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Conservation and Contrariety: The Logical Foundations of Cartesian Physics

Profiteorque mihi nullas rationes satisfacere in ipsa
Physica, nisi quae necessitatem illam, quam vocas
Logicam sive contradictoriam, involvant.
(Descartes to Henry More, Feb. 5, 1649; AT V, 275)

2.1 Introduction

The general theory of matter presented by Descartes in the second book of the *Principia Philosophiae* is the first well founded systematic physical theory of modern science; for it explicitly introduces the logical presuppositions necessary for a system of causal explanations of physical phenomena using equations. While it is true that Descartes himself takes very little advantage of the possibilities created by the introduction of these prerequisites (there is, for instance, very little mathematics, no formal equations, and few proportions in the *Principia* itself), he nonetheless determines basic requirements of such a system of explanations and provides conceptual instruments adequate for the formation of such a physical theory.

These requirements consist: (1) *first and foremost* in the formulation of conservation principles which define the nature of the systems studied, guarantee their identity over time, and put strong constraints on the kinds of assertions that can even be recognized as legitimate attempts at causal explanation.

(2) *Second*, the development of a physical explanation of qualitative and quantitative changes within a system or of the interactions between systems requires a conceptualization of the nature of change and interaction. Descartes conceives physical systems to consist ultimately of particles whose interactions are exclusively collisions. Therefore, he must determine the nature of this fundamental interaction and explain the behavior of particles in collision and in the absence of collision. Descartes explains the latter by a logical principle of inertia and in one-dimensional two-body cases he explains the former by a logical calculus of contrary predicates. These principles guarantee that the asserted causal connections in the material world correspond to the semantic connections of the terms in his conceptual system.

(3) *Third*, in order to apply the theory not merely to interactions in the same line, which are accounted for by the logic of contrary predicates, but also to two- and three-dimensional Euclidean space, a method is needed to reduce all possible interactions (e.g., oblique collisions of real bodies or those of point masses moving in intersecting lines) to the pure forms of direct opposition and lack of opposition. Descartes accomplished this reduction by the dynamical interpretation and application of a geometrical operation, the parallelogram rule for the compounding of motions. In the 17th century this technique was used to give a geometrical solution to a number of problems now solved with the help of the vector calculus.

As will be shown below, the conceptual means developed by Descartes to meet these requirements (1. *conservation*, 2. *interaction*, 3. *reduction*) are consistent with one another, but their application leaves the fundamental interactions of bodies which were to be explained by his physics still underdetermined. As we shall argue, the reason for this underdetermination is that Descartes asserted the conservation of “vector” quantities only for individual bodies (not for the system as a whole) and only in the *absence* of interactions. To achieve complete determinism, Descartes had on occasion to specify further the logic of contrary predicates through a minimum principle as well as implicitly to appeal to considerations of symmetry not explicitly introduced into the system of explanations.

In the following sections we shall discuss the role of conservation laws (2.2) in the foundations of science and their function in Descartes’ system, the role of logic (2.3), especially the logic of contrary predicates in providing Descartes with the conceptual means for dealing with physical interactions, Descartes’ application of the logical principles to the interaction of two particles in the same line (2.4), that is, the derivation of the laws of impact, and Descartes’ use of the parallelogram rule to reduce interactions in two-dimensional space to one-dimensional interactions (2.5).

2.2 Principles of Conservation and Conservation Laws

2.2.1 The Foundational Function of Conservation Laws

By a systematic physical theory, we understand a network of laws, usually formulated as equations, the transformations of which predict and explain the outcome of interactions among the phenomena to which the theory applies. We will argue below that every physical interpretation of such equations as stating causal equivalences presupposes some principles of conservation, and hence that the logical foundations of causal explanations in a deterministic physical theory are principles of conservation for certain physical entities. We shall then show

that Descartes explicitly introduced conservation laws¹ appropriate to the theory he developed: these laws specify the conservation of matter, and of the quantity of motion, which fulfill much the same function as the laws of conservation of mass and energy in classical physics. The role of momentum in classical physics is played by contrariety and “determination” in Cartesian physics.²

Physics, and every science, presupposes the invariance or conservation of some entities, but not only specifically scientific knowledge relies on the conservation of some of its objects: if the invariance in manipulation of some objects could not be presupposed or if their variations were not at least predictable, then regular human activities as we know them, as well as the formation of concepts to describe or explain them, would be impossible. Alice in Wonderland cannot plan her next stroke if the croquet mallet unexpectedly turns into a flamingo and the ball becomes a hedgehog. Playing croquet, as well as forming concepts of the game’s equipment or formulating the rules of the game, is impossible in such a situation. But the conservation laws of physics need not refer to entities identical with the objects of human practice, nor to the time spans relevant to our activities. Moreover, the formation of the concepts of the conserved entities in daily life and in science rests on different processes, which we will briefly discuss below.

In physics, the change of a material system under investigation can be defined only in relation to a determined state of the system considered unchanged over time. Searching for an external cause to explain change presupposes that the system is invariant and would not have changed without interference from outside. Thus in order to have a physical theory of change in a material system, one must already at least implicitly have defined what “staying the same” means in physical terms.³ Hence the quest for a causal explanation of change presupposes not only that a determinate description of a system in different states at different times is possible – and thus that the standards of measurement are invariant over

¹We shall speak of “*principles* of conservation” as the logical or philosophical prerequisites of a physical theory, and of “*conservation laws*” as the specific fulfillment of the requirements in a particular physical theory.

²An extensive discussion of the relation between conservation principles and the principle of causality in mechanics (with special reference to Descartes’ and Leibniz’s contributions) can be found in Wundt 1910, pp. 84-114. The principle of equivalence of cause and effect is the last of the six axioms or hypotheses on which mechanics is founded (according to Wundt). For a discussion of conservation principles with many historical examples, but from a point of view very different from ours, see Meyerson 1962.

³The conceptual link between change and invariance is discussed by Kant under the heading “The Permanence [of Substance]” in the *Critique of Pure Reason* (A182-189). He later explicitly refers to this discussion in his proof of the law of conservation of mass in the *Metaphysical Foundations of Natural Science* (1786). The law states: “*First Law of Mechanics*. In all changes of corporeal nature, the total quantity of matter remains the same, neither increased nor diminished” (*Gesammelte Schriften*, vol. 8, p. 541; Kant 2002, p. 249).

time – but also that the entities in the system which define its identity over time are conserved in the absence of interactions with other systems.

For example, in order to conceive force as the cause of the acceleration of a body and acceleration as a change in its velocity, one needs a conservation principle for the state of motion, i.e., a principle of inertia and a definition of state of motion, e.g., in terms of velocity or speed and rectilinear direction. Force as cause of change in general is considered as external to the system defined by conservation laws. If the cause, too, is considered as a physical entity, then a change in the system under observation implies an interaction with another system, and if the interaction is to be expressed in a causal law, then there must be a complementary change in the other system. A causal law in physics which expresses (in an equation) the possibility of transforming a system through interaction with another, implies a qualitative and quantitative invariance in such a transformation, i.e., that a change in one system will have a constant qualitative and quantitative relation to the change in the other. If both interacting systems are taken together as one, then their combination will be a closed system to which the conservation laws for the magnitudes defining the system apply.

Insofar as a physical law is expressed in an equation it fulfills the philosophical requirement that the cause should be equal to the effect (*causa aequat effectum*). However, the fact that two concrete systems are causally equivalent, in respect to a particular effect based on certain physical magnitudes, does not mean that they are causally equivalent in respect to every possible effect they may produce on the basis of these magnitudes.

The insight into this possibility on the example of the motions of bodies, which can be quantitatively equivalent in different manners according to different effects (measured by mv and mv^2), led Leibniz to distinguish between two kinds of equivalence and between two kinds of measurement: measurement by *congruence* and measurement by *equipollence*.⁴ “Congruence” denotes the simplest kind of measurement, in which a standard is iterated and compared with the object measured, as a yardstick is used to measure the length of an object. In measurement by congruence, two systems are compared with respect to some property possessed by both, and the quantitative relations of the two in this regard are determined. As the name suggests, this type of measurement is originally derived from the *geometrical* properties of systems, where the additivity of the physical magnitudes consists in spatial juxtaposition of parts (i.e., to what were traditionally called “extensive” quantities). Leibniz also considered *counting* other standardized units to be measurement by congruence, so that a body (i.e., a standard

⁴“Equipollence” is a synonym for equivalence used in logic at least up to the time of Carnap. Leibniz gives it the particular technical meaning that we have adopted in a letter to l’Hopital (Jan. 15, 1696). Leibniz, GM II, 305-307, translated in document 5.2.1. See also GP IV, 370-72; and in general “Initia rerum mathematicarum metaphysica,” GM VII, 17-29. For a discussion of the development of Leibniz’s views on equipollence and a collection of sources, see Fichant’s commentary in Leibniz 1994, pp. 277–302.

unit of mass) or a spring (i.e., a standard unit of force) can be iterated, and the number of such units compared.

Equipollence, on the other hand, was also used to measure intensive qualities such as velocity and force.⁵ Here a phenomenon is measured by transforming it (or a representative sample of it) causally into an extensive effect, which can then be measured by congruence and counting. Leibniz measured the force (*vis viva*) of a body in motion by letting it ascend in the gravitational field of the Earth and measuring (by congruence) the height it could reach or (by counting) the number of standard (congruent) springs it could compress.

Equivalence or equality in the sense of congruence is hence given when the left- and right-hand sides of the equation are expressed in terms of the same physical magnitudes. If, however, the two phenomena to be equated (or compared by measurement) are not or cannot be expressed by means of the same physical magnitudes, they can only be equated as *cause and effect*. In this case they can be said to be equipollent or causally equivalent. The equipollence of two phenomena means that both can be produced by the same causes or that both can produce the same effects. The equation of *work* and *heat*, for instance, expresses an equipollence.

It is of course possible that an equation expressing a causal equivalence seems to be a simple congruence, i.e., if the cause and effect happen to be expressed in the same magnitudes. This form of representation disguises the fact that considerations of equipollence are ultimately the basis of the definition of the magnitudes. Moreover, even when an equation actually displays an equipollence, the *invariance* in transformation that it expresses can be interpreted as the conservation of a new entity and a new concept can be introduced for it. The concept does not refer to a new empirical phenomenon but to what is invariant in the transformation of phenomena, and the original phenomena can now be conceived as instances or embodiments of the same entity, e.g., energy, which is not an observable phenomenon or even a definable entity, but is identified in and distinguished from many specific phenomena, e.g., heat, work, electricity, etc.⁶

⁵Traditionally, velocity was the most important intensive magnitude; and as late as Kant *mass* was also sometimes taken as an intensive magnitude.

⁶“The law of energy directs us to coordinate every member of a manifold with one and only one member of any other manifold, in so far as to any *quantum* of motion there corresponds one *quantum* of heat, to any *quantum* of electricity, one *quantum* of chemical attraction, etc. In the concept of *work*, all these determinations of magnitude are related to a common denominator. If such a connection is once established, then every numerical difference that we find within one series can be completely expressed and reproduced in the appropriate values of any other series. The unit of comparison, which we take as a basis, can arbitrarily vary without the result being affected. If two elements of any field are equal when the same amount of work corresponds to them in any series of physical qualities, then this equality must be maintained, even when we go over to any other series for the purpose of their numerical comparison.” (Cassirer 1923, p. 191) Cassirer calls the one-to-one correspondence of values in different series “equivalence” (p. 197).

It should be stressed that, in spite of the operational simplicity of congruence, the conceptually really fundamental form of measurement for physical explanation is equipollence. For, physical equivalence refers essentially to the possibility of intersubstitution of the entities equated. Substitution is of course relative to various purposes, and various forms of equivalence can be imagined; but at least for scientific explanation causal equivalence is decisive. Congruence can have the semblance of being *a priori*, since it is obvious that $5x = 2x + 3x$. But the concept of the magnitude whose units are called *x*'s, and the physical meaning of equivalence and addition, is ultimately determined by equipollence.

Our discussion of the concepts equivalence and equipollence suggests that the scientific manipulation of physical entities and the transformation of propositions about concepts referring to them, are the basis of the formation of concepts of further physical entities which do not directly refer to objects of daily experience but to invariances stated in controlled manipulations of physical objects in experiment and in transformations of propositions about previous scientific concepts. If this approach is applied to concept formation in general, then on each level invariance in manipulations of objects and in transformations of propositions precedes and is prior to the notion of identity and equivalence on the next higher level.⁷

These considerations on causal equivalence and conservation imply two relevant consequences as to the relation of the principle of causality to conservation laws in science. Scientific conservation laws cannot be derived from a philosophical principle like *causa aequat effectum* or *ex nihilo nihil fit*, since the actual scientific problem is to determine *what* remains invariant in transformations supposing that something is conserved.⁸ On the other hand, it is the conservation laws in a given physical theory that determine the specific meaning of the principle "cause equals effect" by defining what counts as a cause and an effect and how their measures are to be expressed. These laws guarantee the observance of the causal principle in that any candidate for the status of natural law must conform to them, if the whole framework of the discipline is not to be reworked.⁹

For the philosophical foundations of science it is not so important to ask what particular magnitudes are determined as invariant but rather *whether* they fulfill the requirements mentioned; the conservation laws may change and do change with the development of science. But as long as science states causal laws in

⁷We have intimated that the formation of concepts for objects that are invariant in our manipulation of them is similar in structure to the formation of concepts of scientific entities and conservation laws. We cannot, however, further discuss the extent and significance of the similarity between these two processes of concept formation.

⁸Robert Mayer, for instance, argued that the conservation of energy follows from this philosophical principle. See Freudenthal 1983.

⁹As E. P. Wigner maintained in his Nobel Prize Lecture of 1963, the most important function of invariance principles is "to be touchstones for the validity of possible laws of nature. A law of nature can be accepted as valid only if the correlations which it postulates are consistent with the accepted invariance principles" (Wigner 1967, p. 46).

equations, the abandoning of one set of conservation laws will necessarily lead to an attempt to formulate others because of their foundational role in scientific theory.¹⁰

The fact that conservation laws define an isolated system means that they define the system in reference to which causal laws apply. Applying conservation laws to the Universe as a whole expresses the claim that the applicability of causal laws is universal, i.e., that a change in any particular material system is to be explained by its interaction with other material systems and that no other causes are admissible. This claim was of paramount importance in the 17th century for the establishment of a scientific world view in opposition to the feudal-religious view dominant at the time, which allowed a transcendent immaterial deity to act physically in the material world.¹¹

2.2.2 Descartes' Conservation Laws

Before we turn to Descartes' physics and examine whether the conservation laws he specifies can, in principle, serve as the foundation of a deterministic quantitative physical theory, we must first determine what *kinds* of conservation laws are necessary and which *particular* laws proved historically to be sufficient for this purpose.

In classical mechanics, in order to provide a deterministic description of a system of elastic particles in motion which act on each other by *impact only*, one needs conservation laws for mass, momentum, and kinetic energy as well as information on the positions and velocities of all particles in one instant. On the basis of this knowledge, Laplace's demon could calculate the evolution of the system and pinpoint the state of each and every particle at any given future time. There are two significant differences between the concepts of kinetic energy and

¹⁰“We can put the question: what would be changed in physics if a *perpetuum mobile* were to be discovered today? Our conviction of the universal subjection of nature to law would not be shaken ... the validity of the law of the conservation of energy would be restricted to certain limits, and perhaps we could hope to recognize it ultimately as a special case of a still more general law” (von Weizsäcker 1952, p. 64-5).

¹¹This does not mean, however, that any particular scientist had to realize that science demands a scientific world view nor that he subscribed to such a view. But even scientists who did not conceive of the universe as a closed system determined by conservation laws had nonetheless to presuppose (in practice) a constant quantity of the relevant magnitudes. In such a case, only inconsistency between the philosophy espoused and the science actually practiced allowed science to be pursued on the basis of unscientific presuppositions. Newton provides a good example of this state of affairs; see Freudenthal 1986, pp. 44-76. The politically and ideologically motivated attempt to construe the universe as a closed system, the states of which can be related only to physical entities, in conjunction with the attempt similarly to construe society was, according to Lefèvre (1978, pp. 45-79), an important factor in the construction of modern science.

momentum. On the one hand, their measures are different: $\frac{1}{2}mv^2$ for kinetic energy and mv for momentum. On the other hand, kinetic energy is a *scalar* magnitude, while momentum is a *vector*, i.e., for our purposes, a magnitude with a determined direction. Mass, too, is scalar in classical physics.

Assuming that all interactions within a system can be reduced to two-body impacts, the information needed to determine the state of a system consists in the positions, masses, and velocities of the particles. Assuming, furthermore, the conservation of mass in each particle in interaction, the state of any given particle depends on its velocity. If we want to determine the velocities of *two* particles after collision, we need (for purely formal reasons) at least two independent equations. Furthermore, at least one of these (or a new equation) must guarantee the invariance of the system as a whole, i.e., that whatever the changes within the system, the magnitudes defining it remain the same. And at least one of the equations must be able to deal quantitatively with directions. Thus the *minimal* requirements for a deterministic idealized impact theory of elastic particles (assuming the conservation of the mass of each particle) are two equations, of which at least one defines the system as closed and at least one operates with a vector magnitude or some other entity capable of expressing direction. In classical mechanics these are conservation laws for kinetic energy (scalar) and for momentum (vector).

The difference in status between scalar and vector conservation laws in classical mechanics and the philosophical relevance of this difference has, to our knowledge, never been seriously studied, and we cannot go into detail on the differences.¹² However, we should point out that the attribution of a certain (vector) momentum to a system *of given mass* entails the attribution of a particular amount of (scalar) energy but not vice versa since (directionally) different momenta are compatible with the same amount of kinetic energy. In the 17th century, the system was *defined* by the “real and positive” (i.e., scalar) magnitudes not by those that are “merely modal” (e.g., directional), even though the latter are necessary for calculating the trajectories of colliding particles.

According to Descartes, material reality consists in “extended substance” (matter) and its “modes” (shape and motion or rest) (II, §25).¹³ Descartes con-

¹²Max Planck began his book, *Das Prinzip der Erhaltung der Energie* (1913, p. 1) by isolating two conservation laws as somehow more fundamental than other laws: “There are two propositions which serve as the foundation of the current edifice of exact natural sciences: the principle of the conservation of matter and the principle of the conservation of energy. They maintain undeniable precedence over all other laws of physics however comprehensive; for even the great Newtonian axioms, the law of inertia, the proportionality of force and acceleration, and the equality of action and reaction apply only to a special part of physics: mechanics – for which, moreover, under certain presuppositions to be discussed later they can all be derived from the principle of conservation of energy.” Planck seems to mean that the fundamental character of the conservation of energy and matter has to do with their being scalar magnitudes and thus not limited to mechanics.

¹³Numbers in parentheses refer to part and paragraph of the *Principia philosophiae* (AT VIII); a translation of bk. II, §§36-53 is given in document 5.2.2.

ceived of substance in traditional terms as something that can exist of itself (*quae per se apta est existere*);¹⁴ this definition applies strictly only to God, and all created substances depend for their existence on God's concurrence (I, §51). The conservation of matter seems to be presupposed by Descartes as evident, i.e., as immediately following from its being a substance. He stresses the conservation of the *modes* of matter and mentions the conservation of matter only in passing (II, §36). However, the proposition that no matter is created or annihilated by natural causes was hardly in dispute. Since Descartes has defined matter as the extended in three dimensions, the quantity of matter is determined by volume (II, §4). Although this measure is different from the measure of mass in classical physics, it is nevertheless obvious that both concepts refer to the same experiences with material objects and serve comparable functions in the respective theories, namely to account for the resistance of a body to a change in its state of motion.

