use n=15 elements for simple illustration, and sometimes n=60 or n=100 elements for more sophisticated examples of beam dynamics. Each node has three degrees of freedom: horizontal displacement, x, vertical displacement, y, and in plane rotation, θ. In total it has 3(n–1) degrees of freedom. The beam is 150 cm long, with a cross-section of 1 cm^2. The external (filled) nodes are clamped. The beam mass and stiffness matrices for n=15 are given in Appendix C.2.

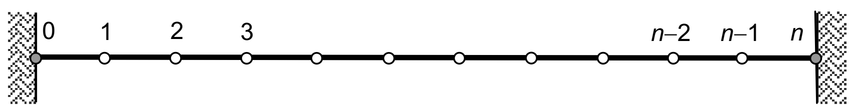

Figure 1.4. A beam divided into n finite elements.

1.1.5 The Deep Space Network Antenna

The NASA Deep Space Network antenna structure illustrates a real-world flexible structure. The Deep Space Network antennas, operated by the Jet Propulsion Laboratory, consist of several antenna types and are located at Goldstone (California), Madrid (Spain), and Canberra (Australia). The Deep Space Network serves as a communication tool for space exploration. A new generation of Deep Space Network antenna with a 34-m dish is shown in Fig. 1.5. This antenna is an articulated large flexible structure, which can rotate around azimuth (vertical) and elevation (horizontal) axes. The rotation is controlled by azimuth and elevation servos, as shown in Fig. 1.6. The combination of the antenna structure and its azimuth and elevation drives is the open-loop model of the antenna. The open-loop plant has two inputs (azimuth and elevation rates) and two outputs (azimuth and elevation position), and the position loop is closed between the encoder outputs and the rate inputs. The drives consist of gearboxes, electric motors, amplifiers, and tachometers. For more details about the antenna and its control systems, see [59] and [42], or visit the web page http://ipnpr.jpl.nasa.gov/. The finite-element model of the antenna structure consists of about 5000 degrees of freedom, with some nonlinear properties (dry friction, backlash, and limits imposed on its rates, and accelerations). However, the model of the structure and the drives used in this book are linear, and are obtained from the field test data using system identification procedures.

Figure 1.5. The Deep Space Network antenna at Goldstone, California (courtesy of NASA/JPL/Caltech, Pasadena, California). It can rotate with respect to azimuth (vertical) axis, and the dish with respect to elevation (horizontal axis).

In the following we briefly describe the field test. We tested the antenna using a white noise input signal of sampling frequency 30.6 Hz, as shown in Fig. 1.7(a). The antenna elevation encoder output record is shown in Fig. 1.7(b). From these records we determined the transfer function, from the antenna rate input to the encoder output, see Fig. 1.8(a),(b), dashed line. Next, we used the Eigensystem Realization Algorithm (ERA) identification algorithm (see [84], and Chapter 9 of this book) to determine the antenna state-space representation. For this representation we obtained the plot of the transfer function plot as shown in Fig. 1.8(a),(b), solid line. The plot displays good coincidence between the measured and identified transfer function.

The flexible properties are clearly visible in the identified model. The identified state-space representation of the antenna model is given in Appendix C.3.

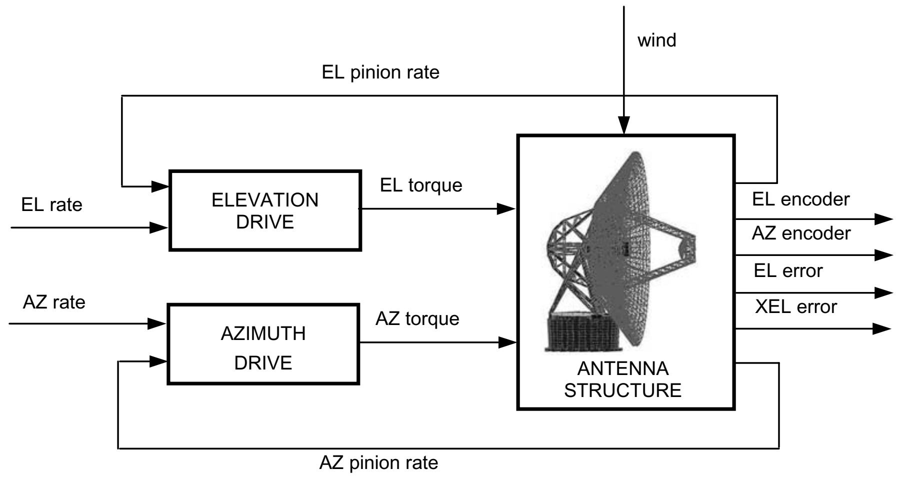

Figure 1.6. The open-loop model of the Deep Space Network antenna (AZ = azimuth, EL = elevation, XEL = cross-elevation): The AZ and EL positions are measured with encoders, EL and XEL errors are RF beam pointing errors.

