Preface to the First Edition

The prehistory of both the theory and the practicalities of mathematical finance can be traced back quite some time. However, the history proper of mathematical finance – at least, the core of it, the subject-matter of this book – dates essentially from 1973. This year is noted for two developments, one practical, one theoretical. On the practical side, the world's first options exchange opened in Chicago. On the theoretical side, Black and Scholes published their famous paper (Black and Scholes 1973) on option pricing, giving in particular explicit formulae, hedging strategies for replicating contingent claims and the Black-Scholes partial differential equation.

Both Black's article (Black 1989) and the recent obituary of Fischer Black (1938–1995) (Chichilnisky 1996) contain accounts of the difficulties Black and Scholes encountered in trying to get their work published. After several rejections by leading journals, the paper finally appeared in 1973 in the *Journal of Political Economy*. It was alternatively derived and extended later that year by Merton.

Thus, like so many classics, the Black-Scholes and Merton papers were ahead of their time in the economics and financial communities. Their ideas became better assimilated with time, and the Arbitrage Pricing Technique of S.A. Ross was developed by 1976–1978; see (Ross 1976), (Ross 1978). In 1979, the Cox-Ross-Rubinstein treatment by binomial trees (Cox, Ross, and Rubinstein 1979) appeared, allowing an elementary approach showing clearly the basic no-arbitrage argument, which is the basis of the majority of contingent claim pricing models in use. The papers (Harrison and Kreps 1979), (Harrison and Pliska 1981) made the link with the relevant mathematics – martingale theory – explicit. Since then, mathematical finance has developed rapidly – in parallel with the explosive growth in volumes of derivatives traded. Today, the theory is mature, is unchallengeably important, and has been simplified to the extent that, far from being controversial or arcane as in 1973, it is easy enough to be taught to students – of economics and finance, financial engineering, mathematics and statistics – as part of the canon of modern applied mathematics. Its importance was recognized by the award of the Nobel Prize for Economics in 1997 to the two survivors among the three founding pioneers, Myron Scholes and Robert Merton (Nobel prize laudatio 1997).

The core of the subject-matter of mathematical finance concerns questions of pricing – of financial derivatives such as options – and hedging – covering oneself against all eventualities. Pervading all questions of pricing is the concept of arbitrage. Mispricing will be spotted by arbitrageurs, and exploited to extract riskless profit from your mistake, in potentially unlimited quantities. Thus to misprice is to expose oneself to being used as a money-pump by the market. The Black-Scholes theory is the main theoretical tool for pricing of options, and for associated questions of trading strategies for hedging. Now that the theory is well-established, the profit margins on the standard – 'vanilla' – options are so slender that practitioners constantly seek to develop new – nonstandard or 'exotic' – options which might be traded more profitably. And of course, these have to be priced – or one will be used as a money-pump by arbitrageurs ...

The upshot of all this is that, although standard options are well-established nowadays, and are accessible and well understood, practitioners constantly seek new financial products, of ever greater complexity. Faced with this open-ended escalation of the theoretical problems of mathematical finance, there is no substitute for *understanding* what is going on. The gist of this can be put into one sentence: *one should discount everything, and take expected values under an equivalent martingale measure*. Now discounting has been with us for a long time – as long as inflation and other concomitants of capitalism – and makes few mathematical demands beyond compound interest and exponential growth. By contrast, equivalent martingale measures – the terminology is from (Harrison and Pliska 1981), where the concept was first made explicit – make highly non-trivial mathematical demands on the reader, and in consequence present the expositor with a quandary. One can presuppose a mathematical background advanced enough to include measure theory and enough measure-theoretic probability to include martingales – say, to the level covered by the excellent text (Williams 1991). But this is to restrict the subject to a comparative elite, and so fails to address the needs of most practitioners, let alone intending ones. At the other extreme, one can eschew the language of mathematics for that of economics and finance, and hope that by dint of repetition the recipe that eventually emerges will appear natural and well-motivated. Granted a leisurely enough approach, such a strategy is quite viable. However, we prefer to bring the key concepts out into the light of day rather than leave them implicit or unstated. Consequently, we find ourselves committed to using the relevant mathematical language – of measure theory and martingales – explicitly. Now what makes measure theory hard (final year material for good mathematics undergraduates, or postgraduates) is its proofs and its constructions. As these are only a secondary concern here – our primary concern being the relevant concepts, language and viewpoint – we simply take what we need for granted, giving chapter and verse to standard texts, and use it. Always take a pragmatic view in applied mathematics: the proof of the pudding is in the eating.

The phrase ‘equivalent martingale measure’ is hardly the language of choice for practitioners, who think in terms of the *risk-adjusted* or – as we shall call it – *risk-neutral measure*: the key concept of the subject is risk-neutrality. Since this concept runs through the book like a golden thread (*roter Faden*, to use the German), we emphasize it by using it in our title.

One of the distinctive features of mathematical finance is that it is, by its very nature, interdisciplinary. At least at this comparatively early stage of the subject’s development, everyone involved in it – practitioners, students, teachers, researchers – comes to it with his/her own individual profile of experience, knowledge and motivation. For ourselves, we both have a mathematics and statistics background (though the second author is an ex-practitioner), and teach the subject to a mixed audience with a high proportion of practitioners. It is our hope that the balance we strike here between the mathematical and economic/financial sides of the subject will make the book a useful addition to the burgeoning literature in the field. Broadly speaking, most books are principally aimed at those with a background on one *or* the other side. Those aiming at a more mathematically advanced audience, such as the excellent recent texts (Lamberton and Lapeyre 1996) and (Musiela and Rutkowski 1997), typically assume more mathematics than we do – specifically, a prior knowledge of measure theory. Those aiming at a more economic/financial audience, such as the equally excellent books (Cox and Rubinstein 1985) and of (Hull 1999), typically prefer to ‘teach by doing’ and leave the mathematical nub latent rather than explicit. We have aimed for a middle way between these two.

We begin with the background on financial derivatives or contingent claims in Chapter 1, and with the mathematical background in Chapter 2, leading into Chapter 3 on stochastic processes in discrete time. We apply the theory developed here to mathematical finance in discrete time in Chapter 4. The corresponding treatment in continuous time follows in Chapters 5 and 6. The remaining chapters treat incomplete markets and interest rate models.

We are grateful to many people for advice and comments. We thank first our students at Birkbeck College for their patience and interest in the courses from which this book developed, especially Jim Aspinwall and Mark Deacon for many helpful conversations. We are grateful to Tomas Björk and Martin Schweizer for their careful and scholarly suggestions. We thank Jon McLoone from Wolfram UK for the possibility of using Mathematica for numerical experiments. Thanks to Alex Schöne who always patiently and helpfully explained the mysteries of LaTeX to the second author over the years – *ohne Dich, Alex, würde ich immer noch im Handbuch nachschlagen!*

It is a pleasure to thank Dr Susan Hezlet and the staff of Springer-Verlag UK for their support and help throughout this project.

And, last and most, we thank our families for their love, support and forbearance while this book was being written.

London, March 1998 *N.H. Bingham* *Rüdiger Kiesel*

http://www.springer.com/978-1-85233-458-1

Risk-Neutral Valuation
Pricing and Hedging of Financial Derivatives
Bingham, N.H.; Kiesel, R.
2004, XVIII, 438 p., Hardcover
ISBN: 978-1-85233-458-1