The conservation of motion on the other hand, a scalar magnitude measured by *lmv*, is explicitly stated and argued by Descartes both for the world as a whole and, in the absence of interactions, for every single body. The conservation of the total quantity of motion and rest (as modes of matter) in the world, is introduced after the fundamental concepts of the theory of matter have been developed but before the laws of nature are discussed. Since Descartes attempts to ground his physics on principles whose validity is independent of the particular theory they are introduced to ground, he cannot justify these principles by appealing to the success of their instantiations. The origin and conservation of both matter and motion are logically prior to the physical theory, and therefore the arguments that Descartes adduces in support of conservation principles are not physical. As presuppositions of physical theory they need a qualitatively different kind of justification. The justification that Descartes actually gives is that the *constancy* and *immutability* of God demands that he conserve unchanged what he originally created, and that it is "most consonant with reason" that the creator be constant and immutable (II, §36). The fact that the "different kind" of argument takes on a theological form is not important here; for the subsequent analysis it is however important that the argument rests neither on any physical proposition, nor on logical or mathematical necessity.

The further specification, that the quantity of motion for *every single body* is conserved in the absence of interaction with others, is introduced as the *First Law of Nature* and is dependent on a logical principle (to be discussed in section 2.3.4 below). The law states that every thing, insofar as it is *simple*, will persist in its state, i.e., conserve its modes, if not acted on from without. Thus it will retain its shape (and presumably its volume) and its state of motion or rest. That is, retaining properties needs no explanation, changing or losing properties needs an explanation: a moving body does not come to rest of its own accord, "for rest is

¹⁴AT VII, 40 (*Meditations*, III).

the contrary of motion and nothing moves by virtue of its own nature towards its contrary or its own destruction" (II, §37).

The *Second Law of Nature* further specifies the preservation of the state of motion. Each body in motion is determined to continue its own motion in a straight line, since motion is conserved exactly as it is in an instant without considering previous instants.¹⁵ The magnitude involved here is called *determinatio*, and its conservation is asserted only for a *single* body in motion. Descartes does not maintain that determination is conserved in a body during or after interactions nor that the sum of the determinations of the interacting bodies after collision stands in any determinate quantitative relation to the sum before collision. Determination, as he elsewhere makes clear,¹⁶ has a direction *and* a magnitude both of which are conserved only in the absence of interactions. Its measure turns out to be matter times directed motion (mv taken as a vector).

It is easily seen that the consistency of a theory containing the concepts quantity of motion (lmv), which is added arithmetically, and determination (mv), which is added vectorially, is going to be problematical any time two determinations which do not coincide in direction are added together. The length of the diagonal of a parallelogram is always shorter than the sum of the lengths of two adjacent sides except in the limit where the angle is zero ($\cos a = 1$). Thus it would seem that the use of the parallelogram rule, which as we have indicated is one of the essential elements of Descartes' system, must constantly lead to internal contradictions. However, as we shall show below, the two measures are only inconsistent under certain assumptions which hold in classical physics but do not hold in Descartes' theory. Specifically, it is not assumed in Descartes' theory that the two magnitudes, motion and determination, are independent, in fact this is explicitly denied: motion (lmv) is a mode of matter, determination (mv) is a mode of this mode. We shall show in section 2.5 that when interpreted within Descartes' conceptual system, there is no contradiction. The vectorial composition and resolution of *motions* is not permitted, and neither the (vectorial) composition and resolution of determinations nor the treatment of the oblique impact of two or three bodies results in any inconsistency – when carried out according to the rules Descartes presents.

¹⁵*Principia*, II, §39. It should be noted, however, that Descartes' concept of "speed" (*celeritas*, *velocitas*) does not refer to an instantaneous quantity but rather denotes the space traversed in a *finite* time. Determination, on the other hand, is introduced explicitly as an instantaneous magnitude: "...that each part of matter, considered in itself, always tends to continue moving, not in any oblique lines but only in straight lines ... For [God] always conserves it precisely as it is at the very moment when he conserves it, without taking any account of the motion which was occurring a little while earlier. It is true that no motion takes place in an instant; but it is manifest that everything that moves is *determined* in the individual instants which can be specified as it moves, to continue its motion in a given direction along a straight line, and never along a curved line" (emphasis added). On the development and the systematic consequences of Descartes' concept of speed, see Chap. 1, above.

¹⁶In the *Dioptrics* and in a series of letters for Fermat and Hobbes; section 2.5 of this chapter discusses the concept of determination in great detail.

This is not to say, however, that Descartes' theory is analogous to classical mechanics or that it differs from it only in the measure of the scalar magnitude: motion as opposed to kinetic energy. On the contrary, as indicated already in the statement that determinations may not be considered as *independent* magnitudes like momenta (in fact only the directional aspect of determination is independent while its magnitude depends directly on and changes with the quantity of motion), his theory differs from classical mechanics in many significant respects. This will become even more clear when we examine Descartes' rules of impact. Our comparison of the conservation laws of classical mechanics with those of Descartes serves only the purpose of examining whether Descartes' theory possesses the minimum of conservation laws necessary for a deterministic impact theory of the behavior of elastic particles that expresses causal relations as equations.

In conclusion we can say that Descartes recognized the importance of conservation principles for the foundations of natural science and that he also saw that the principles must apply both to scalar and directed magnitudes. Furthermore, the conservation laws and the physical theory erected on them are internally consistent. However, as we shall show in section 2.4, his theory remains underdetermined and is unable unequivocally to predict and explain the outcome of some two body interactions, since the directed magnitude conserved is not independent and is conserved only in the absence of interactions.

2.3 The Logic of Contraries and its Application in Physics

Although Descartes, as is well known, often spoke disparagingly about logic, what he rejected was not logic as such but what he considered an unproductive logic, i.e., a logic that merely administered the known instead of discovering the new. As an alternative to unproductive syllogistics he sought a productive logic that could function as an *ars inveniendi*. In the letter to his translator that served as a preface to the French edition of the *Principia*, he advised that one "must study that logic which teaches how to use one's reason correctly in order to discover the truths of which one is ignorant,"¹⁷ and in the letter to Henry More quoted in the motto to this chapter he states unequivocally that he is looking in physics for truths as necessary as those in logic.

If the standard by which logic is to be judged is the ability to produce or guide the production of new knowledge about the world, it is not surprising that Aristotelian syllogistics or even immediate inference according to the law of non-contradiction are found wanting. Even Aristotelian physics did not base physical explanation on this kind of logic. The basic explanatory principle for motions in

¹⁷AT IX, 13-14.

Aristotelian physics was not the exclusion of contradiction but the opposition of logical contraries. Both in Aristotle and in the Aristotelian physics codified in Scholasticism the central principle of explanation was the actualization of potential in motion from one contrary to the other. Mutation between the contradictory affirmation and negation was confined to generation and corruption, while motion (in the general sense of change) was derived from contrariety.¹⁸ We shall see that a logic of contraries also forms the basis of Cartesian physics as well as that of the systems of his contemporaries, whether mechanistic or scholastic.

In this section we shall first of all review some basic characteristics of logic in the works of Aristotle (2.3.1) and systematically explicate the logic of contraries (2.3.2) in order to be able to explain Descartes' program. Then, we shall briefly sketch the application of this kind of logic in one part of Aristotle's physics (2.3.3). Finally, we shall illustrate the prevalence of the logic of contraries in the 17th century (2.3.4), before we present Descartes' use of this logic to derive the basic categories of his physical system (2.3.5).

2.3.1 Contrariety and Contradiction

The law of noncontradiction is the fundamental principle of logical inference, although it itself, as we shall argue, is based on a more fundamental semantic principle of the exclusion of inconsistency. We shall claim (i) that in Aristotle the principle of noncontradiction is based on the principle of inconsistency of contrary predicates and (ii) that this contrariety is in turn based on an ontology of real opposition or real incompatibility of properties of things.

The classical formulation of the law of noncontradiction is to be found in Aristotle's *Metaphysics*, where three versions of the principle are presented, which later philosophers have classified as "logical," "ontological," and "psychological," although it seems that Aristotle did not distinguish rigorously among them.¹⁹

[The *logical* formulation:] The most certain of all [principles] is that contradictory statements are not true at the same time (1011b13f).

[The *ontological* formulation:] It is impossible that the same thing should both belong and not belong to the same thing at the same time and in the same respect (1005b19f).

[The *psychological* formulation:] No one can believe that the same thing can [at the same time] be and not be (1005b23f).

¹⁸See Aristotle's *Physics* V, 1, 225a34ff and the commentaries of Aquinas (1963, §670) and Scotus (*Opera*, 1891, vol. 2, p. 326).

¹⁹Our discussion of Aristotle relies heavily on H. Maier, *Die Syllogistik des Aristoteles* and J. Lukasiewicz, "Aristotle on the Law of Contradiction." For a contemporary grounding of the principle of noncontradiction on the more general principle of inconsistency, see Strawson 1952.

Aristotle believed that the logical principle could be shown to be valid because it expresses the ontological: “To say of what is that it is and of what is not that it is not, is true” (1011b26f).²⁰ The psychological principle he reduced immediately to the ontological: Since belief in something is a property of the believer, if someone were to believe two inconsistent statements, he would have two contrary properties at the same time, which is (ontologically) impossible (1005b16-32).

While the psychological version bases the relation of contradiction directly on that of contrariety and thus on real incompatibility, the logical version does so only *indirectly*. Aristotle presents what can best be called a *semantic* argument for the validity of the law of noncontradiction: it is argued that the law is the prerequisite for meaningful discourse. A predicate that signifies something determinate or conveys information not only includes some things it also excludes some things. If I maintain that “X is a man” is true, then I say at the same time at least that he is not, for instance, a trireme and that statements like “X is not a man” or “X is a non-man” are false.

However, a negative statement cannot be warranted on the basis of direct experience alone but also depends on an inference. We cannot experience what is not the case, but only what is the case. From the fact that a subject possesses a property incompatible with the one we are interested in, we infer that it “lacks” the latter; and since “privation is the denial of a predicate to a determinate genus” (1011b19), we infer from the truth of the ascription of the privation the falsehood of the ascription of the positive property.²¹ Thus a contradiction between two statements about empirical matters of fact presupposes that we have two

²⁰See also *Categories*, Chap.10, 12b16f: “Nor is what underlies affirmation or negation itself an affirmation or negation. For an affirmation is an affirmative statement and a negation a negative statement whereas none of the things underlying an affirmation or negation is a statement ... For in the way an affirmation is opposed to a negation, for example, ‘he is sitting’ – ‘he is not sitting’, so are opposed the actual things underlying each, his sitting – his not sitting.”

²¹We believe that Lukasiewicz has the same argument in mind when he writes that “it is in general impossible to suppose that we might meet with a contradiction in perception; for negation, which is part of any contradiction, is not perceptible. Really existing contradictions could only be inferred” (Lukasiewicz 1979, §19b). The most prominent opponent of this point of view is Geach (1972, p. 79), who remarks: “What positive predication, we might well ask, justifies us in saying that pure water has no taste? Again, when I say there is no beer in an empty bottle, this is not because I know that the bottle is full of air, which is incompatible with its containing beer.” While it is clear that one need not know what else is in the bottle in order to state that there is no beer in it, positive knowledge is nonetheless necessary to justify this statement. One cannot simply see or feel that there is or isn’t beer in the bottle. *Mistaken* judgments which involve no optical illusions illustrate this clearly; and many of the standard tricks of magicians depend precisely on such mistaken inferences (e.g., that the hat is empty or that there is nothing up his sleeve). Although we may readily admit that a gestalt can be built up over time so that we begin to *perceive* immediately what we originally had to infer (e.g., the absence of something), this psychological process does not effect the inferential justification of the statement.

positive statements ascribing incompatible predicates to the same subject, and that we then transform one of them into the formal negation of the other.²²

Although knowledge about the incompatibility of properties is empirical, this does not mean that this knowledge depends each time on actual empirical experience. Aristotelian logic is a logic of natural language, and natural language embodies at any given time a great deal of knowledge about the incompatible properties of things; in fact, the meaning of many words is learned just by identifying them as the contraries of others. Their meanings thus depend on the meaning of the name of the genus, the meaning of the term for the property and the meaning of the relation of contrariety. Hence the application of the principle of the inconsistency of contrary predications depends on the knowledge conveyed by language of what properties are incompatible. This is the basis of Aristotle's opinion that to know a predicate is to know its genus and its contrary (see 1004a9; 105b33; 109b17; 155b30; 163a2).

To sum up: at any given time the concepts in a language incorporate a great deal of empirical knowledge about those properties of objects that are incompatible with each other.²³ Any application of the principle of noncontradiction to empirical statements depends on this knowledge about contraries.²⁴

²²This enables Aristotle then to turn around and draw inferences about the state of the world from the consistency of language: "If then it is impossible to affirm and deny truly at the same time, it is also impossible that contraries should belong to a subject at the same time ..." (1011b13f).

²³See the following observations of Sigwart: "Which representations are incompatible cannot be derived from a general rule, but rather is given with the factual nature of the content of the representations and their relations to one another. We can conceive our sense of sight as so constituted that we could see the same surface shine in different colors, as it in fact emits light with different refractibility, just as we hear different overtones in a single tone and distinguish different tones in a chord; it is purely factual that the colors are incompatible as predicates of the same visible surface, but different tones as predicates of the same source of sound are not, no more than sensations of pressure and temperature, which can be ascribed to the same subject in very different combinations (cold and hard, cold and soft, etc.)" (Sigwart 1904, vol. I, p. 179).

²⁴The relation between the real incompatibility of properties and the logical inconsistency of propositions attributing them to the same subject caused considerable difficulties in the early development of Logical Empiricism. The problem arose, why not all "elementary" or "atomic" propositions are compatible with one another. For instance, Ludwig Wittgenstein in his *Tractatus logico-philosophicus* (6.3751) wrote: "For example, the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour." In his "Some Remarks on Logical Form" (1929), Wittgenstein then introduced a distinction between contrariety and contradiction in his own terminology: "I here deliberately say 'exclude' and not 'contradict,' for there is a difference between these two notions, and atomic propositions, although they cannot contradict, may exclude each other" (p. 35). The process of exclusion is then described: "The propositions 'Brown now sits in this chair' and 'Jones now sits in this chair' each, in a sense, try to set their subject term on the chair. But the logical product of these propositions will put them both there at once, and this leads to a col-

2.3.2 The Logic of Contraries

There are two aspects of Descartes' dissatisfaction with the use of traditional logic in science which we can distinguish analytically without maintaining that Descartes himself clearly distinguished between them. The first aspect is the inadequacy of traditional syllogistic logic to provide the *formal* techniques of inference necessary for modern science. This is connected to the introduction into science of more powerful mathematical techniques of inference which were able to expand the scope of scientific knowledge. The second aspect is the search for a logical calculus of *material* propositions, for a source of logically necessary statements which nonetheless have empirical content. This is a basic characteristic of the rationalist program from Descartes to Kant.

Descartes' alternative to syllogistic inferences and immediate inference based on the law of noncontradiction can be seen as the attempt to solve both the above mentioned problems through a logical calculus of exhaustively contrary predicates and through a calculus of contraries with quantifiable intermediates.

Logically *contrary* predicates are used to designate mutually incompatible physical or moral properties, i.e., properties that cannot be possessed and thus not be ascribed to the same subject in the same regard at the same time, such as hot and cold, just and unjust, red and green. In order to be logically incompatible such predicates must however share certain common features, they must be mutually exclusive species of a common genus, or as contemporary logicians put it, they must belong to the same "range of incompatibility": only colored things, such as apples but not symphonies, can be red or green; gasses but not molecules can be hot or cold, competent adults but not infants can be just or unjust.

By *exhaustively* contrary predicates we mean two predicates which are incompatible and which together exhaust the possibilities of a given range of incompatibility, such as odd and even.²⁵ These can be distinguished from another kind of contrary predicates: those predicates which have *intermediates*, i.e., which con-

lision, a mutual exclusion of these terms" (p. 36; emphasis added). Wittgenstein's "collision" and "exclusion" correspond to the "struggle" and "expulsion" adduced by most scholastic logicians up to the 17th century to explain the incompatibility of contraries (see the quotation from Toletus in section 2.3.4 below). The difference between Wittgenstein's position and that of a scholastic logician like Toletus lies of course in Toletus' insight that the relevant struggle takes place between physical entities, not between propositions, and that their incompatibility is an empirical fact and therefore stated on the basis of empirical knowledge. For an analysis of the role of this problem in Wittgenstein's eventual rejection of the *Tractatus*, see Allaire 1966.

²⁵Such predicates have often been called *contradictory* predicates. This can lead to confusion since, strictly speaking, terms cannot be contradictory; only statements or propositions can be contradictory. Sigwart (1904, pp. 23-25) deals with some of the problems associated with the use of such predicates; and Wundt (1906, vol. II, pp. 62f. and 80f.) gives examples of their use in empirical sciences.

stitute the extremes of a continuous scale, such as the temperatures between hot and cold.

However, any range of incompatibility can be divided into two areas, any genus divided into two species by a simple and purely formal process of negation. Red and nonred exhaust the colors, Europeans and non-Europeans exhaust *homo sapiens*, etc. Such “non”-predicates formed by negating a term were called infinite or indefinite terms in traditional logic. What Descartes is looking for, however, is not a definite and an indefinite term which exhaust a range of incompatibility, but for two definite predicates which either exhaust the possibilities or define the extremes of quantifiable intermediates. These two predicates should be independently determined, i.e., neither should be generated solely by the negation of the other species within the genus or defined primarily by this relation.

In contrast to contrariety, a contradiction is a special kind of logical incompatibility characterized since Aristotle by the opposition of affirmation and negation. Its importance lies in the automatic applicability of the so-called law of the excluded middle or law of the excluded third and hence in the possibility of obtaining a true statement by negating a false one. This also holds for the negation of one of two exhaustively contrary predicates, if their common genus is necessarily predicated of the subject. If two statements contradict each other in the strict sense, then both of them cannot be true at the same time, but since both of them also cannot be false, one of them *must* be true. From the falsity of the one statement one can infer the truth of the other and vice versa, thus making a two-valued calculus of propositions possible. However, the law of noncontradiction cannot guarantee that *both* statements have determinate meanings. This is because the negation involved in contradiction is indeterminate; we learn nothing in particular about anything by this kind of negation, which tells us only that a given predicate, whose meaning we understand, cannot be ascribed to a particular subject (or that an entity’s membership in one particular class does *not* imply its membership in another particular class) but says nothing else. For instance, “X is not red” tells us neither what if any color X has, nor even whether X is visible; the sentence is true of salt, prime numbers, and moral virtues among other things, and in almost every imaginable context says nothing determinate about them.

In contrast, a calculus of contrary predicates would, if successful, allow the inference of statements with some determinate meaning from the negation of statements with determinate meanings. Even as a purely formal calculus, the contrary of a predicate is at least as determinate as the genus or range of incompatibility common to them both. Furthermore, whether two exhaustively contrary predicates both have meanings more determinate than that of the common genus depends on the semantics of the conceptual system to which they belong. To take a simple example: in an arctic environment being white means to blend into the background of snow and ice; being nonwhite, i.e., colored, means to contrast with the background. Thus, whatever the *linguistic form* of the two predicates white and nonwhite (colored), they would both have a quite determinate meaning, we suppose, in the conceptual system of arctic inhabitants. Conse-

quently, knowing it to be false that a thing of a particular kind is white tells us something quite determinate about it, namely, that it contrasts with the background.²⁶ Thus, the calculus of contraries provides a formal technique for drawing inferences which always have as much material content as the common genus of the predicates and can under certain circumstances have as much content as the negated species: natural numbers are even or odd, motion in a vertical line is upwards or downward, etc.