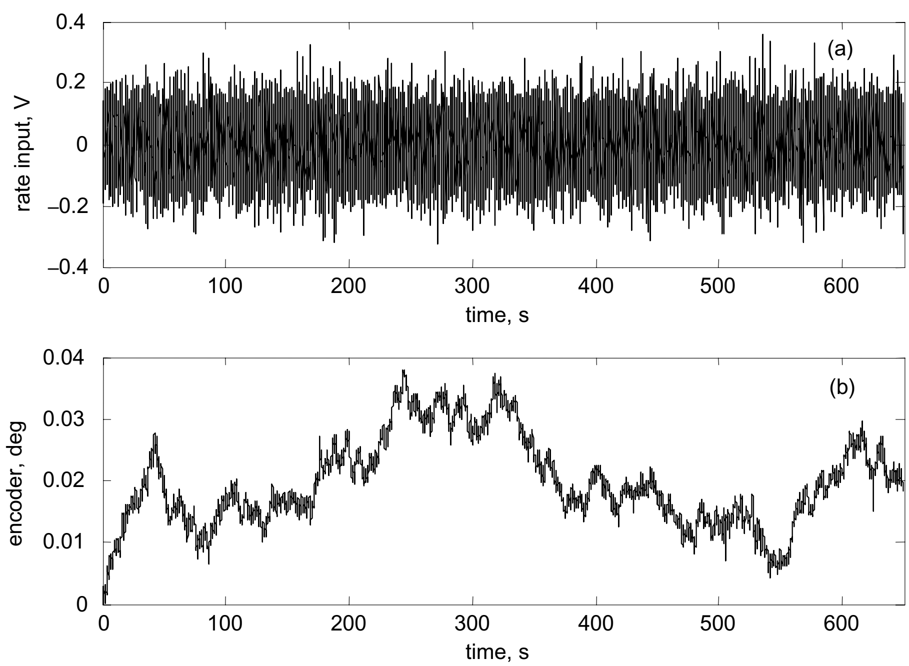

Figure 1.7. Signals in the identification of the antenna model: (a) Input white noise (voltage); and (b) output–antenna position measured by the encoder.

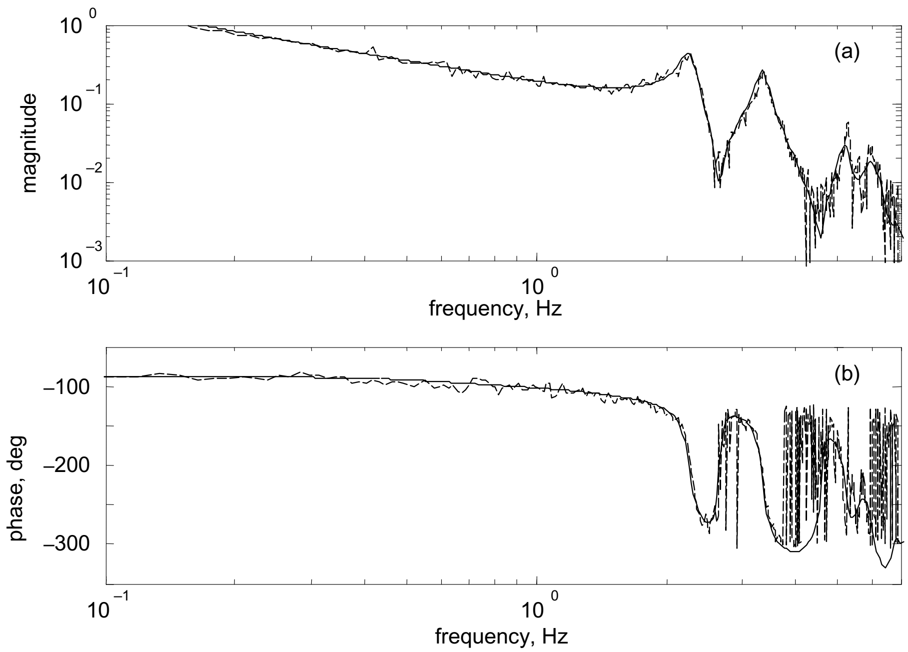

Figure 1.8. The antenna transfer functions obtained from the data (dashed line), and obtained from the identified model (solid line): (a) Magnitude; and (b) phase.

