

Preface

The Journey Ahead

At the heart of transcendental number theory lies an intriguing paradox: While essentially all numbers are transcendental, establishing the transcendence of a particular number is a monumental task. Thus transcendental numbers are an enigmatic species of number: We know they are all around us and yet it requires enormous effort to catch one. More often than not, they slip through our fingers and disappear back into the dense jungle of numbers. Here we will venture to tame a few of these incredible creatures. In the pages ahead we offer an approach to transcendence that not only includes the intricate analysis but also the beautiful ideas behind the technical details.

The phrase “classical transcendental number theory” in the title of this book refers to the most widely known results that were obtained in the nineteenth and early twentieth centuries. The reason for this focus is threefold. Firstly, this body of work requires only the mathematical techniques and tools familiar to advanced undergraduate mathematics students, and thus this area can be appreciated by a wide range of readers. Secondly, the ideas behind modern transcendence results are almost always an elaboration of the classical arguments we will explore here. And finally, and perhaps more importantly, this early work yields the transcendence of such admired and well-known numbers as e , π , and even $2^{\sqrt{2}}$.

While the theory of transcendental numbers is a fundamental and important area of number theory, it is not widely known. Part of the reason for its relative obscurity is that upon first, or even second or third inspection, the arguments and techniques employed appear to be formidable if not impenetrable. In truth, however, the underlying principles upon which the entire discipline is based are both straightforward and central to all of number theory.

Our quest is to see beyond the complexity and intricacies of the subject and develop some intuition into the delicate ideas that are at the center of transcendental number theory. Thus we endeavor not only to present the proofs of the transcendence results we will encounter, but more importantly to make those results and their justifications natural and intuitively sound. Our desire is to bring some of the beautiful ideas from one of the richest areas of mathematics to life, and to inspire further explorations.

Making Transcendence Transparent

An intuitive approach to classical transcendental
number theory

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