It is evident that the ideal case combining both certitude and informativity would be possible if a genus comprising two and only two positively determined species were necessarily predicated of an object, such that the negation of the statement attributing one of them to an object yields the affirmation of the statement attributing its contrary to the object.²⁷ And it is this logic that Descartes wants to apply. Examples of such pairs of contraries actually used by Descartes are: *divine* and *created* (substance), *thinking* (active) and *extended* (passive) substance, *motion* and *rest*, and *determination to the right* and *determination to the left*.

Whereas in Aristotle's time a calculus was available in logic only for exhaustively contrary predicates, where the principle of *tertium non datur* applied, by the time of Descartes a calculus had been developed that was also capable under certain circumstances of yielding determinate results when dealing with contraries that admitted of *intermediates*. This new method of dealing with intermediates, introduced by medieval logicians in Oxford and Paris, was the calculus of "latitudo formarum" (discussed in section 1.2.3), which also allowed for the simultaneous representation of contrary predicates. If, say, the heat in a subject is uniformly diffused terminated at no degree and is represented by a right triangle, then the quantity of the contrary quality, cold, could be represented by a complementary right triangle. Thus the heat and cold in a subject could be represented by a rectangle.²⁸

²⁶See also Strawson 1952, p. 8.

²⁷The informativity of this logic is of course not confined to cases where *two* species exhaust the genus. If there are a limited number of determinate species, a limited number of negations are necessary to establish the sole remaining species.

²⁸Contrary qualities were conceived in general as incompatible. It seems that Buridan was the first to allow that some contrary qualities, for instance, hotness and coldness, may coexist in the same substance and that their sum is constant. See Maier 1952, pp. 304f. Oresme first represented this thesis by means of the method of configuration of qualities and motions:

2.3.3 Aristotle's Physics of Contraries

We have seen in the preceding sections that the law of non-contradiction in Aristotle was based on the ontological principle that contraries cannot coexist in the same substance and that a logic of contrary predicates (both exhaustively contrary predicates and contraries with quantifiable intermediates) could in principle be productive under certain circumstances. We will now show that both these variants of the logic of contraries were applied by Aristotle to generate the basic doctrines of his physics.

A considerable part of Aristotle's scientific knowledge is derived from and justified by the logic of contraries sketched above. One central element of Aristotle's physics is the application of the logic of contraries to the explanation of motion, i.e., qualitative change, quantitative change, and locomotion, each of which involves a transition from one state to its contrary (*Physics* I, 6-10, 189a10ff, *Metaphysics* XII, 2; 1069b3ff). The elements, too, are defined by contrary predicates and each can be transformed into another, if one of its properties is changed into its contrary (cf. *De gen. et corr.* II, 2; 329b18ff). Natural motion along a radius of the world sphere allows for two contrary directions: *up* and *down*, which allow no intermediates but which correspond to the two opposite ends of the radius representing the qualities *light* and *heavy*. Four of Aristotle's five elements are based on contraries: Fire and Earth move up and down and being absolutely light or heavy respectively therefore occupy the periphery and the center. In between are two elements which are relatively light and heavy: Air and Water. Hence, Air moves up until it reaches Fire, and Water moves down until it reaches Earth.

Another way to define elements by contraries is derived from basic tactile qualities: heat, cold, dryness and wetness. These four qualities can be combined in

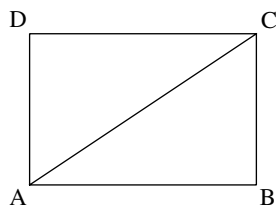


Fig. 2.1 (Oresme 1968, p. 214)

tion of the one is changed, so the figuration of the other will be equivalently changed in a contrary fashion enough to make up the uniformity of the whole aggregate. Whence it is evident that if one of the contraries is imagined by a convex figure, the other existing [with it] at the same time is to be imagined by a concave figure, and vice versa ..." (Oresme 1968, pp. 212-14). Similar figures applied to contraries can also be found in later treatises. See Clagett's Introduction to Oresme 1968, pp. 75 and 81ff. See, e.g., Paul of Venice 1503 p. 16v.

"And so let the hotness of subject AB be uniformly difform terminated at no degree in point A and at the highest degree in point B, or at the least degree with which the coldness cannot stand. Therefore the hotness will be imagined by $\triangle ABC$, and so the coldness is to be imagined by $\triangle DCA$. Then let the figure be inverted and let A be put in place of D and B in place of C. And then it is evident that the coldness is uniformly difform terminated at no degree in point B. And it is the same for all the contraries which exist together, so that however the figura-

four possible ways to define the elements: hot and dry (Fire); hot and wet (Air); cold and wet (water); cold and dry (Earth) (cf. *De gen. et corr.* II,2, 329b18ff; *Meteor.* IV,1, 378b10f). (The combinations hot/cold and wet/dry are excluded since none of the elements themselves may contain incompatible properties.)

Now, if all material bodies or substances contain all the elements (see *De gen. et corr.* II,8, 334b31), then a logical and physical problem arises: Each of the four adjacent pairs of the elements (Air and Fire, Fire and Earth, Earth and Water, Water and Air) contains a pair of shared properties and a pair of contraries. Each of the opposite pairs Air and Earth, Fire and Water has two pairs of contrary properties. Any time we are constrained to conceive of elements (or bodies) which have incompatible properties as constituting one body, i.e., any time we have a “true mixture”²⁹ not a mere aggregate, we will obtain a subject with inconsistent predicates: a logical contradiction. According to Aristotle, however, many combinations of elements are true mixtures. The solution to the problem lies in the fact that these fundamental contraries admit of intermediates, so that the new mixed substance can be conceived to have intermediate qualities, e.g., between wet and dry.

The logic of contraries thus allows the conclusion that the new substance will either have an *intermediate* quality (if the contraries are conceived as extremes of a continuous scale) or that one of the original qualities will *prevail* (if the contraries are conceived as exhausting the range of possibilities).

It might seem that all knowledge embedded in natural language is simply empirical. However, even though the knowledge that two given qualities are contraries with intermediates is acquired empirically, the knowledge that the mixture of two substances bearing the qualities will generate a substance with an intermediate property need not be empirical. Rather it can be inferred according to the deductive rules of the conceptual system from the definition of intermediates as composed of the extremes and from the definitions of continuous magnitudes. Hence, even if the concepts of the new qualities generated out of contrary ones arise as generalizations of empirical knowledge, nonetheless, once a conceptual system using and defining the contraries exists, the logic and semantic relations of the system can determine assertions for which no adequate empirical basis is available. This is why Aristotle and his successors could and did derive new information about the world from language.

It is a *logical* problem that an inconsistency arises in a conceptual system which characterizes the mixture of two materials by the attribution of two contrary predicates to the same subject; and it is similarly a *logical* consideration that in order to avoid inconsistency, one has to demand a change in one or more of the contrary qualities so as to render them compatible and the description of their mixture consistent.

²⁹In a true mixture two or more materials generate a new one with determinate qualities out of the contrary qualities of the components (327b10-238b25). For a discussions of these topics in scholastic natural philosophy see Maier 1952, pp. 1-140.

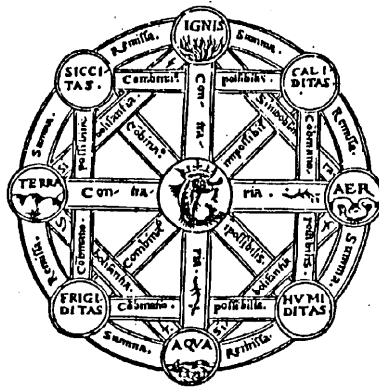
2.3.4 Contraries in 17th Century Logic and Physics

That the contrary predicates in language provide the basic principles of explanation in physics was part of the standard knowledge in science up to the time of Descartes. The four elements and the contrary qualities which allow one to be transformed into the other were basic to physics as codified in textbooks in the early 17th century. In many works³⁰ the elements and basic qualities were presented in “Pythagorean tetrads” or squares of oppositions analogous to the standard presentation in logic of the square of oppositions of judgments. Scholastic logic texts and commentaries on Aristotle’s *Physics*, including those probably used by Descartes in his studies at La Flèche,³¹ carried on the tradition of deriving physical knowledge from the logic and semantics of language. One of the most prominent of these authors, Franciscus Toletus, analyzed contrariety as follows³²:

Three [conditions] are necessary for those which are called contraries. *First, that they belong to the same genus*: because those which belong to diverse

³⁰See Heninger 1977, esp. pp. 103-108, for a number of such tetrads. A tetrad almost identical to the one given here from the *Cosmographia* of Oronce Fine can be found in Leibniz, *Dissertatio de arte combinatoria*, (1666) GP IV, 34. The Square of Opposition is taken from De Soto *Summulae* (1554), p. 52.

Oronce Fine, *Cosmographia* (1542)



De Soto, *Summulae* (1554)

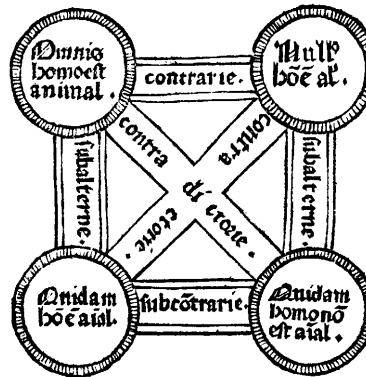


Fig. 2.2 (Heninger 1977, p. 106)

Fig. 2.3 (Soto 1554, p. 52)

³¹See Cronin (1966, pp. 32-3) for a discussion of the texts used at the Jesuit school La Flèche.

³²Toletus, *Logicam* (1589), p. 105r (emphasis added); almost verbatim the same discussion can also be found in the *Summa* of Eustace of St. Paul (Part I, pp. 38-39), which Descartes once recommended as a competent scholastic presentation (AT III, 251). For an analysis of relevant late scholastic doctrines see Des Chene 1996, esp. pp. 55-64.

genera or predicaments are not called contraries but diverse. *Second, that they be positive forms.* Insofar as one is a privation of something that is, they are not called contraries because blindness and vision are not contraries: these two [conditions] do not suffice since man and horse fulfill these two but are not contraries. A *third* [condition] is necessary, namely, *that they struggle with one another and act in the same subject*, from which by their mutual interaction one expels the other, as heat expels cold from the hand. This action takes place according to the forms themselves or according to those of which they consist, as heat struggles against cold according to itself, while white and black act by qualities of which they intrinsically consist ... *Let this suffice, the rest is Physics and Metaphysics.*

In the same text Toletus distinguishes between contraries that have intermediates (e.g., black and white) and those that do not (even and odd). Furthermore, by pointing out that authorities sometimes disagreed as to whether a particular pair of contraries (e.g., sickness and health) had intermediates, he makes it clear that empirical knowledge is involved in determining which is which. He also stresses that the terms must be predicated of a “proper subject,” i.e., a subject that may receive one or the other predicate. For instance, a “mixed body” may be black or white; but an angel is neither sick nor healthy, nor is it even or odd.³³ The logic of contraries as a tool for acquiring knowledge about the material world and for constructing theories about it was not only an integral part of the philosophical tradition, it was also part of standard text book knowledge used in scholastic education in the 17th century and was thus immediately available to anyone with a certain amount of academic training.

The use of contraries as basic principles of physics in Descartes’ time was not confined to the scholastics. Mechanistic science distinguished itself from scholasticism not by abandoning the logic of contraries but rather by determining different concepts as contraries, for instance by including circular motions (of the celestial bodies) which according to Aristotle had no contraries. Moreover, since dynamics was developed by means of concepts and techniques adapted from statics, the logic of contraries was also applied to these concepts. Such contemporary “mechanical philosophers” as Galileo, Hobbes, and Marci relied heavily on the use of the contrary predicates in language to enable them to represent causal necessity in the world by logical necessity in their systems.

The representation of physical oppositions by contrary concepts combined with an extension of reasoning from statics typical of the mechanists of the 17th century is well illustrated in the following passage from Hobbes³⁴:

That nothing can hinder motion but contrary motion, That the motion of the water when a stone falls into it, is point blancke contrary to the motion of the stone, for the stone by descent causeth so much water to ascend as the bignesse of the stone comes to. ... and this rising upwards is contrary to the

³³Toletus 1589, p. 176.

³⁴Hobbes, letter to Cavendish, Jan. 29/Feb. 8, 1641 (HC I, 82-83; *Works* 7, pp. 458-459). For Galileo’s use of contraries, see Chapter 3 below, sections 3.2.1, 3.2.3, 3.3.1, and 3.7.3.

descent; and is no other operation then wee see in scales, where, of two equall bullets in magnitude, that which is of heavier metall maketh the other to rise.

In his treatise, *De proportione motus*, Marcus Marci not only uses the logic of contraries in argument, he bases his system explicitly on this logic. The first of the definitions with which his work begins is a definition of contrariety³⁵:

[Def.] 1. Contraries are said to be those which remove or impede their contraries.

He then distinguishes degrees of contrariety from absolute contrariety and applies this distinction to motion:

[Def.] 4. Motions are absolutely contrary, which conduct the same mobile from the same point towards the opposite direction in the same right line.

[Def.] 5. Motions are contrary *secundum quid*, which from that same point or origin of motion make an angle greater or smaller than a right angle but less than two right angles.

Marci's seventh definition points out that, if the two motions make an angle neither greater nor smaller than a right angle (i.e., exactly 90°), they are not opposed at all but constitute a "perfectly mixed" motion. In the second postulate, which follows the definitions, Marci then establishes the connection between the logic of contraries as applied to motions and the analysis of forces in statics.³⁶

Although these three mechanist contemporaries of Descartes all apply a logic of contraries to physics, they all also differ from Descartes in taking motion as essentially directed. Thus the contrary of motion is not rest, but rather the contrary of motion (in one direction) is *motion* (in the opposite direction).

2.3.5 Descartes' Physics of Contraries

Descartes' theory of matter is constructed according to the logic of contraries discussed above. His first step is to divide substance as the bearer of properties and subject of predicates into two kinds: *contingent* (created, finite) and *necessary* (divine, infinite) substance. Created substance is then divided into *thinking* (active) and *extended* (passive) substance. Physics is the study of created, extended substance. On this basis Descartes then introduces the three pairs of contrary predicates from which he constructs his physics. These are: (1) *matter* and *vacuum* (something and nothing), (2) *motion* and *rest*, and (3) determination *towards one side* and determination *toward the opposite side*.

³⁵Marcus Marci, *De proportione motus* (1639), no pagination, sigs. A3v, A4v, B1v-B2r.; see also Gabbey 1980, p. 245.

³⁶Marci 1639, B1v-B2r. On the quantification of contraries by means of statics, see our discussion below in 2.4.3.

These pairs of predicates are introduced in the second book of the *Principia*. Using the first pair, matter and vacuum, Descartes argues that “space” and “material substance” refer to the same entity, for there cannot be empty space or a vacuum, “that is, a space in which there is absolutely no substance ... because it is entirely contradictory for that which is nothing to possess extension (*quia omnino repugnat ut nihili sit aliqua extensio*)” (II, §16). Since nothingness can have no predicates, what has extension cannot be a vacuum. Thus the subject of extension is substance and extended substance is matter. This pair of contraries (matter and vacuum) is used to analyze the concept of extended substance and to show that it cannot be further specified.

After introducing the conservation laws of the system of matter, Descartes presents three laws of nature. The first two laws define invariants in the behavior of single bodies, the third governs two-body interactions. Each of the first two laws of nature is based on one of the pairs of contrary predicates introduced above.

1. From the contrariety of *motion* and *rest* Descartes derives his *First Law of Nature*, the law of inertia. Rest and motion are both positive and contrary predicates,³⁷ and “nothing can move by its own nature towards its contrary or its own destruction.” Thus, in general, if a substance is to lose a property and take on a contrary property, then there must be an *external* cause.

The first of these [laws] is that each thing, insofar as it is simple and undivided, always remains in the same state as far as it can and never changes except as the result of external causes. Thus, if a particular part of matter is square, we can be sure without more ado that it will remain square forever, unless something coming from outside changes its shape. If it is at rest, we hold that it will never begin to move unless it is pushed into motion [*ad id*] by some cause. And if it moves, there is equally no reason for thinking that it will ever cease this motion of its own accord and without being checked by something else. Hence we must conclude that whatever moves, so far as it can, always moves. (II, §37; AT VIII, 62)

The physical striving of a body to remain in its state of motion is derived from a logical principle: the logical opposition of the predicates motion and rest. We can see that the concept state of motion (encompassing motion and rest) which Descartes bequeathed to classical physics is developed to name the genus or range of incompatibility of the two contrary predicates, rest and motion.

2. Since motion as the opposite of rest is defined as a scalar magnitude without reference to direction, Descartes needs to introduce a *Second Law of Nature* to guarantee that the conserved motion keeps the same direction. Here he introduces for the first time (in this systematic exposition) the concept of determination. Determination is a *mode* of motion, which in turn is a mode of bodies. As a specification of the concept of motion, which has a quantity, determination is

³⁷Descartes criticized scholastic philosophers who “attribute to the least of these motions a being much more solid and real than they do to rest, which they say is nothing but the privation of motion. For my part I conceive that rest is just as much a quality, which must be attributed to matter while it remains on one place, as is motion, which is attributed to it while it is changing place” (*Le Monde*, AT XI, 40).

supposed to have a magnitude as well as a direction. Its magnitude is entirely dependent on that of the motion it “determines”; its direction is independent of the quantity of the motion.³⁸ A thorough analysis of the concept of determination must be put off until section 2.5.

The *Third Law of Nature* containing the rules of impact will be analyzed in depth in the next section (2.4). To sum up, we have seen thus far that the basic concepts of Cartesian physics are derived by means of a logic of contrary predicates adopted from traditional logical and physical thought.

2.4 Descartes' Derivation of the Rules of Impact

2.4.1 The Means of Representation

We have argued that Descartes formulated conservation laws functionally similar to those laws used in classical physics for the deterministic description of a mechanical system, and we have already indicated that his concepts are very different from those of classical physics. This will become clear in the following discussion of his rules of impact which describe the basic interactions of bodies.

In classical mechanics the result of a central collision of two perfectly elastic bodies under extremely idealized conditions is calculated by simultaneously solving two equations based, respectively, on the conservation laws for (1) momentum and (2) kinetic energy. The two equations derived, (3) and (4), determine the velocities of each body after the collision.

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 , \quad (1)$$

$$1/2 m_1 v_1^2 + 1/2 m_2 v_2^2 = 1/2 m_1 u_1^2 + 1/2 m_2 u_2^2 , \quad (2)$$

$$u_1 = \frac{(m_1 - m_2) v_1 + 2 m_2 v_2}{m_1 + m_2} , \quad (3)$$

$$u_2 = \frac{(m_2 - m_1) v_2 + 2 m_1 v_1}{m_1 + m_2} . \quad (4)$$

³⁸Although determination is defined as a mode of motion, it is also sometimes *used* as if it were a mode of the bodies themselves. For instance, in the section of the *Principia* in which Descartes first introduces the concept it seems that a body can have a determination in an instant (“in that instant at which it is at point A”), although it can only have motion during some finite length of time: “It is true that no motion takes place in an instant; but it is manifest that everything that moves is *determined* [*determinatus esse*] in the individual instants which can be specified as it moves, to continue its motion in a given direction along a straight line, and never along a curved line” (II, §39; AT VIII, 64; emphasis added).