1.1.6 The International Space Station Structure

The Z1 module of the International Space Station structure is a large structure of a cubical shape with a basic truss frame, and with numerous appendages and attachments such as control moment gyros and a cable tray. Its finite-element model is shown in Fig. 1.9. The total mass of the structure is around 14,000 kg. The finite-element model of the structure consists of 11,804 degrees of freedom with 56 modes, of natural frequencies below 70 Hz. This structure was analyzed for the preparation of the modal tests. The determination of the optimal locations of shakers and accelerometers is presented in Chapter 7.

1.2 Definition

The term *flexible structure* or, briefly, *structure* has different interpretations and definitions, depending on source and on application. For the purposes of this book we define a structure as a linear system, which is

- finite-dimensional;
- controllable and observable;
- its poles are complex with small real parts; and
- its poles are nonclustered.

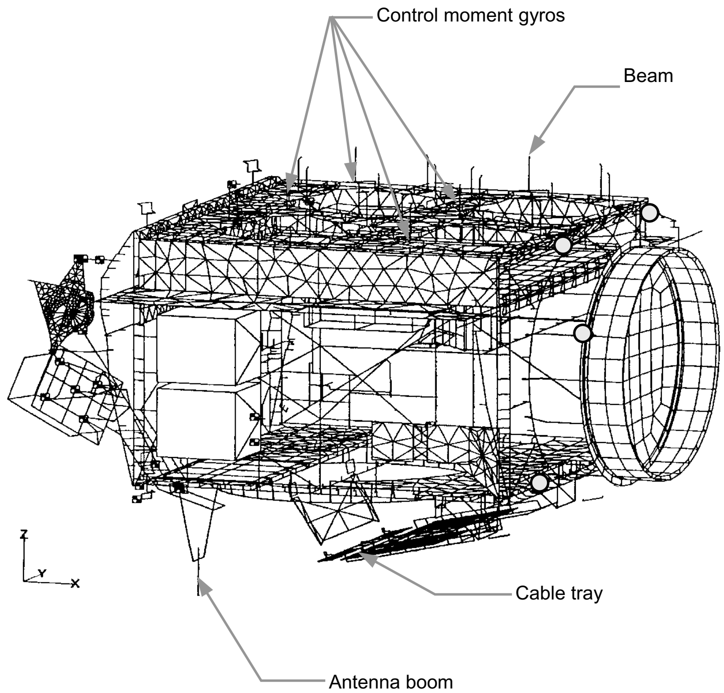

Figure 1.9. The finite-element model of the International Space Station structure.

Based on this definition, we derive many interesting properties of structures and their controllers later in this book.

The above conditions are somehow restrictive, and introduced to justify the mathematical approach used in this book. However, our experience shows that even if these conditions are violated or extended the derived properties still hold. For example, for structures with heavy damping (with larger real parts of complex poles), or with some of the poles close to each other, the analysis results in many cases still apply.

1.3 Properties

In this section we briefly describe the properties of flexible structures. The properties of a typical structure are illustrated in Fig. 1.10.

- Motion of a flexible structure can be described in independent coordinates, called modes. One can excite a single mode without excitation of the remaining ones. Displacement of each point of structure is sinusoidal of fixed frequency. The shape of modal deformation is called a modal shape, or mode. The frequency of modal motion is called natural frequency.
- Poles of a flexible structure are complex conjugate, with small real parts; their locations are shown in Fig. 1.10(a).

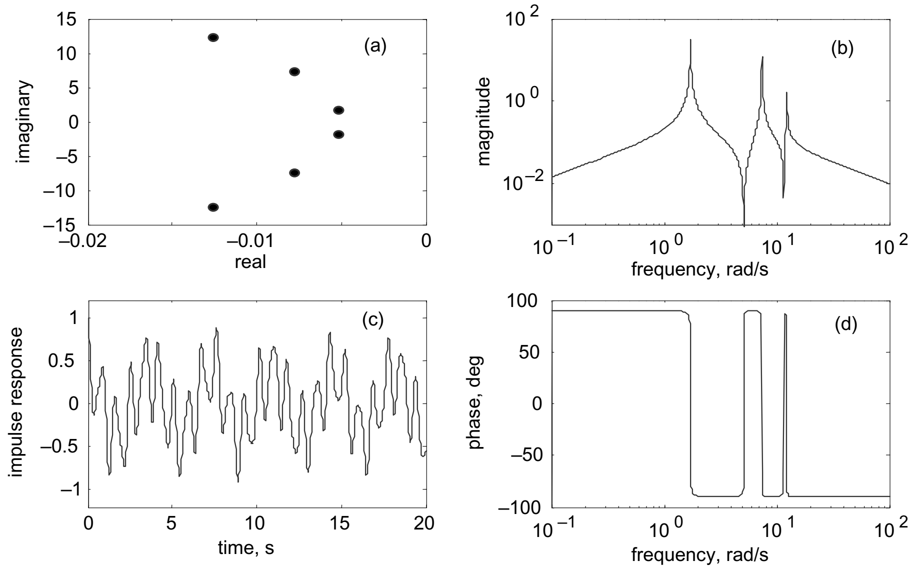

Figure 1.10. Properties of a typical flexible structure: (a) Poles are complex with small real parts; (b) magnitude of a transfer function shows resonant peaks; (c) impulse response is composed of harmonic components; and (d) phase of a transfer function displays 180 deg shifts at resonant frequencies.

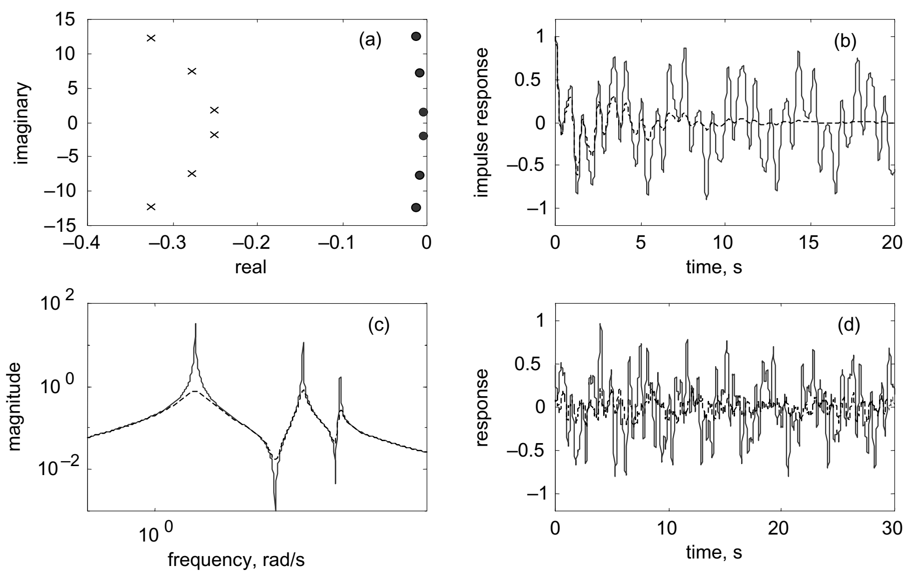

Figure 1.11. Structure response depends strongly on damping: (a) Poles of a structure with small (•) and larger (×) damping – damping impacts the real parts; (b) impulse response for small (solid line) and larger (dashed line) damping – damping impacts the transient time; (c) magnitude of the transfer function for small (solid line) and larger (dashed line) damping – damping impacts the resonance peaks; and (d) response to the white noise input for small (solid line) and larger damping (dashed line) – damping impacts the rms of the response.

- The magnitude of a flexible structure transfer function is characterized by the presence of resonance peaks; see Fig. 1.10(b).
- The impulse response of a flexible structure consists of harmonic components, related to complex poles, or to resonance peaks; this is shown in Fig. 1.10(c).
- The phase of a transfer function of a flexible structure shows 180 degree shifts at natural frequencies, see Fig. 1.10(d).