Note that here the conservation of mass in the system need not be stated explicitly because the much stronger statement, that the mass of each body is conserved, is incorporated into the mathematical representation by using the same variable for the mass of each body before and after collision.

In early classical mechanics, although both quantity of motion (momentum) and *vis viva* (kinetic energy) were defined by the same basic magnitudes (mass and speed), they were nonetheless dealt with according to different algorithms (parallelogram rule for addition of mv and arithmetical addition of mv^2), and this difference required justification and led to controversies. In later (19th century) classical mechanics, on the other hand, no justification was needed and no controversies arose, since momentum and kinetic energy could already be defined on the basis of a presupposed mathematical theory as vector and scalar magnitudes, respectively, and the two different kinds of magnitudes could be represented by different kinds of symbols (mv^2 and mv). The equations give us both speed and direction (i.e., velocity) of the bodies after collision based on a knowledge of speed and direction before collision.

For Descartes the situation is quite different. His two fundamental magnitudes, quantity of motion and determination, have the same absolute value $|mv|$. The concept of vector had not yet been invented, and he compounded and resolved determinations by means of the geometrical parallelogram rule. In the geometrical representation (Fig. 2.4) the quantity of motion or (abstracting from the size of the body) the speed was represented by the radius of a circle, i.e., by the

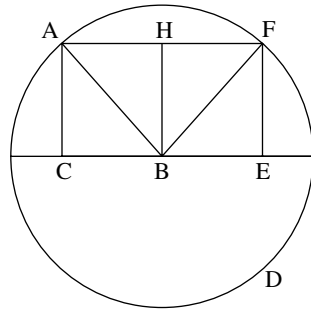


Fig. 2.4

undirected distance between center (B) and circumference (AFD). The *particular* radius as actually drawn (e.g., AB) connects the positions of the body at the beginning and end of the time interval considered and represents, therefore, *both* the quantity of motion and the determination. But the distinction between quantity of motion and determination was of paramount importance since the determinations can be resolved and then recombined according to the parallelogram rule, whereas the motions are added and subtracted arithmetically. For instance, if we resolve the determination according to the parallelogram rule, the sides of the parallelogram (AC and AH) represent

components of the determination (AB) but *do not represent motions or components of motions*. Consequently, the *sides* of the parallelogram may not be considered to represent actual motions, although the *diagonal* represents not only determination but also motion. Thus, it was in practice not always possible to distinguish between motions and the determinations of motions on the basis of the geometrical representation alone; and furthermore, it had constantly to be borne in mind that the radius actually drawn represented both. In order to refer in language to what the radius represents, the linguistic representation, e.g., of the

quantity of motion (*motus*) had to be evoked and further qualified “in a particular direction” (e.g., *determinatus versus certam partem*). Thus one must keep in mind the *content of the physical argument*, part of which is represented only in language and its logical structure, in order to understand the meaning of the elements of the geometrical representation and to apply the appropriate calculus (arithmetical addition or vector addition). (This will be illustrated in detail in section 2.5, in which Descartes’ derivation of the inverse sine law of refraction is analyzed.)

Furthermore, in the study of collision, which involves two bodies moving in a straight line in opposite directions, the motions (*lmv*) of the two bodies are added to find the total quantity of motion conserved in the system, but since the *determinations* of the bodies are not conserved in collision, their behavior cannot be predicted by any purely computational algorithm. And, as we shall see, the impact rules are presented verbally with *pictures* of the bodies but without geometrical representation. Thus, the mathematical representation is not sufficient for deriving the outcome of physical interactions.

2.4.2 Contrary Predicates and Opposing Modes

Since the mathematical techniques for dealing with motions and determinations are insufficient to determine unequivocally the outcome of collisions, further qualifications of a non-mathematical nature must be introduced explicitly, and their introduction must also be argued for. The argument must appeal to some form of accepted knowledge in order to have any foundation; this knowledge, however, cannot consist simply of empirical experience. Descartes does not claim that his impact rules apply to empirical bodies; i.e., although the motions say of billiard balls are *ultimately* governed by these rules, billiard balls rolling on a smooth surface in an air medium are not considered an adequate realization of the theoretical ideal to which the rules apply. He also states explicitly that the motions of the bodies in fluid media (like air) is different from (though of course ultimately governed by) the ideal case;³⁹ but he offers no way of reducing a complex empirical interaction to the two-body ideal case governed by the theory. Thus there is *factually* no means of testing the theory. Furthermore Descartes cannot appeal to an accepted system of *theoretical* or *scientific* knowledge about material reality itself, since he was consciously rejecting the accepted Aristotelian system of knowledge and developing an alternative to it.⁴⁰ But since the physical phenomena with which he was dealing were conceptualized in natural language, he could and did appeal to the necessary relations embedded in the language between the concepts used, i.e., broadly speaking, to *logic*. The logic

³⁹*Principia*, II, §§53 and 56; AT VIII, 70 and 71.

⁴⁰The one area of scientific knowledge, to which Descartes could and did appeal, namely statics, had to be interpreted and its adaptation itself had to be justified.

applied by Descartes to derive his impact rules is the logic of contrary predicates discussed in section 2.3. The logical contrariety of concepts is taken to express the physical opposition of properties of bodies; the contrary predicates correspond to incompatible modes of bodies and the inconsistency of the predicates is resolved when one or both incompatible modes undergoes change. Thus the collision of *bodies* is represented by the contrariety of *predicates*.⁴¹

Not every concept that has a conceptual contrary denotes an entity which can be opposed to another entity. The central concept, quantity of matter, for instance, refers to material substance, and substance is the contrary of nothing (II, §16); but a substance cannot be physically opposed to another substance or even to nothing. It is only the properties or modes of a substance or a body that can be opposed to one another.⁴² The relevant modes of bodies that can be incompatible are *motion* and *rest*. Since motion as such is without direction, the possible opposition between directions, e.g., left and right is expressed as an opposition between further specifications of the concept motion, or in Descartes' terminology: *modes of the mode of motion*; i.e., the determinations of two motions can be opposed.⁴³ The title of §44 states, "That motion is not contrary to motion, but to rest; and that determination in one direction is contrary to determination in the opposite direction."

Both oppositions admit of degrees and can be quantified. From the opposition between motion and rest, an opposition between "rapidity of motion and slowness of motion" can be derived "to the extent that this slowness partakes of the nature of rest." Thus not only motion and rest can be opposed but also to a certain extent *slow* and *swift* motion. Determinations, too, can also be more or less opposed, i.e., the opposition can be direct or more or less oblique, but since Descartes' impact rules deal explicitly only with direct oppositions of determinations, i.e., collisions in one dimension, we shall deal with oblique collisions only in a later section (2.5.3).

⁴¹Leibniz clearly saw that Descartes was deriving physical theorems by applying the logic of contraries and presented one of them (concerning firmness) in syllogisms; in these syllogisms the common relation of the predicates is "maxime adversatur." Leibniz's criticism of Descartes is not directed against this procedure in general (in fact Leibniz applies it too), but against two specific points. On the one hand, Leibniz doubts that the characterization of certain concepts as most opposed is correct (thus he remarks that contrary motion is more opposed to a specific motion than is rest) and on the other hand he doubts the validity of Descartes' implicit axiom, that the cause of what is most opposed to something is also most opposed to the same thing. (see. Leibniz, "Critical Thoughts on the General Part of the Principles of Descartes," on Articles 54, 55, GP IV, 385-388, PPL, 403-407.)

⁴²See Aristotle, 225b10-11 and 3b24-27.

⁴³There have been numerous studies of Descartes' physics, which we shall not be able to deal with here. For a discussion of various previous interpretations of Descartes' physics consult Gabbey 1980, which has set qualitatively new standards of analysis for the study of Cartesian science. Gabbey's interpretation emphasizes especially the opposition of modes as a key to understanding Descartes' system. Our debt to his work will be evident both in this and the next section.

Descartes' general rule governing the interactions of bodies, i.e., governing the resolution of the incompatibility of their modes whenever they "encounter" (*occurrere*) one another, is stated as the *Third Law of Nature*:

The third law is this: when a body that moves encounters another, if it has less force to continue in a straight line than the other has to resist it, it is deflected in another direction, and retaining its quantity of motion, it gives up only the determination of the motion. If, however, it has more force, it moves the other body with it and loses as much of its motion as it gives to the other. (II, §40; AT VIII, 65)

This law of nature is then further qualified by the explanation of the measure of force:

And this force should be measured not only by the size of the body in which it is and by the surface which separates this body from the other, but also by the speed of its motion and by the nature and [degree of] contrariety of the mode in which the different bodies encounter one another. (II, §43; AT VIII, 65)

Note that in addition to the size and speed of the body also the size of the surface of collision is considered relevant to the question of how much force one body can apply to another. This is the case because the cohesion of a body consists only in the mutual rest of the parts relative to one another. As Descartes says "we cannot indeed think up any glue that would join together the particles of solid bodies more firmly than their rest."⁴⁴ Therefore those parts of a body that are not immediately bumped by opposing parts of the other body (or are directly behind them in the line of motion) are not involved in the interaction at all and would simply continue in their inertial motion until they actually encounter opposing parts.⁴⁵ Thus to calculate the force that a body can actually bring to

⁴⁴*Princ.* II §55; AT VIII, 71.

⁴⁵Although the context of §43 makes it is unequivocally clear that we are dealing with the interaction of two and only two bodies and although the Latin text itself makes it clear that the surface in question is only the surface *between* the two colliding bodies, most commentators have nonetheless interpreted this passage as making some vague reference to the *entire surface* area of the body, including its back and sides. This has even led them to mistranslate the passage to fit the interpretation. The Latin reads: "Visque illa debet aestimari tum a magnitudine corporis in quo est, et superficiei secundum quam istud corpus ab alio disjungitur; tum a celeritate motus, ac natura et contraritate modi, quo diversa corpora sibi mutuo occurrunt." The key phrase is *ab alio*; Descartes speaks of the surface that separates the colliding body from *the other body*. All three published English translations have Descartes talk about the surface that separates a body from all the surrounding bodies, not just from the one it hits. See Descartes 1964, 1983, 1985f, as well as almost every commentator on the subject. The French translation of the phrase ("séparé d'un autre") appears to be ambiguous and has been cited in support of the usual interpretation, e.g., by Costabel 1967. There are other passages in the *Principles*, *Le Monde*, and an often cited letter to de Beune on "natural inertia" (AT II, 543–544), where Descartes also deals with the physical significance of the surface of a body and may be interpreted to mean more than just the front end. But how much of the surface is significant depends on how much is involved in interactions: the dynamically relevant cross section.

bear in interaction with another body, we need to know how great the surface of collision is. See also our discussion of the pictures of colliding bodies in section 2.4.4.

Thus, when two bodies with incompatible modes encounter one another (collide), the modes of one or both bodies must change in such a way that the incompatibility is removed. The major determinant of the relation of the changes in each body's properties or modes is the relation of the sizes of their *forces*. But the incompatibility of opposed modes in collision can be resolved in different ways, and more than one opposition of modes can be involved: motion versus rest and determination ($\rightarrow$) versus ($\leftarrow$) determination. The unequivocal calculation of the outcome of a collision in more complicated cases demands a rule specifying the relations between changes in the pairs of opposed modes. Since every change in the speed of a body involves a change in (the amount of) determination while not every change in determination (e.g., in direction) entails a change in the scalar speed of the body, determination being only a mode of motion, the relation of changes in both these quantities has to be introduced explicitly. Both the concept of physical change as the solution to an incompatibility of modes and the specification of the ratio of change of quantity of motion and determination are formulated by Descartes as one principle determining all collisions. This principle is used implicitly in the *Principia Philosophiae* to derive some of the impact rules, although Descartes actually stated it explicitly only a year later in a letter of explanation to Clerselier⁴⁶:

That when two bodies encounter each other and have in them incompatible modes, unquestionably there must occur some change of these modes to make them compatible, but this change is always the least possible. That is, if they can become compatible through the change of a certain quantity of these modes, a greater quantity will not change. And it must be noted that there are in motion [*mouvement*] two different modes: one is the motion [*motion*] alone, or the speed, and the other is the determination of this motion [*motion*] in a certain direction. Of these two modes, one changes with as much difficulty as the other.

This principle, which Gabbey has called the "Principle of the Least Modal Mutation,"⁴⁷ guarantees that the result of interactions will be unequivocal by singling out one of the possible results (the smallest amount of change) as the one that must actually occur.⁴⁸

⁴⁶Descartes, letter to Clerselier, Feb. 17, 1645; AT IV, 185-186 (transl. adapted from Gabbey 1980, p. 236). Spinoza argued that this principle directly follows from the first law of nature (inertia); see *Renati Des Cartes Principiorum Philosophiae Pars I & II*, bk. II, §25.

⁴⁷Gabbey 1980, p. 263f.

⁴⁸However, when calculating the amount of change in the various possible results, it should be remembered that the quantity of determination changes whenever the quantity of motion changes. Thus, although each mode changes with equal difficulty, a change in determination involving only change of direction is in fact easier than a change in motion which also necessitates a change in the quantity of determination and thus counts double (i.e., reversing four units of determination without changing

Thus, to derive the rules of impact, Descartes uses (alongside the unstated but assumed conservation of matter) *one* conservation law, the conservation of the quantity of motion, as well as an extremal principle (the principle of minimal modal change), and the qualification, that force is transferred only from a stronger to a weaker body.

2.4.3 The Rules of Impact

There are three relevant possibilities of incompatible modes involving two bodies in one dimension, and these possibilities determine the structure of Descartes' presentation of the rules of impact. There can be an opposition: (1) between the *modes* of bodies, motion and rest, (2) between the *determination* of the motions of bodies in opposite directions, and (3) between the *intermediaries*, slowness and rapidity of the motion of bodies.⁴⁹ The order of the impact rules that Descartes presents is determined by the increasing complexity of opposition between incompatible modes.

In addition to the logic of contraries there is another conceptual system that structures inference in Descartes' physics, namely statics, which since the time of Aristotle had provided a compelling form of material inference. Statics at the time was not merely the Archimedean science of weights in equilibrium, but rather also a systematic study of the application of the simple machines to raise a particular load with a particular force. The analysis of these machines had traditionally been the attempt to reduce them to the law of the lever, which was taken to be the fundamental principle of mechanics.⁵⁰ Whereas the two arms of a balance are symmetrical, the lever introduces a fundamental asymmetry between the two arms: one is the force and the other is the load. And the machine is not intended to be in equilibrium and rest but rather to be in motion and thus to raise the load. Furthermore, statics also provided the prototype of a rigorous theory of forces. According to the law of the lever weights and distances from the fulcrum are inversely proportional. If the lever is tilted by the smallest imaginable force,

motion is equal to transferring two units of speed because two units of determination are attached to them).

⁴⁹In fact there are four possibilities: two *kinds* of opposition, each of which can be *absolute* or admit of *degrees*. As Descartes says: "strictly speaking, only a two-fold contrariety is found here. One is between motion and rest, or also between swiftness and slowness of motion (that is, to the extent that this slowness partakes of the nature of rest); the other is between the determination of a body to move in a particular direction and the encounter in this direction with a body which is at rest or moving in a different manner; and this contrariety is greater or lesser in accordance with the direction in which the body that encounters the other is moving" (*Princ.* II, §44; AT VIII, 67). The fourth possibility – oblique oppositions – will be dealt with in the next section.

⁵⁰See Westfall *Force*, pp. 121–22.

the distances passed through by weights on the lever are proportional to their distances from the fulcrum. Since the two ends of the lever pass through their respective distances in the same time, their velocities are also proportional to their distances from the fulcrum. Thus both velocities and distances are inversely proportional to the weights, so that the magnitude of force could be determined either by means of weight and speed or by weight and distance. The two quantities seemed interchangeable. Thus a concept of force is obtained that is apparently completely general, that is, applicable to balances in equilibrium, to machines raising loads, and to interacting bodies in motion.

This concept of force, though not as universal as the logic of contraries, which was as universal as the application of natural language, was nonetheless the point of departure for almost every scientific attempt to deal with the forces of motion in the 17th century. When applied to the same problem these two means of conceptualization were mutually adapted so as to form a complex conceptual system. Contrary forces could be illustrated on the example of statics where opposing force are balanced, and in turn the mathematical precision of argumentation in statics could serve to quantify the concept of contrariety – initially at least in certain areas on the margins. A good example of the elaboration and synthesis of these two conceptual schemes can be found in *De proportione motus* by Marcus Marci, where the classical principle that contraries remove their contraries is quantified using statics to yield:

Postulate II

An equal contrary removes or impedes its contrary in the same proportion, the whole namely the whole, a part indeed the part equal to it. (B2–B3)

In the explication of this postulate Marci argues for the postulate by mean of weights on opposite sides of a balance illustrating this with a compound balance see (Fig. 2.5). Whenever statics is applied to a problem, the other resources provided by the logic of contraries are of course also available. And in turn the more the area of application of statics becomes central to science, the more the logic of contraries is infused with the quantificational perspective of statics.

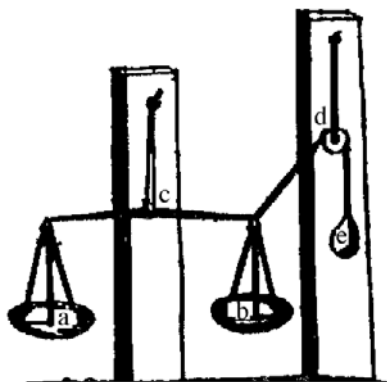


Fig. 2.5, Marci, *De proportione motus*, B2r

The general concept of force derived from the law of the lever – with the concomitant asymmetry between force and load that is peculiar to statics – and the quantified logic of contraries provide the key resources to which Descartes can turn when developing an exact theory of the interaction of bodies in motion. Given the asymmetry between force and load, there are three possible force relations between two interacting bodies: the force can be greater than, less than, or equal to the load. Since Descartes considers three kinds of modal opposition, the synthesis of the logic of contraries with the three possible force relations of statics generates nine cases to be considered – and, if we take into account that Descartes' seventh impact rule actually consists of three variants, he in fact has nine rules of impact.⁵¹

2.4.3.1 The Opposition between Determinations

The rules begin with the simplest case of the interaction of bodies, in which only the determinations of motion are opposed: two *completely equal* bodies (i.e., equal in both size and collision surface), both *in motion* with the same speed (if both are at rest, there is no interaction), collide because their determinations are directly opposed – and they lie in the same line on the appropriate side of one another (see Fig. 2.6). This case is governed by *Rule 1*, which states that the two bodies rebound with unchanged speeds and exchanged or reversed determinations, the quantity of motion in the two-body system being conserved. Rule 1 is the only one of Descartes' impact rules that actually holds for billiard balls and could be confirmed on the billiard table.⁵²

Rules 2 and 3 introduce no new *oppositions*, but each introduces an additional *difference*: either in the *size* of two bodies that are equal in speed (Rule 2) or in the *speeds* of two bodies that are equal in size (Rule 3). In each case one of the bodies is said to be stronger than the other, and the stronger body compels the weaker to change its direction so that both move in the stronger body's direction at the same speed, conserving the quantity of motion in the two-body system.

It should be pointed out that although the stronger body is in fact the one with the greater quantity of motion, Descartes does not use the quantity of motion to define strength, nor does he consider in the discussion of these two rules any cases where the two bodies differ in *both size and speed*. It is also noteworthy that Descartes states *two* rules for what should actually be only *one* case, namely

⁵¹As we shall discuss below, two of the rules Descartes presents actually deal with the same case so that one rule is actually missing.

⁵²Even this, the simplest case and the only “empirically correct” one, is not strictly implied by Descartes' premises. Since the two bodies are equal in force, this case does not really fall under the Third Law of Nature, which holds for the relations between stronger and weaker bodies. See also Spinoza (1925, vol. 1, pp. 211-212) *Renati Des Cartes Principiorum Philosophiae*, bk. II, prop. 24, who appeals implicitly to symmetry considerations when the two bodies are equal.

Descartes' Rules of Impact

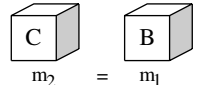
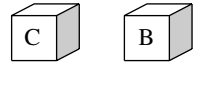
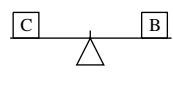
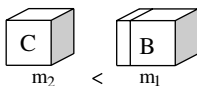
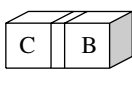
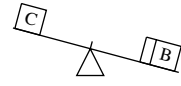
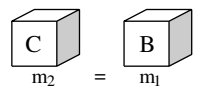
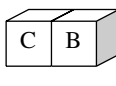
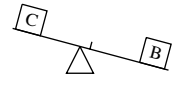
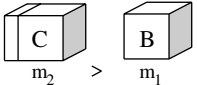
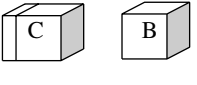
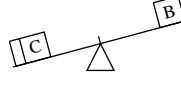
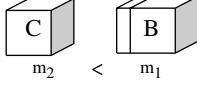
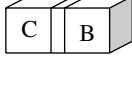
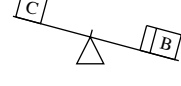
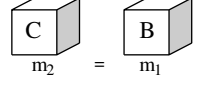
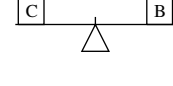
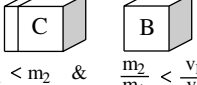
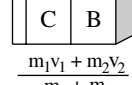
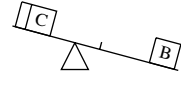
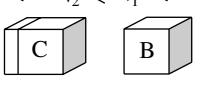
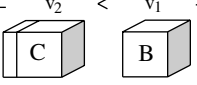
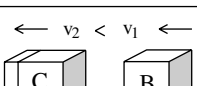
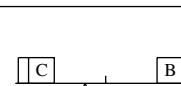
Before Impact	After Impact	Force Relations
1 $\rightarrow v_2 = v_1 \leftarrow$  $m_2 = m_1$	$\leftarrow v_2 = v_1 \rightarrow$ 	
2 $\rightarrow v_2 = v_1 \leftarrow$  $m_2 < m_1$	$\leftarrow v_1$  $\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$	
3 $\rightarrow v_2 < v_1 \leftarrow$  $m_2 = m_1$	$\leftarrow$  $\frac{v_1 + v_2}{2}$	
4 rest $v \leftarrow$  $m_2 > m_1$	rest $v \rightarrow$ 	
5 rest $v \leftarrow$  $m_2 < m_1$	$\leftarrow$  $\frac{m_1 v}{m_1 + m_2}$	
6 rest $v \leftarrow$  $m_2 = m_1$	$\leftarrow \frac{1}{4}v$  $\frac{3}{4}v \rightarrow$ 	
7(a) $\leftarrow v_2 < v_1 \leftarrow$  $m_1 < m_2$ & $\frac{m_2}{m_1} < \frac{v_1}{v_2}$	$\leftarrow$  $\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$	
7(b) $\leftarrow v_2 < v_1 \leftarrow$  $m_1 < m_2$ & $\frac{m_2}{m_1} > \frac{v_1}{v_2}$	$\leftarrow v_2 < v_1 \rightarrow$ 	
7(c) $\leftarrow v_2 < v_1 \leftarrow$  $m_1 < m_2$ & $\frac{m_2}{m_1} = \frac{v_1}{v_2}$	$\leftarrow u_2 > v_2$  $u_1 < v_1 \rightarrow$ 	

Fig. 2.6 (see also Aiton 1973, p. 36)

where the impinging body is stronger than the one it hits, that is, has a larger quantity of motion than the other. This peculiarity proves to be significant for the status of the concept of quantity of motion (see section 2.4.4 below).

On the other hand the asymmetry between force and load in the Third Law requires two impact rules for the nonequilibrium case: one where the force is greater than the load and one where the load is greater than the force. However, instead of also considering a case in which the force is smaller than the load, Descartes considers two ways in which the force can be greater than the load: Rule 2, where B is *larger* than C, and Rule 3, where B is *faster* than C. Thus the required number of impact rules is actually given, since the missing case is compensated for by a superfluous one.

The postil summary of the Third Law of Nature is explicitly asymmetrical distinguishing between the encountering body (i.e., the force) and the body encountered (the load). And although the actual formulation of the law itself seems to be symmetrical insofar as it speaks only of a moving body that encounters another, its application is asymmetric. According to the law, if the moving body in question is weaker than the one it hits, it retains its motion and changes its determination, but if it is stronger, it loses as much motion as it transfers to the weaker. If one views this symmetrically, then whenever one body is weaker the other body must be stronger and thus the weaker body might not just retain its motion but even acquire some additional motion from the other. Both parts of the law should actually apply in all cases. For instance, if the impacting body B is equal in size to the impacted body C but is weaker because it is slower, it must rebound retaining its motion but must also acquire half the difference between its speed and that of impacted body C. This is the case that, based on the conceptual scheme, Descartes should have considered as either second or third rule of impact. But Descartes never considers this case.⁵³

2.4.3.2 Double Opposition: Determination vs Determination and Motion vs Rest

Rules 4 to 6, which we shall consider in some detail below, introduce a *second opposition*, namely, the opposition between the rest of one body and the motion of the other. In these cases the bodies are doubly opposed, both in their modes of motion or rest and in the determinations (or modes) of these modes. Since a stationary body has no motion and therefore no determination, Descartes attributes to the resting body an ability to resist motion and determination in any direction. The second opposition is thus referred to as one between determination and the encounter with another body in its path (cf. II, §44). Descartes treats three possible variants: the body in motion is *smaller* than the body at rest (Rule 4); it is *larger* (Rule 5); they are *equal* (Rule 6).

⁵³For details see McLaughlin 2001

2.4.3.3 Double Opposition: Determination vs Determination and Swiftiness vs Slowness

The three-part *Rule 7* deals with the collisions of bodies moving with different speeds in the *same* direction, in which one body overtakes the other. This is a somewhat more complicated double opposition, in which neither states nor determinations of motion are *directly* opposed but rather are quantitatively incompatible: the swiftiness of one is opposed to the slowness of the other and the determination of the faster motion is opposed to the encounter in its path with the slower body. Descartes considers three cases in which a *smaller and faster* body overtakes and collides with a *larger but slower* body.

Here again three possible differences are taken into account: the quantity of motion of the smaller faster body is (a) *greater than*, (b) *smaller than*, or (c) *equal to* that of the larger slower body. If the overtaking body is stronger than the larger but slower body, it transfers enough motion to the other so that both move with the same speed and determination; if weaker, it rebounds with unchanged speed and reversed determination. And if they are equal in strength, the overtaking body transfers some motion and rebounds with decreased speed and with reversed and decreased determination.⁵⁴

Once again it should be pointed out that Descartes himself does not formulate the rule in terms of quantity of motion but rather in terms of the quantitative relations between the *ratio* of the sizes and the *ratio* of the speeds: for instance, “if C were larger than B but the excess of speed in B were greater than the excess in magnitude of C ...” (II, §52). (See section 2.4.4 below.)

2.4.4 Analysis of Difficulties

The three rules (4-6) governing straightforward double oppositions are particularly revealing of the difficulties of Descartes' system:

Rule 4 states that if a smaller body in motion hits a *larger* body at rest it will rebound with its original speed in the opposite direction. For “a resting body resists a great speed more than a small speed and does this in proportion to the excess of one over the other” (II, §49). This implies that no matter how fast the smaller body may be, it will never be able to move a resting body which is larger than it, no matter how small the difference in size. This follows from the application of the principle of minimal change or “Least Modal Mutation,” as Gabbey has called it.⁵⁵ Descartes explicitly draws this seemingly absurd conclu-

⁵⁴Descartes deals with this last possibility only in the French edition, but it can be derived in analogy to Rule 6.

⁵⁵See Gabbey 1980, p. 269. This rule is further complicated by the fact that a body *at rest* with respect to its contiguous bodies must actually be considered to be *part of*

sion, because for the smaller body to push the larger ahead of it, it would have to transfer *more than half* its own motion (and thus more than half its determination) to the other body; this is a larger change of modes than the *complete* reversal of its own determination. Descartes gives no explanation for this in the *Principia Philosophiae*, but the reasoning behind the position, which is based on considerations from statics, can be found in his letters. For instance, in an exchange with Hobbes, who maintained that nothing that cannot be moved to some extent by the smallest force can be moved by any force at all (*nulla vi amoveri, quod non cedit levissimae*), Descartes replied that a *balance* loaded with 100 pounds on one side cannot be moved at all by 1 pound placed on the other. Nonetheless, it can easily be moved by 200.⁵⁶

Rule 5 states that a larger body colliding with a smaller body at rest will push it ahead of itself in such a way that both move with the same speed and the quantity of motion is conserved. Since this rule is the only one for which Descartes elsewhere calculates a case of *oblique* collision (to which we shall return in section 2.5.3), we shall here give the formula for calculating the speed of the bodies after direct collision: If B is the large body moving with velocity v and C the small resting one and if the quantity of motion (and in this case its direction as well) is conserved in collision, then:

$$Bv_1 = u_1(C + B) \quad \text{or} \quad \frac{v_1}{u_1} = \frac{C + B}{B} = \frac{C}{B} + 1 .$$

Rule 6 deals with the case where the body in motion is neither larger nor smaller than the body at rest. Both bodies are equally large and thus equally strong, the resting body having as much force to resist as the moving body has to drive it forward. Rules 4 and 5 solved this same double contrariety in favor of the larger body, but in this case an indeterminacy arises because both bodies are the same size and thus have equal force. Since there seems to be no logically necessary solution to the problem, Descartes calculates a mean result, as if both Rules 4 and 5 applied simultaneously. Calculated according to Rule 4, the moving body would retain its speed (say 4 units) and reverse its determination (also 4 units). On the other hand, according to Rule 5, the moving body would communicate half its speed and determination (say 2 units of each) to the resting body, retaining half its speed and half its determination in the same direction. As a compromise Rule 6 asserts that the moving body will both reverse its direction and transfer some motion and determination to the resting body. The result will

them, since “our reason certainly cannot discover any bond which could join the particles of solid bodies more firmly together than does their own rest ... for no other mode can be more opposed to the movement which would separate these particles than is their own rest” (II, §55; AT VIII, 71). It is thus hard to see how we can even estimate the *size* of a resting body without contradicting ourselves.

⁵⁶Hobbes’ objection is cited by Descartes in a letter to Mersenne of Jan. 21, 1641, in which he answers the objection; AT III, 289. See document 5.2.11; also 5.2.9.

be the mean between rules 4 and 5 and the two bodies will split the difference between 2 units and 4 units of speed. The moving body B will rebound in the opposite direction with a speed of 3 units (instead of 4), thus losing 1 unit of speed (and determination) and reversing 3 units of determination. The resting body C will move in the direction of B's original motion with a speed of 1 unit.

This last example shows that Descartes algorithm, even with the Principle of Least Modal Mutation, is not unequivocal. the first two possible solutions to the problem involve the same amount of modal change: following Rule 4, four units of determination, and following Rule 5, two units of speed and 2 of determination. But according to Rule 6, five units are changed: 1 unit of speed and 1 unit of determination are transferred and 3 units of determination are reversed. The actual answer that Descartes provides is determined not by the Principle of Least Modal Mutation, which is violated, but rather by an arbitrary decision to take a mean transfer of *motion*.

These three examples of double contrariety show that Descartes algorithms are neither unambiguous nor do they really employ the concepts quantity of motion and determination in the same way as kinetic energy and momentum are employed in classical mechanics. Far more important than the differences in the measures used is the fact that the concepts involved do not yet form the basis of a closed deductive structure. On the one hand there are still indeterminacies (e.g., whenever the forces of bodies are equal) and possibly also inconsistencies between the inferences drawn within this conceptual system, so that ad hoc considerations such as in Rule 6 have to be introduced to assimilate even relatively simple cases into the system.

On the other hand, some of the concepts introduced still bear the stigma of derived concepts, which depend for their interpretation on their constituent concepts. Thus, Rules 2 and 3 are treated as two different cases since the differences in the quantity of motion of the two bodies is due in one case to a difference in size and in the other case to a difference in speed. Rule 7, although it implicitly considers three different relations of the quantity of motion of the two interacting bodies, deals explicitly with different *ratios* of sizes and speeds. Hence, while quantity of motion is, on the one hand, a concept referring to an explanatory physical entity (it determines the sum of speeds of bodies after impact), it does not determine the direction and speed of each single body (not even together with determination) after collision. For this purpose the constituent concepts quantity of matter and speed must be invoked, as well as other considerations such as taking the mean of two possible but incompatible solutions.

Before we conclude our discussion of Descartes' impact rules, let us take a brief look at his illustrations. Many of Descartes' successors (and most historians of science) consider the impact of homogeneous spherical bodies taking these to be adequate models of the collisions of elastic bodies or point masses. But Descartes, who identifies the quantity of matter as volume, cannot use point masses, *and he does not use spheres*. A law governing the fundamental interactions of matter (i.e., collisions) cannot presuppose unexplained forces of cohesion in spheres. The parts of a body are held together not by forces of cohesion

but only by their relative rest with regard to one another (II, §55). Hence, if two spheres were to collide, the parts around the point of contact would not be affected at all but proceed in their motion (the body would break up or be deformed) until they came into contact with the corresponding parts of the other body moving in the opposite direction. Descartes' own illustrations (see Fig. 2.7) picture *cubes* or *parallelepipeds* colliding with their entire surfaces. In one such picture the representation of the difference in size without change in collision surface is achieved by simply extending the length of the larger body. Thus in collision the whole body acts as a unit.⁵⁷

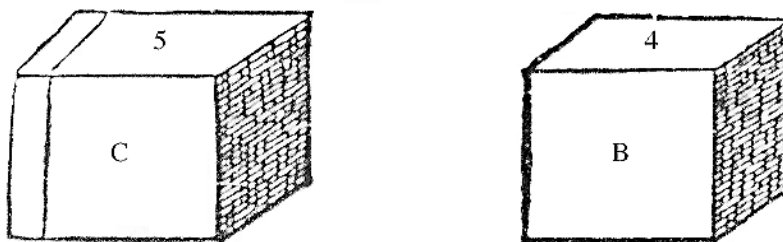


Fig. 2.7 (Descartes 1657, p. 651, 653)

To sum up: A comparison between the concepts of classical mechanics and of Descartes shows two very different results. On the one hand, we see that Descartes introduced and used conservation laws as logical prerequisites for a deterministic causal theory of impact. On the other hand, the concepts introduced and the algorithms used are very different from those of classical physics and it turns out that even some of the simplest cases are underdetermined.

2.5 Determination

The concept of determination is one of the central concepts of Descartes' conceptual system, and it is especially in the use of this concept that inconsistencies and defects in the system come to the surface. Determination refers, on the one

⁵⁷See especially the figures in the letter to Clerselier of Feb. 17, 1645 (Descartes 1657, p. 651). The versions of the figure published in AT IV, 185 and in the first edition of our book are slightly inaccurate: Body B should be smaller and come from the right. See also *Princ. Phil.* II, §46, and AT III, 79; see documents 5.2.2, 5.2.3, and 5.1.12. William Neile (1637-1670), who like Descartes defined as one body all matter sharing the same motion, stated among the presuppositions of his discussion of impact that the colliding bodies are cubes and that "the whole square surface of the one meets in the same instant of time with the whole square surface of the other" (Neile, "Hypothesis of Motion," (May, 1669), in: *The Correspondence of Henry Oldenburg*, vol. 5, pp. 519-524).

hand, to the supposedly *instantaneous* direction of a velocity conceived as *actual* space covered in a unit of time. It must furthermore guarantee the compatibility of the conservation of scalar motion with the application of the parallelogram rule for compounding motions. It is even called upon to explain the action of the mind on the body.⁵⁸ Most of Descartes' contemporaries had great difficulties in grasping what he meant by the term, and his disciples had no fewer problems than did his opponents.

The term determination itself is taken from logic, and it was used by Descartes in the sense in which it was used in logic. Determination refers to the further specification of a concept, for instance, the determination of a species within the genus. In particular, a determination is the modification of a predicate, the qualification of a quality.⁵⁹ It is so described in the major lexicons of philosophy of the 17th and 18th century; but more importantly, the term was simply *used* in logic with this sense, whether or not it was considered a *terminus technicus* that must be included in lexicon or index. Spinoza's famous "determinatio negatio est" exemplifies this use of the term, asserting that any specification of a concept excludes some other specifications.

Any change or motion was said in the Aristotelian tradition to be *determined* by its *terminus a quo* and its *terminus ad quem*. Descartes' innovation in the application of the term to motions (for which he was criticized by Hobbes) consists in maintaining (i) that the determination is "*in*" the motion, i.e., it is a physical magnitude and not merely a specification of direction; (ii) it is a physical entity which has a quantity relevant to its causal efficiency; and (iii) the determination of a motion specifies it as directed towards a *line* or a *plane* perpendicular to the line of motion, never towards a *point*. The expression Descartes uses is always: "vers quelque costé" or "versus aliquam partem." It specifies not a particular line directed to a point but a set of parallel lines, so that parallel motions (which have different points as their *termini ad quem*) have the same determination.⁶⁰

As a first approximation we can define determination as a *quantified* and *directional mode* of motion or motive force.⁶¹ Pragmatically, we can say that determination designates the vector of motion (*mv*), i.e., the most important function

⁵⁸On instantaneous direction see the Second Law of Nature, *Principia*, II, §39; AT VIII, 63-64. The mind or will cannot increase or decrease the amount of motion in the world; it can only determine the motion of the body. Descartes uses the term "determine" on a number of occasions to describe the action of the mind on the body (see especially AT VII, 229 and XI, 225-226). This is an adaptation of a traditional way of speaking about freedom of the will: in decision the will determines *itself* to action. See Suarez, *Opera* vol. 10, pp. 459ff; Descartes (*Passions de l'âme*, §170; AT XI, 459) also uses the term in a similar sense; see McLaughlin 1993.

⁵⁹Gabbey (1980, p. 248) provides many examples of the logical use of "determination" and points out that it was derived from the Greek *προσδιορισμός* meaning division, distinction, definition.

⁶⁰Hobbes objects that "towards a certain *side*" does not unequivocally *determine* a motion (letter to Mersenne, March 30, 1641; AT III, 344-5). See document 5.2.12.

⁶¹See Gabbey 1980, p. 258; Schuster 1977, p. 288

of the concept is to give a justification for computing the results of physical interactions among bodies by mathematical means corresponding to vector addition.