Poles of a flexible structure are complex conjugate. Each complex conjugate pair represents a structural mode. The real part of a pole represents damping of the mode. The absolute value of the pole represents the natural frequency of the mode.

Consider two different structures, as in Fig. 1.11(a). The first one has poles denoted with black circles (•), the second one with crosses (×). The locations of the poles indicate that they have the same natural frequencies, but different damping. The structure with poles marked with black circles has larger damping than the one with poles marked with the crosses. The figure illustrates that structural response depends greatly on the structural damping. For small damping the impulse response of a structure decays slower than the response for larger damping, see Fig. 1.11(b). Also, the magnitude of the response is visible in the plots of the magnitude of the transfer function in Fig. 1.11(c). For small damping the resonance peak is larger than that for larger damping. Finally, the damping impacts the root-mean-square (rms) of the response to white noise. For example, Fig. 1.11(d) shows that for small damping the rms response of a structure is larger than the response for larger damping.

When a structure is excited by a harmonic force, its response shows maximal amplitude at natural frequencies. This is a resonance phenomenon – a strong amplification of the motion at natural frequency. There are several frequencies that structures resonate at. A structure movement at these frequencies is harmonic, or sinusoidal, and remains at the same pattern of deformation. This pattern is called a mode shape, or mode. The resonance phenomenon leads to an additional property – the independence of each mode. Each mode is excited almost independently, and the total structural response is the sum of modal responses. For example, let a structure be excited by a white noise. Its response is shown in Fig. 1.12(a). Also, let each mode be excited by the same noise. Their responses are shown in Fig. 1.12(b),(c),(d). The spectrum of the structural response is shown in Fig. 1.13(a), and the spectra of responses of each individual mode are shown in Fig. 1.13(b),(c),(d). Comparing Fig1.13a with Fig.1.13b,c,d we see that the resonance peak for each natural frequency is the same, either it was total structure excited, or individual mode excited. This shows that the impact of each mode on each other is negligible.

The independence of the modes also manifests itself in a possibility of exciting each individual mode. One can find a special input configuration that excites a selected mode. For example, for the simple structure presented above we found an excitation that the impulse response has only one harmonic, see Fig. 1.14(a), and the magnitude of the transfer function of the structure shows a single resonance peak, see Fig. 1.14(b). However, there is no such input configuration that is able to excite a single node (or selected point) of a structure. Thus structural modes are independent, while structural nodes are not.

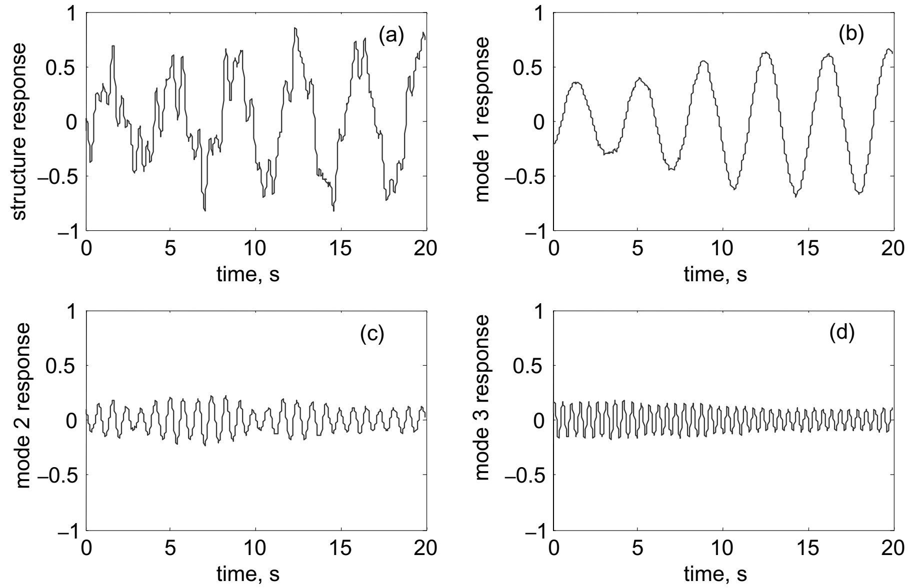

Figure 1.12. Response to the white noise input: (a) Total structure response is composed of three modal responses; (b) mode 1 response of the first natural frequency; (c) mode 2 response of the second natural frequency; and (d) mode 3 response of the third natural frequency.

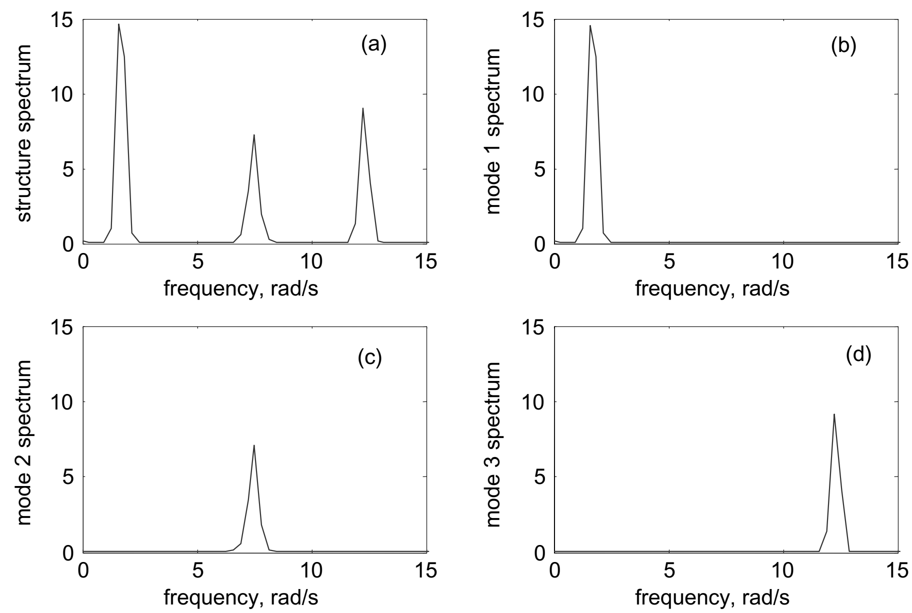

Figure 1.13. Spectra of the response to the white noise input: (a) Total structure spectrum consists of three modal spectra; (b) mode 1 spectrum of the first natural frequency; (c) mode 2 spectrum of the second natural frequency; and (d) mode 3 spectrum of the third natural frequency.

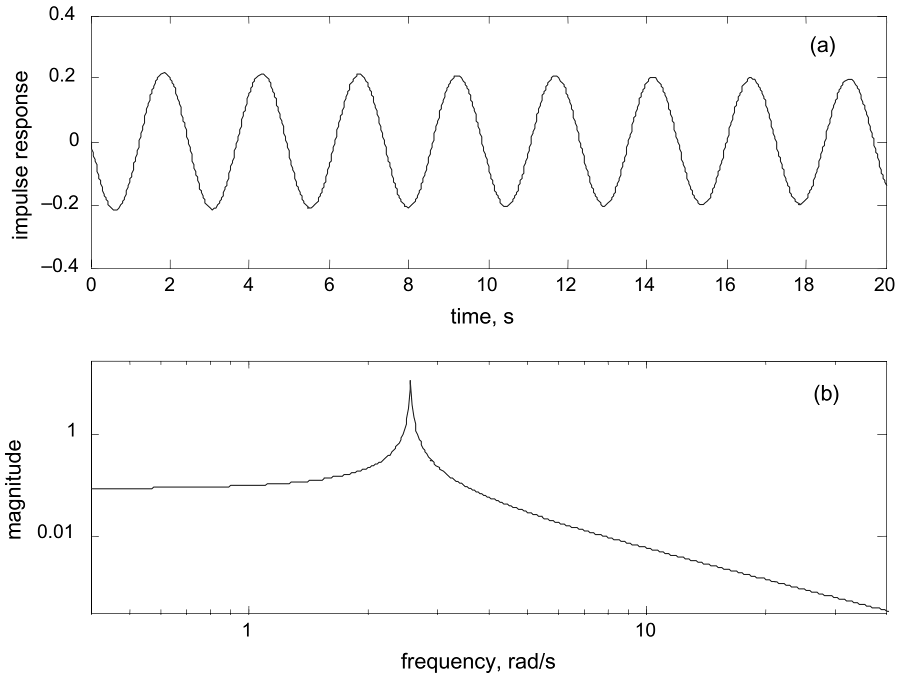

Figure 1.14. An input configuration that excites a single mode: (a) Impulse response; and (b) magnitude of the transfer function.

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