Technically speaking, vectors were an invention of the 19th century,⁶² but the *geometrical* technique embodied in the parallelogram of forces was well known in the 17th century under the title “composition of motions” and presented no technical difficulties.⁶³ It also presented no serious philosophical difficulties in either statics or kinematics. In statics the interpretation of the sides of the parallelogram as component forces, whose resultant is the diagonal, raised no philosophical problems, since it is easy to conceive of two forces pushing or pulling a body from different directions at the same time or to conceive that these two forces are at equilibrium with a third force equal to their resultant and acting in the opposite direction. The technique had been used by Stevin in the analysis of the inclined plane.⁶⁴ A purely kinematic interpretation also presented no insurmountable problems. At first it seems contradictory to speak of a body as moving simultaneously in two different directions, but there were two alternative interpretations which could avoid this formulation: the sides of the parallelogram could represent consecutive trajectories, so that the distance between the beginning and end of the resultant diagonal would represent only the *displacement*; or a point could be conceived to move along one side of the parallelogram while the side itself is moved sideways in some direction so that the compound motion of the point on the line and the line in the plane describes the diagonal. However, although the geometrical technique was easily mastered and could be applied without great difficulty in *statics*, the science of motionless forces, as well as in *kinematics*, the science of forceless motions, it proved to be somewhat troublesome to apply the technique to the motions of forceful bodies or the forces of bodies in motion, i.e., in dynamics. The troubles involved in the dynamical application of the parallelogram rule and some of the solutions proposed by Descartes will be discussed in this section.⁶⁵

The established interpretations of the parallelogram rule in both kinematics and statics were presented by Fermat in patient detail and applied to dynamics in the second letter of his exchange with Descartes over the *Dioptrics*.⁶⁶ After distinguishing “two sorts of compounded motions,” Fermat described the first sort as follows:

⁶²See Crowe 1967, chap. 1.

⁶³There were, however, serious technical difficulties involved in applying the compounding of motions to infinitesimals and accelerations, e.g., in calculating tangents to curves and ellipses. See Costabel 1960.

⁶⁴See Dijksterhuis 1970, pp. 48–63.

⁶⁵Other contemporary attempts to deal with the problems that arise from the notion of compounding and resolving the forces of bodies in motion show them to be inherent in the shared knowledge of the scientific community of the time, not just peculiar to Descartes. For two later examples, John Wallis and Honoré Fabri, see Freudenthal 2000.

⁶⁶Fermat, letter to Mersenne, Nov. 1637; AT I, 464–74. See document 5.2.6.

Let us imagine a heavy body [*un grave*] at point A [Fig. 2.8], which descends in the line ACD at the same time that the line advances along AN such that it always makes the same angle with AO ... (AT I, 468-9).

The second sort of compound motion deals with forces:

Let us suppose in the same figure a heavy body at point A, which is pushed at the same time by two forces, one of which pushes it along AO and the other along AD ... (AT I, 469-70).

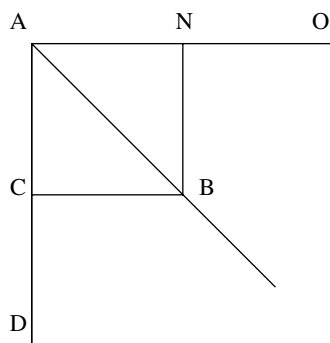


Fig. 2.8 (AT I, 468)

Descartes was of course well aware that a body in motion can be part of a system in motion which in turn is part of another system in motion. He illustrates this state of affairs with the example of the motion of the hand of a watch worn by a (moving) sailor on a moving ship on a moving ocean current on the moving earth. However, while the hand of a watch takes part in various motions, it has (strictly speaking) only one proper motion, which is defined by the relation to contiguous bodies (II, §§30-32).

Descartes saw two major problems for a physical interpretation of the parallelogram rule: (1) It cannot be interpreted as a compounding of *motions* since a body cannot execute two different motions with respect to the same frame of reference at the same time. If “motion in a certain direction” has any determinate meaning, then it excludes motion in other directions. Properly speaking, “each individual body has only one motion which is peculiar to it” (II, §30). The sides of the parallelogram cannot be interpreted as two different motions of one and the same body at one and the same time, this being a contradiction in terms.⁶⁷ (2) If the

⁶⁷Hobbes later applied the same reasoning to determinations, which he took in the traditional sense of the *points* from which and to which the motion is directed:

“Thirdly, it is to be objected that *one motion* cannot have *two determinations*; for in the figure drawn, let A be a body which begins to move towards C, having the straight path AC. If someone tells me that A is moved in a straight line to C, he *has determined* for me this motion; I can myself trace the same path as unique and certain. But if he says A is moved along a straight path toward the straight line DC he has not shown me the *determination* of the motion, since there are infinitely many such paths. Thus the motions from ABD to DC and from AD to BC are not determinations of *one single motion* of the body A towards C, but rather the

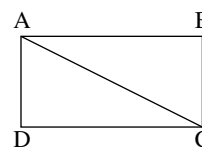


Fig. 2.9 (AT III, 344)

diagonal (or whatever it represents) is to be conceived of as *compounded* out of the sides of the parallelogram (or whatever the sides represent), i.e., if we are to conceive of the operation as genuine *addition*, then the component sides must be conceivable as genuine *parts* of the resultant.⁶⁸ But they cannot be conceived as parts if they are proportional to motions since in Descartes' system motions are scalar magnitudes whose sum is not equal to the resultant derived from the parallelogram rule. Descartes' solution is to introduce a new entity.

The physical magnitude whose parts are added and subtracted according to the parallelogram rule is called *determination* by Descartes. The concept was first introduced in Descartes' published works in the *Dioptrics*, where it is used to derive the law of refraction, and it is explicated in some detail in the subsequent debates with Fermat, Bourdin, and Hobbes, which Mersenne instigated and mediated.

In this section we shall analyze the use of the concept of *determination* to deal with the problem of directed quantities. We shall first of all present the derivation of the inverse sine law of refraction. Then we shall take up some of the difficulties which arise from the particular nature of the concept of *determination* and which were brought to light in the debates with Fermat and Hobbes. For the sake of intelligibility some of these later clarifications will be incorporated into the presentation of the derivation of the inverse sine law of refraction.

2.5.1 The Argument of the *Dioptrics*

The inverse sine law of refraction is the showcase piece of Cartesian physical argument. Using his oppositions of contraries and his conservation laws as well as some at least plausible assumptions on the nature of light,⁶⁹ he inferred rigor-

determinations of two motion of two bodies, one of which goes from AB to DC, the other of which from AD to BC." (AT III, 344-5; see document 5.2.12.)

⁶⁸The significance of the question, whether the entities represented by the sides of the parallelogram may be considered as *parts* of that represented by the diagonal, can be illustrated by the consequences drawn by Bertrand Russell. In his *A Critical Exposition of the Philosophy of Leibniz* (p. 98) he wrote: "It has not been generally perceived that a sum of motions, or forces, or vectors generally, is a sum in a quite peculiar sense – its constituents are not parts of it. This is a peculiarity of all addition of vectors, or even of quantities having sign. Thus no one of the constituent causes ever really produces its effect, the only effect is one compounded, in this special sense, of the effects which *would* have resulted if the causes had acted independently." In *The Principles of Mathematics* (p. 477) he formulated a "paradox of independent causal series": "The whole has no effect except what results from the effects of the parts, but the effects of the parts are non-existent."

⁶⁹These assumptions are: (1) that light is transmitted instantaneously; (2) that it is an instantaneous action or inclination to move that can be taken to follow the same laws as an actual motion in time; (3) that the amount of impetus necessary to traverse a particular medium instantaneously with a particular intensity is analogous to the speed

ously the “correct” law of refraction. Although many contemporaries sought and some seem to have stumbled upon the law of refraction,⁷⁰ only Descartes presented a conceptual system from which the empirical law could be inferred. It would be hard to exaggerate the historical and systematic significance of the discovery of the law of refraction for the foundations of Cartesian physics. The historical development of Descartes’ theories will not concern us here, since this has been quite adequately discussed by recent research.⁷¹ The systematic significance, which we shall be dealing with, can be seen by the following considerations.

From the definition of some general concepts and the application of simple logical and mathematical operations, Descartes derives a nontrivial (and long sought) functional relation between the states of a system before and after a physical interaction: the angle of incidence and the angle of refraction. This functional relation could be presented mathematically as a fixed proportion and needed only the empirical measurement of a constant (for the index of refraction between two media) in order to be formulated in an equation as an *empirical* law of nature. This law could be tested by experiment, and we know that Descartes did in fact try to have the necessary apparatus constructed to verify the law experimentally.⁷² Thus Descartes not only proposed one of the first nontrivial empirical physical laws expressible as an equation (actually expressed as a proportion) and conceived one of the first fully scientific experiments to confirm a theoretical hypothesis expressible in such a form; he also derived the hypothesis from a comprehensive theory that was not primarily formulated to account for this particular phenomenon. The significance of the stated law is thus much greater than just the fact that it can explain a particular phenomenon, since it is presented as an instance of a comprehensive theory.

2.5.1.1 Reflection

Before deriving the law of refraction, Descartes first explicates his general procedure on the example of the law of reflection, a trivial case known since an-

with which a body with a particular force traverses the medium, so that the *speed* of a ball is comparable to the *ease* of passage of a light ray.

⁷⁰Harriot, Snel, and Mydorge were all working on the problem. See Schuster 1977, pp. 308-321.

⁷¹See Gabbey 1980; Sabra 1967, chaps. 3 and 4; and Schuster 1977, chap. 4. The standard work on Descartes’ *Dioptrics* has long been the excellent analysis in Sabra 1967. While we follow in basic outline much of Sabra’s presentation, there are important differences, especially in the analysis of Fermat’s objections. A recent monograph on Descartes’ *Dioptrics* (Smith 1987), while somewhat unclear about the concept of determination, also presents some interesting material on the background in perspectivalist optics.

⁷²See Ferrier’s letter to Descartes, Oct. 26, 1629 (AT I, 38-52) and Descartes’ letter to Ferrier, Nov. 13, 1629 (AT I, 53-59); also Descartes to Golius, Feb. 2, 1632 (AT I, 259); and Descartes to Huygens Dec., 1635 (AT I, 335-6).

tiquity: angle of reflection equals angle of incidence. The law was illustrated at least since Alhazen (Ibn al-Haytham) with geometrical figures displaying the composition of motions. Descartes argument is in effect the demonstration that according to his system the *sines* of the two angles are equal and therefore that the angles must be equal.

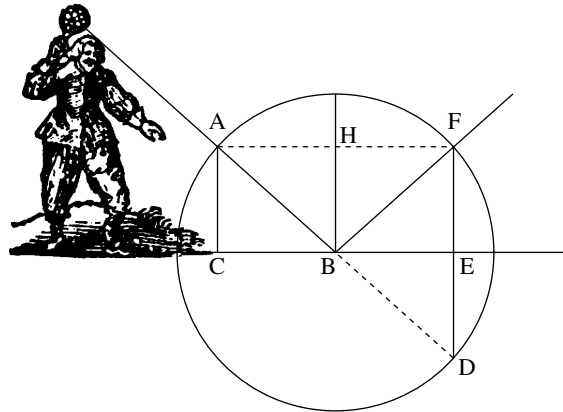


Fig. 2.10 (AT VI, 95)

Descartes takes up the traditional analogy between a light ray and a spherical body in motion, in this case a tennis ball, and illustrates the laws of reflection of light on the example of the reflection of a tennis ball struck by a racket onto the court (Fig. 2.10). He distinguishes first of all between the power (*puissance*) that makes the ball move and that which “determines it to move in one direction rather than another.”⁷³ It is the force (*force*) by which it was struck by the racket that causes the ball to move in the first place and to continue in motion after hitting the ground. But motion as such is directionless, being merely the amount of displacement in a certain time. On the other hand, it is the position (*situation*) of the racket, and later of the ground, that determines in what direction the ball moves. A different position of the racket attached to the same force would determine the motion of the ball in a different direction, just as the position of the ground determines the same motion of the ball in a different direction. On the other hand, a different force could be attached to the same position of the racket giving the ball more motion in the same direction. But, according to Descartes, motion and its determination are not independent magnitudes: the determination, which is defined for a given body by its speed and direction, can change without affecting the quantity of motion (if only the direction changes); but if the scalar quantity of motion changes, the determination also changes. That is, determination has not only a direction which is independent of motion but also a quantity which is dependent on motion. Since determination is a directed magnitude, it

⁷³*Dioptrics*, AT VI, 94.

can be divided into parts which are also directed magnitudes and which can vary in both *size and direction*. Descartes explains⁷⁴:

Moreover, it must be noted that the determination to move in a certain direction just as motion itself, and in general any sort of *quantity*, can be *divided* into all the parts of which we can imagine that it is composed. And we can easily imagine that the determination of the ball to move from A towards B is composed of two others, one making it descend from the line AF towards line CE and the other making it at the same time go from the left AC towards the right FE, so that these two determinations joined together direct it to B along the straight line AB.

Thus, while it is not yet completely clear what a determination is, it is sufficiently clear that the operation embodied in the parallelogram rule is *constitutive* of the meaning of the term: the diagonal represents the whole and the sides represent the parts.

In reflection the scalar motion of a tennis ball (or the ease of instantaneous passage of a light ray through the medium) is conserved so that the space traversed by the ball in a unit of time is the same before and after the collision with the reflecting surface. The scalar speed can be represented by any radius of the circle AFD (Fig. 2.10). The motion of the incident ball or ray has a (vector) determination represented by AB, which is by definition equal in absolute value to scalar speed, i.e., the directed distance AB is equal in length to the radius. It is important to remember that any particular, i.e., determinate, radius represents a determination; its length also represents the scalar speed. The surface of reflection CBE determines a coordinate system, in which opposition to the surface (interaction with the surface) is represented by perpendicular (vertical) vectors and non-opposition to the surface is represented by a parallel (horizontal) line pointed in either direction.

The actual determination AB of a ball's motion can be divided into two components, the one (AC) directly opposed (i.e., perpendicular) to the surface and the other (AH) not at all opposed (parallel) to the surface. Of course, as a purely mathematical exercise there are infinitely many possible divisions of the determination AB which can be obtained either by choosing an angle different from 90° between the two sides of the parallelogram or by keeping the right angle and choosing a different coordinate system. However, the physical theory in this case determines that line AC is the line of opposition and that angle ACB is a right angle; this in turn determines that the parallelogram used for calculation must be a rectangle with one side on the reflecting surface. The actual surface of reflection fixes the coordinate system, and the need for a clear cut distinction between opposition and non-opposition fixes the right angle between the component

⁷⁴AT VI, 94-5; emphasis added. This is the point that caused most of the misunderstandings which Descartes tried unsuccessfully to clarify in his letters. See AT II, 18-20; AT III, 163, 250-51; see documents 5.2.8, 5.2.17, and 5.2.18.

determinations. That is, as Descartes puts it, the interaction with the *real* surface *really* divides the determination into parallel and perpendicular components⁷⁵:

... the determination to move can be divided (I mean really divided and not at all in the imagination) into all the parts of which one can imagine it to be composed; there is no reason at all to conclude that the division of this determination, which is done by the surface CBE, which is a real surface, namely that of the smooth body CBE, is merely imaginary.

Although the motion of the ball is opposed to the rest of the immovable ground, the motion continues unabated, as we know from the conservation law. The perpendicular component determination is also opposed to the encounter with the reflecting surface and must therefore change in some unspecified way in order to resolve the double conflict. On the other hand, the parallel component determination is not at all opposed to the encounter with the surface and therefore does not interact with it, so that it is conserved unchanged. After the collision of the ball with the surface, this component determination can be represented by BE or any parallel line of equal length and in the same direction. Since the scalar motion is also conserved, the actual (compound) determination of the ball's motion must also have the same *absolute value* before and after collision. The perpendicular component determination involved in the interaction must change in such a way that it remains compatible with the conservation of scalar speed and the conservation of parallel determination; i.e., the vector sum of the parallel BE and whatever the new perpendicular determination turns out to be must be equal in absolute value to the radius of the circle, since the resultant actual determination is equal in absolute value to scalar speed. In geometrical terms, if the parallel determination (AH) is unchanged and the absolute value of the compound determination is also unchanged, then the ratio of the two must remain the same: $AH/AB = BE/BX$. This ratio expresses the *sine* of the angle made by the compound determination with the normal to the surface, i.e., what we now call the angle of incidence or reflection.⁷⁶ Since the sines of these angles are necessarily equal, the angles themselves are necessarily equal.

Returning to the figure, it can be seen that there are only two points where the conserved parallel determination and conserved scalar speed coincide: points F and D. Only one of these points lies *above* the surface. Here Descartes has used two conservation laws (motion and unopposed determination) and the opposition of determinations plus the qualitative consideration, that the ball or ray does not penetrate the surface, to derive unequivocally the angle of reflection. While it is clear that the result itself is nothing new, the conceptual and geometrical rigor of the inference is new; i.e., the law of reflection is shown to be a necessary consequence of the way the appropriate entities and their interactions were defined in

⁷⁵Descartes, letter to Mersenne, Oct. 5, 1637; AT I, 452. See document 5.2.6. The italics indicating where Descartes is quoting himself have been removed.

⁷⁶Descartes and his contemporaries sometimes used the term "angle of incidence" for the angle made by a ray with the *surface* and "angle of refraction" for the *deviation* of a refracted ray from its original line. See documents 5.2.4 and 5.4.3.

Descartes' conceptual system. The same kind of argument is then used to derive the law of refraction, which was entirely new.

Before we turn to refraction, one peculiarity of this argument should be stressed. Unlike the quantities of motion and matter, determinations are not conserved in the interactions of bodies, but only *in the absence of interactions*. Or only that *part* of a determination that is not involved in an interaction is conserved. An interaction involves a conflict of determinations and some or all of the conflicting determinations must undergo change. Exactly how an interacting determination or that part of it that interacts must change is not immediately given. It is not necessarily conserved, but there is no algorithm directly governing its change, except for the dependence of the magnitude of the compound determination on scalar speed.⁷⁷

2.5.1.2 Refraction

In refraction the surface separating two media fixes the coordinate system and the direction of opposition. We are to imagine that the refracting surface acts like a second tennis racket, that at the point of passage from one medium to the next increases or decreases the scalar speed of a tennis ball by a certain amount and also interferes with its downward determination. As in the case of the original tennis racket we must distinguish between the force imparted by the racket's power and the determination conferred by its position. The ball has a different speed in each medium, and the light ray requires a different amount of force or impetus to travel instantaneously through each different medium.⁷⁸ Whereas in reflection the scalar speed was conserved, in refraction a change in speed occurs that is specific to the relation between the two media. This change in speed at the point of transition is characterized by Descartes as an arithmetic compounding of motions in contradistinction to the compounding of determinations according to the parallelogram rule.⁷⁹ But nonetheless, in Descartes' example, if a ball moves with half its original air speed in water, we have a problem that is structurally similar to that of reflection.

⁷⁷In the original formulation of the Third Law of Nature in *Principia philosophiae*, cited in section 2.4.2 above, Descartes even says merely that the colliding body *loses* (*amittit*) its determination and only later adds that it acquires a new one.

⁷⁸In the *Dioptrics* Descartes says that light passes "more easily" in one medium than in another. As early as the manuscripts now known as the *Cogitationes Privatae* (1619-1621) Descartes had maintained that light passes more easily through a dense medium than through a rare medium (AT X, 242-3). Later, Descartes avoids the terms *dense* and *rare* when speaking of the different media. Since in his physics there is no vacuum, all matter should be equally dense. The media differ in that one is "more solid" (*durior*) or more fluid (*fluidior*) than the other. In a later letter for Hobbes (Jan. 21, 1641; AT III, 291) Descartes explains that less *impetus* is required in water than in air. See document 5.2.11.

⁷⁹Letter to Mydorge, March 1, 1638 (AT II, 19-20). See document 5.2.8.

To represent the *new* speed of the ball after entrance into the second medium, we can either construct a second circle with an appropriately altered radius,⁸⁰ or we can (with Descartes) use the same circle and radius but change the scale of measurement; the same length now represents, e.g., half or twice the motion and determination, and an unchanged determination is represented by a line of twice or half the length depending on whether the ball moves half or twice as fast in the second medium (Fig. 2.11).

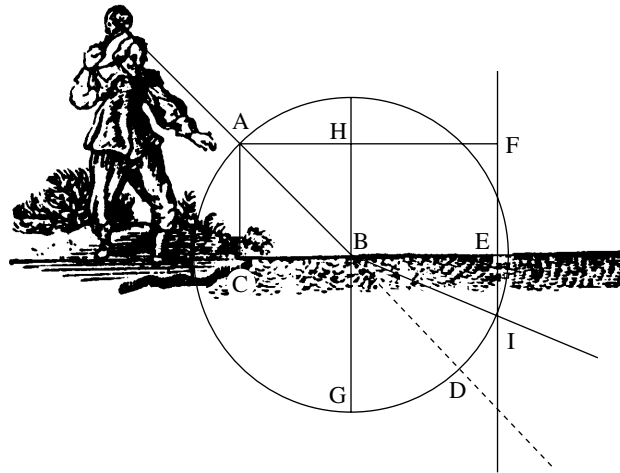


Fig. 2.11 (AT VI, 97)

Again, the horizontal component of the determination is not at all opposed to the encounter with the surface, and not being involved in any interaction, it is conserved without change. In Descartes' example, where the speed of the ball after refraction is half its speed before, the unchanged determination is represented by the line BE (which is *twice* as long as AH due to the change in scale). The vertical determination alone is opposed to the surface and must change in such a way as to be compatible with the new speed and the conserved parallel determination; i.e., the vector sum of changed perpendicular determination (whatever it turns out to be) and conserved parallel determination must be equal in absolute value to the new speed. There is only one point *below* the surface where this is true, namely I.

The new scalar speed and thus the absolute value of the new compound determination is dependent only on the medium; it is independent of the angle of inci-

⁸⁰Historical evidence strongly suggests that Descartes, like Harriot, Snel, and Mydorge originally worked with an altered radius, formulating the law of refraction in terms of a constant ratio of the lengths of the radii (cosecant form). See Schuster 1977, pp. 268-368, for a highly plausible reconstruction of the original path of discovery.

dence. Once again the ratio of the original parallel determination AH to the actual compound determination AB expresses the sine of the angle of incidence, and here the ratio of the unchanged parallel determination BE (differently represented) to the changed speed and compound determination BI (represented by the unchanged radius) in the new medium expresses the sine of the angle of refraction. The proportion between the two sines depends only on the factor of the change in scale of measure, which in turn depends on the ratio between the speeds in each medium; and this ratio is constant for any two given media. Thus, there is a constant relation between the sine of the angle of incidence ($\sin i = AH/AB$) and the sine of the angle of refraction ($\sin r = BE/BI$) for any two given media: $AH/AB = k BE/BI$. The constant ($k = \sin i/\sin r$) is an empirical magnitude that can in principle be ascertained by a single accurate measurement.

Having derived the inverse sine law for the refraction of a tennis ball, Descartes proposes an analogous argument for a ray of light. The inference from the behavior of the tennis ball to that of light is carried out in four steps. First, the surface of the tennis court in the example of reflection is replaced with a finely woven sheet which is punctured by the ball and in the process takes away half its speed. The sheet is then replaced by the surface of a body of water which is said to do much the same thing. Third, Descartes asks us to imagine that the surface of the body of water *increases* the speed of the ball instead of decreasing it, as if a second tennis racket acted at the surface to add (scalar) speed to the ball – in Descartes' example one-third more.⁸¹ Finally, the refraction of light is compared to this process.

Whereas the ball that moved more slowly in the water was supposed to break *away from* the perpendicular, light breaks *towards* the perpendicular. Descartes cannot argue that light moves faster in the second medium, since he has already defined it as the *instantaneous* transfer of action; but he does make a similar kind of argument by asserting that light passes more easily (i.e., needs less force or impetus) through the “more solid” medium than through the “more fluid” one.⁸² Thus, if light passes through water one-third more easily than through air, and if the *changed* ease of passage after interaction with the surface is represented by the *unchanged* radius of circle AFD (see Fig. 2.12), then the *unchanged* parallel component of the determination will be represented by the length BE (or GI), which is one-third shorter than AH. The intersection of line FEI below the surface with the circumference of circle AFD determines the path of the refracted ray BI. Since the relation between AH and GI is constant for any two given media

⁸¹Descartes does not stipulate that the racket “hits the incident ball perpendicularly, thus increasing its perpendicular velocity,” as Sabra (1967, p. 124) assumes; he says nothing about the slant of the racket, asserting merely that it should be thought to increase the scalar speed and not to affect the parallel component of the determination.

⁸²Descartes does not use the terms *dense* and *rare* for the different media since there is no empty space; but in this context he also avoids his own terms, *more solid* and *more fluid*. He just speaks of air and water.

and since AH and GI are proportional, respectively, to the sine of the angle of incidence and the sine of the angle of refraction, the ratio of the two sines is also constant.

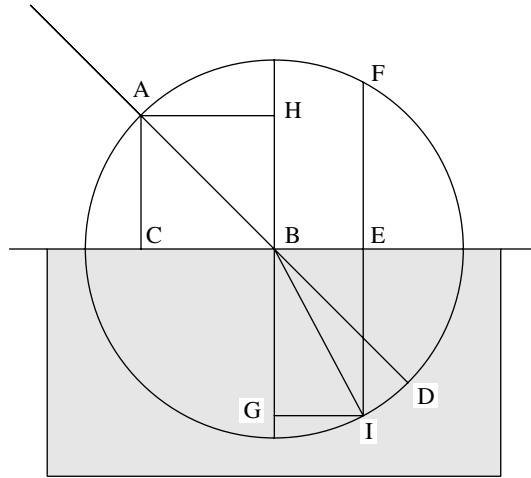


Fig. 2.12 (AT VI, 100)

Before turning to the two most important contemporary critics of Descartes' arguments in the *Dioptrics*, we should point out that although Descartes' results for light are correct from the point of view of classical mechanics, his analysis of the refraction of the *tennis ball* is incorrect. A material particle passing from air to water and continuing to move in the water with less than its former velocity breaks *towards* the perpendicular not away from it, just as light does. The reason for this is that the relevant magnitude is not simply the velocity of the particle, but rather its *momentum* in reference to the *density* of the medium; the energy needed to continue at half the speed in a denser medium can be considerably more than that associated with the full speed in the rarer medium.⁸³

2.5.2 Criticisms of the *Dioptrics*

After Descartes had completed the *Dioptrics*, Mersenne arranged for a critical review by Pierre Fermat in 1637 (in fact prior to the actual publication), and three years later he organized a second debate with Thomas Hobbes on the expla-

⁸³For a detailed explanation and the relevant equations see Joyce and Joyce (1966), who also give a list of modern physics textbooks that argue (wrongly) along the lines of Descartes.

nation of reflection and refraction. Both the exchanges with Fermat and Hobbes served to clarify the concept of determination simply through the fact that Descartes was compelled to explicate the concept more than he had in the *Dioptrics*, and we have already incorporated many of these clarifications into our presentation. But there were also new elements in the two debates that deal with questions of the philosophical legitimacy of concepts and techniques of inference used in the *Dioptrics*. Fermat's objections point out problems in the relation of geometry to physical theory; Hobbes' critique forces Descartes to explicate the logical status of the concept of determination.

2.5.2.1 Pierre Fermat

Fermat's difficulties with Descartes' theory, aside from very real misunderstandings which were never cleared up,⁸⁴ centered on the Descartes' use of geometry in making physical arguments. To put his objection simply: A line drawn on a sheet of paper can be merely an instrument for geometrical construction, in which case it need not have a physical meaning. Or else it can represent directed distance covered in a given time, in which case it stands for a *motion*. Fermat maintained that Descartes' determinations were either simply projections of motions on an arbitrary line of direction having no physical meaning, or else they were actual motions. If they are simply projections, then they cannot "make" the tennis ball or the light ray *do* anything at all. If the lines drawn and called determinations are supposed to have physical meaning and are resolved and compounded according to the parallelogram rule for the compounding of motions, then they are in fact really motions and must be treated as such. Fermat's first letter stresses the point about projections; his second letter expounds at length on the second point, the parallelogram rule.

In his first letter Fermat denies that Descartes' argument is a genuine proof or demonstration. He sees that determination cannot mean *simply* direction since it has a quantity and can be divided into parts, but direction remains the primary aspect in his interpretation. He takes determination to be the projection of the motion onto a particular direction. He does not take it to be a component of the motion, *and he does not apply the parallelogram rule*. In fact in the second letter he even accuses Descartes of having confused determinations with motions

⁸⁴The exchange with Fermat consisted of three letters: Fermat to Mersenne April or May 1637 (AT I, 354-363); Descartes to Mersenne Oct. 5, 1637 (AT I, 450-54); Fermat to Mersenne Nov. 1637 (AT I, 463-74). Descartes later discussed Fermat's comments in a letter to Mydorge March 1, 1638 (AT II, 15-23). Fermat continued to consider motion and determination as independent magnitudes, specifically, he thought that motion can change without the determination's changing. He admitted (at least for the sake of argument) this misunderstanding 20 years later in his correspondence with Clerselier and Rohault. But even in a letter after this admission, he still interprets Descartes' determination as direction. See Fermat 1891, vol. 2, pp. 397-8, and 486; see also Sabra 1967, p. 129, fn. 77.

because he applied the parallelogram rule to determinations.⁸⁵ He thus conceives determination as a measure of how far a body advances in a certain direction, and this measure is taken by dropping a perpendicular onto the line of direction (Fig. 2.13). Fermat then points out that the two directions onto which a body's motion

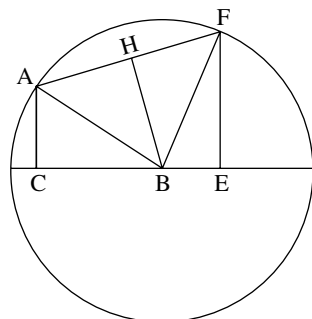


Fig. 2.13 (AT I, 359)

may be projected need not be at right angles. Instead of projecting the motion (AB) onto the vertical and the horizontal directions, Fermat suggests that we project it onto the vertical (AC) – since he does not doubt that the vertical represents the line of opposition – and onto a line (AF) that slants *away* from the surface and thus is also not opposed to it. This, he asserts, conforms to Descartes' specifications just as well as does the projection onto the horizontal. In fact there are an infinite number of directions which are unopposed to the surface and make an acute angle with the line of incidence AB.⁸⁶ If we extend the projection of the motion onto any

one of these lines the same length (from H to F) in the second unit of time, we arrive at a point on the circumference of the circle, which must according to Descartes' specifications show the location of the ball or light ray. The actual determination of a motion can be divided into all the parts one can imagine it to be composed of; it can be projected onto infinitely many directions, and it is simply arbitrary to single out the one line of direction which gives us the law of reflection we already know. These geometrical constructions have no physical meaning. Fermat sums up his critique⁸⁷:

It is therefore evident that, of all the divisions of the determination to motion, which are infinite, the author has taken only that which can serve to give him his conclusion; and thus he has accommodated his *medium* [i.e., middle term] to his conclusion, and we know as little about it as before. And it certainly

⁸⁵Fermat, letter to Mersenne for Descartes, Nov. 1637; AT I, 466 and 468. See document 5.2.7.

⁸⁶"Whereby it should be obvious that AF makes an acute angle with AB; otherwise, if it were obtuse, the ball would not advance along AF, as is easy to understand" (AT I, 359). Fermat's insistence that the angle BAF be acute makes it clear that he is talking about projections and not about the parallelogram rule. A line can only be projected on another line that makes an acute angle with it. This restriction does not apply to the side and the diagonal of a parallelogram; here, the angle made by the diagonal with either side may be obtuse as long as their *sum* is less than 180°. On this point we differ significantly with Sabra's interpretation. Sabra attempts to interpret Fermat (Fig. 2.13) as applying the parallelogram rule; this compels him to treat line HB as the line of opposition to the surface. Not only is there no textual basis for this, but it represents a position that would have been unique in the 17th century. See document 5.2.5.

⁸⁷Fermat, letter to Mersenne, April or May 1637; AT I, 359. See document 5.2.5.

appears that an imaginary division that can be varied in an infinite number of ways never can be the cause of a real effect.

In his second letter Fermat takes up the use of the parallelogram rule in the explanation of refraction. He argues that if two independent forces act on a body, they will “interfere with and resist one another.”⁸⁸ Thus the size of the resultant force or motion of the body will depend on the angles at which the forces impinge on it; the greater the angle, the smaller the velocity of the compound motion.

If the angle of refraction is to be found by applying the parallelogram rule, then it is actually forces or motions that are being resolved and compounded, not simply projections onto lines. If the surface of refraction (CBE), say, adds mo-

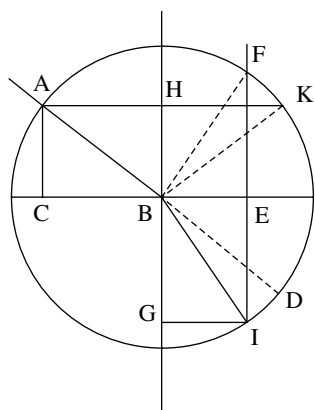


Fig. 2.14 (AT I, 473)

tion to the tennis ball at the point of transition from one medium to the next and only acts in the perpendicular, then the resultant force is to be calculated as the diagonal of the parallelogram whose sides are the (continued) original motion BD and the (added) perpendicular motion BG (Fig. 2.14). The diagonal of this parallelogram will be longer the more acute the angle GBD between the two motions; and the size of the resultant or compound motion of the body will depend on the angle of incidence. Thus the resultant speed of the body cannot be constant for the medium and independent of the angle of incidence as Descartes had maintained.⁸⁹

Fig. 2.14 (AT I, 473)

To sum up Fermat's critique: If the lines Descartes has drawn are simply projections onto directions, they have no physical meaning; but if they are to be compounded and resolved according to the parallelogram rule for the compounding of motions, they must represent actual motions, and operating with them leads to results quite different from those arrived at by Descartes. The geometrical techniques of inference used by Descartes may, according to Fermat, only legitimately be applied to actual motions.

In subsequent responses to Fermat's criticisms Descartes says that the same determination may be "attached" to different motions thus apparently lending credence the misunderstanding that determination is merely direction. However in both passages in question the context makes it clear that he is talking only about component determinations: the same component determination that is not involved in an interaction may indeed be attached to the altered motion after

⁸⁸Fermat, letter to Mersenne, Nov. 1637; AT I, 470. See document 5.2.7.

⁸⁹Fermat, letter to Mersenne, Nov. 1637; AT I, 473-474.

interaction, but of course the compound determination changes in magnitude with the motion.⁹⁰

2.5.2.2 Thomas Hobbes

Descartes' exchange of letters with Hobbes from early 1641 sheds light on the logical status of the concept of determination. We have already seen that determination is a mode of motion, which is itself a mode of bodies. One of the major points of this new discussion was the logical-ontological status of the entities resolved and compounded according to the parallelogram rule; and Hobbes also raises the problem of the compatibility of this rule with the conservation of motion.

Hobbes raised a number of objections to Descartes' concept of determination, many of them similar to those of Fermat, in a long letter to Mersenne (Nov. 5/15 1640) sent just before he departed from England. This letter included some 56 folio pages of a treatise on some optical questions, parts of which Mersenne had copied and forwarded to Descartes by way of Constantin Huygens in two installments: 3 leaves containing criticisms of Descartes' *Dioptrics* and 8 leaves containing Hobbes' own explanation of refraction.⁹¹ Both extracts as well as the original letter are lost. However, a chapter on refraction in Mersenne's *Optica* (1644) ascribed by Mersenne to Hobbes, gives some indication of what was contained in the second installment;⁹² and a Latin manuscript on optics that Hobbes had had copied for his patron Cavendish in 1640 some months before departing for the continent contains most of the criticisms to which Descartes replied in his first letter.⁹³ The content and vocabulary of this reply clearly reflect that of

⁹⁰Descartes, letter to Mydorge, March 1, 1638; AT II, 17-18. Letter to Mersenne, July 29, 1640; AT III, 113. For an analysis of this problem see McLaughlin 2000.

⁹¹The exchange with Hobbes consisted of eight letters, starting with the two extracts from the lost letter of Hobbes made by Mersenne and sent to Descartes by way of Huygens: Mersenne to Huygens (received Jan. 20 and Feb. 18, 1641); Descartes to Mersenne, Jan. 21, 1641 (AT III, 287-392); Hobbes to Mersenne, Feb. 7, 1641 (AT III, 300-313); Descartes to Mersenne, Feb. 18, 1641 (AT III, 313-318) – this letter deals with Hobbes's own optical work (see Shapiro 1973); Descartes to Mersenne, March 4, 1641 (AT III, 318-333); Hobbes to Mersenne, March 30, 1641 (AT III, 341-348); Descartes to Mersenne, April 21, 1641 (AT III, 353-357). This exchange overlapped with Hobbes's "Objections" to Descartes' *Meditations*.

⁹²The *Optica* is part of Mersenne's compilation *Universae Geometriae ... synopsis* (Mersenne 1644b) OL V, 215-248. The actual title given by Mersenne to Hobbes's work is "Opticae, liber septimus," but since it was published by Molesworth in Hobbes's *Opera Latina* under the title *Tractatus opticus*, it is now known under that name. We have been very much helped in sorting out a number of the technical details concerning Hobbes's manuscripts by the sound advice of Frank Horstmann.

⁹³A preliminary transcription of this manuscript was published by Ferdinand Alessio in 1963 under the title *Tractatus opticus*, but since another work of Hobbes published by Mersenne in 1644 had long been known under that title, this manuscript has usually been referred to as *Tractatus opticus II*. Scholars were long uncertain as to its

parts of the Hobbes manuscript. Furthermore, there is also a passage in Mersenne's *Ballistica* (1644) that presents an argument not contained in the 1640 optical manuscript, which deals with questions raised in Descartes' response to the first letter.⁹⁴ The content of Hobbes's critique can thus be reconstructed from these writings.

To the first installment of Hobbes's remarks Descartes replied that the sides of the parallelogram should not be called motions or even determinate motions as suggested by Hobbes, lest this lead to confusion, since motions are added arithmetically and not according to the parallelogram rule, which was to be reserved for determinations.⁹⁵ Hobbes responded that the confusion can easily be avoided and the diagonal sum reconciled with an arithmetical sum. Each of the sides of a parallelogram can be examined to see how much it actually *contributes* to the resultant diagonal (Fig. 2.15).

In effect, Hobbes resolves the directed motion BC into two components, BA and AC; he then resolves BA and AC in turn into two components each (BF and FA; AD and DC), one of which is parallel to the diagonal BC and one of which is perpendicular to it. The two subcomponents that are parallel to the diagonal

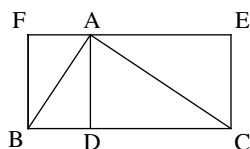


Fig. 2.15

(FA and DC) add up as vector and as scalar magnitudes to the diagonal; these diagonal subcomponents can be seen as the *real contribution* of the component motions to the resultant motion. (Hobbes does not say what happens to the other subcomponents (BF and AD), which are perpendicular to the diagonal, but they are in fact equal and opposite and could have been seen to cancel each other out.) What Hobbes is actually saying is that the diagonal can be conceived as (vectorially) compounded of the sides

actual date, though most of them, following Brandt 1928, dated it later than the exchange with Descartes. There is, however, a great deal of evidence both internal and external indicating that the *Tractatus opticus II* dates from 1640: The manuscript is in the hand of a scribe not in the regular employment of the Cavendish family, who also copied a number of other manuscripts in 1640; as pointed out by G. C. Robertson in 1886, the figures and many corrections to the manuscript are in Hobbes's own hand; and some of the corrections on the basis of their *content* can only have been made by Hobbes, who left England in Nov. 1640 for ten years. For details about this manuscript, see Malcolm in HC I, liv–lv, and Tuck 1988; for a reconstruction of the content of the lost letter, see Schuhmann 1998. Some important passages from this manuscript can be found in document 5.2.9. Our translation there is based on a new transcription made by Karl Schuhmann to be published in the near future, which he has generously made available to us prior to publication.

⁹⁴The *Ballistica* is part of Mersenne's compilation *Cogitata physico-mathematica* (Mersenne 1644a), some parts of which are unquestionably derived from Hobbes. The editors of Mersenne's *Correspondance* (MC 10, 577) attribute the content of this argument to Hobbes, but this remains conjectural. See document 5.2.10.

⁹⁵Descartes, letter to Mersenne for Hobbes, Jan. 21, 1641, AT III, 288. Figure 2.15 is reconstructed from the diagram in Descartes' response. See documents 5.2.11–5.2.15.

because it is actually (vectorially and arithmetically) compounded of the effective subcomponents of the sides. The sides compose the diagonal because each contributes a part of itself to it.⁹⁶ Descartes rejects this argument as patently absurd: a subject must actually consist of what it is said to be compounded of. Hobbes is in effect asserting that a motion may be said to be compounded of two other motions even if it contains merely a *part* of (a contribution from) each of them. This is, Descartes remarked, “just the same as to say that an axe is composed of the forest and the mountain because the forest contributes wood for the handle and the mountain [contributes] iron dug up from it.”⁹⁷ If the entity represented by the diagonal is to be compounded out of the entities represented by the sides, then it must *contain* them, not just part of them but all of them.

This argument by Descartes may well show that *motions* may not be resolved and compounded according to the parallelogram rule, but it certainly does not explain why anything else could be so compounded and resolved. It must still be explained why component *determinations* can be conceived of as genuine parts of the resultant determinations.

Descartes makes two comparisons which indicate how he wants the modes of modes to be conceived and how determinations can be considered to have *parts*. First of all, one can compare the motion of a body to the other fundamental mode of bodies, namely, shape and compare the determination of motion to that of shape.⁹⁸ Just as any real motion is a determinate motion, any real shape is a particular, determinate shape. But just as one can distinguish between a particularly shaped body, e.g., a flat or round body, and its flat or round *surface* (its “flatness” or “roundness”), so too can one distinguish between a determinate motion and the *determination* of the motion. A determination can be divided into parts just as a surface can be so divided. Let a body have a determinate shape, say it is a cube. Although the surface of the cube can be divided into six faces out of which it can be said to be compounded, this division of the surface does not divide the body itself. The parts (six square faces) of the surface of the cube are not the same as the surfaces of parts of the cube. Thus, too, the parts of the determination of a motion are not the determinations of the parts of the motion.⁹⁹ The

⁹⁶Hobbes, letter to Mersenne, Feb. 7, 1641; AT III, 304-5: “In as much as the motion from A to B [i.e., B to A; see Fig. 2.15] is composed of the motions from F to A and from F to B [i.e. B to F], the compounded motion AB does not *contribute* more speed to the motion from B towards C than the components FA, FB can *contribute*; but the motion FB *contributes* nothing to the motion from B towards C: this motion is determined downwards and does not at all tend from B towards C. Therefore only the motion FA *gives* motion from B to C ...” (emphasis added). Hobbes makes a number of minor technical mistakes in this letter (which Descartes harps on and corrects); they do not however affect the substance of his argument (to which Descartes also replies). See documents 5.2.12 and 5.2.13.

⁹⁷Descartes, letter to Mersenne, March 4, 1641; AT III, 324.

⁹⁸Descartes, letter to Mersenne, March 4, 1641, AT III, 324-5.

⁹⁹This point is made more clearly in an earlier letter to Mersenne (July 29, 1640; AT III, 113), from which the details of the example are taken. See document 5.2.16. and for a more detailed analysis of this argument see McLaughlin 2000.

general principle to which Descartes is implicitly appealing here is that *the parts of a mode of a subject are not the modes of the parts of the subject*. Therefore, the sides of the parallelogram, which represent the component parts of the determination of the body's actual motion cannot be taken independently as if they were determinations of component parts of a motion. They have meaning only in relation to a particular diagonal.

In order further to explicate the difference between a determinate motion and the determination of the motion, Descartes takes up an example from logic suggested by Hobbes which also gives some indication of how a determination can be divided into components.¹⁰⁰ The example involves proper names and singular terms, cases of what medieval logic sometimes called discrete or determinate supposition.¹⁰¹ Although every man is a determinate man, for instance, Socrates, and thus "man" and "Socrates" can refer to the same entity or *suppositum*, the two terms have different meanings. The meaning of Socrates contains the "individual and particular differences" which individuate Socrates. Furthermore, one can distinguish conceptually between the individual denoted by Socrates and the determination of being Socrates, i.e., the individual properties ascribed to him. If one of the individual differences that make up what it is to be Socrates were to disappear, the man would still be a determinate man but would no longer have the determination of being Socrates. Descartes' argument implies that the determination of being Socrates is composed of component determinations such as, "for instance, the knowledge he had of philosophy." The parts of the determination of a motion can no more be ascribed to parts of the motion than can Socrates' knowledge of philosophy be ascribed to one of his parts, e.g., his arm or leg. One such component determination can change without affecting the other *component* determinations. The component parts of the determination of a motion are to be conceived as elements of the definite description of that particular motion.¹⁰²

2.5.3 Oblique Impact

Descartes' comparison of a determinately moving body with a cubical body is systematically important because it shows that it is logically precluded that the component determinations be taken as determinations of parts of the motion itself. Thus, the sides of the parallelogram have no meaning independent of the resultant and may not be detached and operated upon separately. No more than Socrates' knowledge of philosophy can be predicated of his elbow, can the verti-

¹⁰⁰Letter to Mersenne, April 21, 1641, AT III, 354-6. See document 5.2.15.

¹⁰¹See Kneale and Kneale 1969, p. 258ff.

¹⁰²Letter to Mersenne, April 21, 1641, AT III, 354-6. J. M. Keynes (1906, p. 469) still calls the components of a "complex term," e.g., "A and B and C ..." *determinants* of the term.

cal component of the determination of a motion be ascribed to a part of that motion. No contradiction can occur between the scalar conservation law and the parallelogram rule, as long as the sides of the parallelogram are interpreted as components determinations, as parts of a mode of motion.

The only other case in which a direct conflict could arise between these two fundamental elements of Cartesian physics would be in the oblique collision of bodies. Here the sides of the parallelogram could represent the motions of *two different bodies* caused by the motion of a third body (represented by the diagonal), which collides with them at an angle and *causes* them to move as the sides of a parallelogram while itself ceasing to move. Such a case would indeed lead to a contradiction with the conservation of scalar motion, but only if such a collision were governed by the impact rules of classical mechanics. Let us conclude with a look at Descartes' handling of oblique impacts.

Descartes never deals explicitly and quantitatively with the kind of three-body collision sketched above. All his impact rules apply to two-body collisions. On a number of occasions, however, he did deal with a similar phenomenon in the transfer of the action of light.¹⁰³ He often uses the example of a body *pushing* against *two* others obliquely which in turn both push obliquely against a fourth body. It is clear that he considers this to be equivalent to having one body push a second which pushes a third. But he is dealing with the transfer of action through bodies which are in some way constrained by their surroundings (see Fig. 2.16), not with the free motions of the bodies themselves. At one point, however, he

does maintain that a three-body collision is in fact two successive two-body collisions,¹⁰⁴ so that it can at least in principle be reduced to the two-body collisions governed by the impact rules. It remains to be seen whether an interpretation of a two-body oblique collision can be given that is consistent with the scalar conservation law, the impact rules of the *Principia philosophiae*, and Descartes' use of the concept of determination.

In one letter to Mersenne¹⁰⁵ Descartes does deal in a quantitatively precise way with a case of oblique impact. He considers the case of a large moving ball colliding obliquely with a small resting ball; the case should thus be subsumed under impact Rule 5 discussed above. Although a moving body cannot normally make another body move faster than it itself moves, this applies, Descartes tells us, only to motion in the same straight

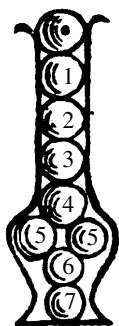


Fig. 2.16 (AT XI, 100)

¹⁰³See AT II, 370; VIII, 187; and XI, 100.

¹⁰⁴“Since the continuous motion of these [balls] brings it about that this action is never, in any period of time, received simultaneously by two, and that it is transmitted successively, first by the one and then by the other” (AT VIII, 187; *Principia*, III, §135).

¹⁰⁵April 26, 1643, AT III, 648-655; see document 5.2.19.

line. It does not necessarily apply when the motions are in different lines (see Fig. 2.17 and Fig. 2.18).¹⁰⁶

Thus, for instance, if the large ball B coming from A to D in a straight line encounters the small ball C from the side, which it makes move towards E, there is no doubt whatsoever that even if these balls are perfectly solid [*dures*], the little one ought to leave more quickly than the large one moves after having encountered it; and constructing the right angles ADE and CFE, the proportion which holds between the lines CF and CE is the same as holds between the speed of the balls B and C.

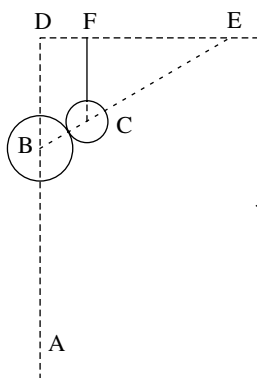


Fig. 2.17 (AT III, 652)

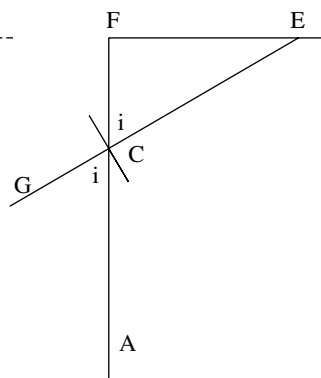


Fig. 2.18 (our figure)

This must of course be interpreted. We assume that Descartes meant to say that the lines CF and CE also represent the actual paths along which the two balls move after collision.¹⁰⁷ We also assume that the line CE is the line of direct opposition between the two bodies, i.e., the normal through their centers of gravity. Both assumptions are suggested by Descartes' illustration. Although Descartes does not explicitly mention determinations in this passage, it can be seen that they do indeed play a role; and the relation of determinations to motion was directly thematized one page earlier in the letter. While the *length* of the line CF represents the speed of the larger ball B after collision, the line CF itself can be taken to represent not only the actual determination of B but also the vertical component determination of C. Thus although the speed of C is greater than that of B, their determinations in B's original direction are equal. Thus B transfers enough motion to C to make it move with the same vertical determination as B has but in the direction determined by the line of direct opposition, i.e., along CE. The proportion between CF and CE, which Descartes says is also the proportion between the speeds of B and C, expresses the cosine of the angle FCE,

¹⁰⁶AT III, 651-2; document 5.2.19.

¹⁰⁷This condition implies that both bodies can be conceived as points (i.e., that Fig. 2.17 [Descartes'] and Fig. 2.18 [ours] are equivalent); it is a conclusion that is difficult to reconcile with Descartes' definition of material bodies.

which is equal to angle ACG, the angle of incidence of ball B. Thus the ratio of the two speeds after collision is a function of the angle of incidence. The speeds can then be calculated.

Let v_1 , u_1 be the speeds of ball B before and after collision; and let u_2 be the speed of ball C after collision ($v_2 = 0$) and let i be the angle FCE, which is equal to the angle of incidence ACG, then: $u_1/u_2 = CF/CE = \cos i$; i.e., the proportion between the speeds of the two balls is determined by the angle of incidence, or $u_1 = u_2 \cos i$.

Assuming that the quantity of motion in the two-body system is conserved, then:

$$Bv_1 = Bu_1 + Cu_2 = u_1 \left(B + \frac{C}{\cos i} \right)$$

and:

$$\frac{v_1}{u_1} = \frac{B}{B} + \frac{C}{B \cos i} = \frac{C}{B \cos i} + 1.$$

The fifth impact rule, as we saw in section 2.4.4, can be formulated as

$$Bv_1 = u_1 (C+B) \text{ or } \frac{v_1}{u_1} = \frac{C+B}{B} = \frac{C}{B} + 1.$$

Thus, impact Rule 5 can be seen as a special case of $\frac{C}{B \cos i} + 1$ where $i = 0$ and accordingly $\cos i = 1$.

Thus, at least in the one case where Descartes actually calculates an oblique collision, there need be no contradiction to the scalar conservation law. And, in fact, assuming the conservation law allows us to derive an even more general formula for the impact rule governing the collision. Since the conservation of scalar motion is one of the construction principles of any rule within the system for calculating impacts, this is not surprising. It merely shows that instead of a contradiction in the system between conservation of scalar momentum and empirically adequate rules of impact, we find a consistent system, from which impact rules are derived that are empirically inadequate.

2.6 Conclusion

Our discussion of Descartes' application of conservation laws, the logic of contraries, the parallelogram rule, and the concept of determination has shown that he successfully inferred by means of the conceptual system they constituted not only consistent laws of impact but also a nontrivial and empirically adequate law of refraction. And there can be little doubt that his system provided the basis for the later introduction of the concepts of (instantaneous) velocity and momentum and for the generalized application of the parallelogram rule in mechanics.

However, we have also seen the great difficulties that Descartes' contemporaries had in applying some of his concepts.

Descartes is probably the only person who ever used his concept of determination productively in physics. His followers were unable to make any advances in science by applying the concept to explain new cases or to integrate disparate elements more coherently into a conceptual system. Although the *term* itself was retained and used on into the 18th century, due primarily to the popularity of Jacques Rohault's textbook, *Traité de Physique*, it came to mean simply *direction*. Rohault's presentation is faithful to Descartes' use of the concept – in fact it is simply a paraphrase –, but he applies the concept only to those cases already handled by Descartes. When Spinoza in his presentation of Descartes *Principia* attempted to deal with a somewhat more complex case of oblique collision (in which *both* bodies are in motion), he introduced a “*force* of determination” thus making determination independent of the “*force* of motion.”¹⁰⁸ But perhaps the most illustrative case of the later use of the concept of determination is given by Clerselier, who a dozen years after Descartes' death renewed the debate with Fermat on reflection and refraction. When he tried to explain in his own way what happens in reflection, he did not derive the change in the perpendicular determination from the conservation of parallel determination and scalar motion. He simply asserts that the determination itself is *reversed* and thereupon derives the *compound* determination: when balls collide with the surface of reflection, “they will be constrained to *change the determination* which they have to go [downwards] ... *into that* to go or reflect [upwards].”¹⁰⁹ Clerselier is compounding and resolving independent entities according to the parallelogram rule; he does not need, nor does he in this case use, Descartes' circles representing scalar motion.

It proved almost impossible to apply the concept of determination to cases which Descartes had not already explained. It was not an everyday tool that could be used by any competent scientist; it was an idiosyncratic instrument that could be employed only by its inventor. One could not simply manipulate the representations of determinations and draw inferences: the slightest deviation from Descartes' terminology (the linguistic representation of the conceptual relations in the theoretical system) led to errors of reasoning. But these errors were not simply at random: they generally involve either taking determination as mere *direction* or as an *independent* magnitude. Both errors are allowed or even suggested by the mathematical means of representation. There is nothing in the geometrical representation itself of determination nor in the construction rules of geometry that prevents the sides of a parallelogram from being treated as independent magnitudes. The conceptual dependence of determination on motion is represented only in language not in the geometrical means of inference. Keeping the non-

¹⁰⁸Spinoza 1925, vol. 1, pp. 213-216; *Renati Des Cartes...*, II, Prop. 27.

¹⁰⁹Clerselier, letter to Fermat, May 13, 1662; *Oeuvres de Fermat*, vol. 2, pp. 478-9. See the Epilogue (section 4.2) and document 5.4.2.

represented conceptual relations in mind while operating with the geometrical representations is a *tour de force*, not the application of an algorithm.

While such virtuosi as Hobbes and Fermat were able to argue with Descartes about the philosophical background of his theories, other scientists either accepted or rejected his explanations without real comprehension. Descartes' exchange with Bourdin is a good case in point. Bourdin sees that Descartes' explanation of refraction makes use of right triangles for which the Pythagorean relations hold; a determination of "4 to the right" combines with one of "3 downwards" to make a resultant of 5. But instead of saying with Descartes: a *ball* covers 4 units to the right due to its determination, he says the *determination* covers 4 units or the determination carries the ball 4 units.¹¹⁰ Descartes maintains that such seemingly minor changes totally misrepresent the theory; he points out that according to his theory it is not the determination that conducts but the force itself as determined (*sed virtus ipsa ut determinata*). He has, he thinks, explicitly excluded the possibility of manipulating the geometrical representation in the manner suggested by Bourdin. In fact Descartes' refrain every time Bourdin interprets the geometrical representation in his own words is: "quod ineptum est." However, Bourdin was not alone; even the Latin translator of the *Dioptrique* only a few years later translated similar mistakes into the Latin text. Where Descartes, using the language of indirect causality says the determination "makes" the ball move (*la fait aller, fait descendre, faisoit tendre*), the Latin simply says it moves the ball: *agebat, propellit, ferebatur*.¹¹¹

But even Descartes was not virtuoso enough for his own system; for in one of his later comments on Bourdin he, too, makes a mistake: "But I believe that what perplexes him is the word *determination*, which he wants to consider without any motion, which is chimerical and impossible; in speaking of the determination to the right, I mean all that *part of the motion* that is determined towards the right ... [*qu'en parlant de la determination vers la droite, j'entens toute la partie du mouvement qui est determinee vers la droite*]." ¹¹² It should by now have become clear that a component determination is neither a *part* of a motion nor even the determination of such a part.

Now even if Descartes' genius were taken as an explanation of his ability to invent and apply fruitfully the conceptual system of which determination was an essential component, it is the fact that genius was actually required to apply the concept fruitfully that explains why the system had no future in science. Whether or not genius is necessary to invent a new concept, the new concept must be integrated into a representational system whose use does not require genius, if it

¹¹⁰Descartes cites Bourdin's remarks in a letter to Mersenne (July 29, 1640; AT III, 105-119). For Descartes comments on Bourdin, see documents 5.2.16-5.2.18.

¹¹¹See AT VI, 95-97 and 591-92. On the terminology of indirect causality in the 17th century, see Specht 1967, pp. 29-56.

¹¹²Letter to Mersenne, Dec. 3, 1640 (AT III, 251; second emphasis added). See document 5.2.18. Unaccountably, Gabbey (1980, p. 259) cites this passage as "the nearest Descartes came to a clear definition of the notion."

is to become part of a scientific tradition. The scientific adequacy of conceptual means is inversely proportional to the genius required for their application. The virtuosity needed to work with Descartes' system of explanations is a reliable indicator of the deficiency of the system.

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