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## Preface

This book is a comprehensive and accessible introduction to the *cross-entropy* (CE) method. The CE method started life around 1997 when the first author proposed an adaptive algorithm for rare-event simulation using a cross-entropy minimization technique. It was soon realized that the underlying ideas had a much wider range of application than just in rare-event simulation; they could be readily adapted to tackle quite general combinatorial and multi-extremal optimization problems, including many problems associated with the field of learning algorithms and neural computation.

The book is based on an advanced undergraduate course on the CE method, given at the Israel Institute of Technology (Technion) for the last three years. It is aimed at a broad audience of engineers, computer scientists, mathematicians, statisticians and in general anyone, theorist or practitioner, who is interested in smart simulation, fast optimization, learning algorithms, image processing, etc. Our aim was to write a book on the CE method which was accessible to advanced undergraduate students and engineers who simply want to *apply* the CE method in their work, while at the same time accentuating the unifying and novel mathematical ideas behind the CE method, so as to stimulate further research at a postgraduate level.

The emphasis in this book is placed on concepts rather than on mathematical completeness. We assume that the reader has some basic mathematical background, such as a basic undergraduate course in probability and statistics. We have deliberately tried to avoid the formal “definition – lemma – theorem – proof” style of many mathematics books. Instead we embed most definitions in the text and introduce and explain various concepts via examples and experiments. In short, our goal is to promote a new unified way of thinking on the connection between rare events in simulation and optimization of complex systems in general, rather than burden the reader with too much technical detail.

Most of the combinatorial and continuous multi-extremal optimization case studies in this book are benchmark problems taken from the World Wide Web, and CE was compared with the best known solutions. In *all* examples

tested so far the relative error of CE was within the limits of 1-2% of the best known solution. For some instances CE produced even more accurate solutions. It is crucial to emphasize that for the “noisy” counterparts of these test problems, which are obtained by adding random noise to the objective function, CE still performs quite accurately, provided the sample size is increased accordingly. Since our extensive numerical experience with different case studies suggests that CE is quite reliable (by comparing it with the best known solution), we purposely avoided comparing it with other heuristics such as simulated annealing and genetic algorithms. This of course does not imply that one will not find a problem where CE performs poorly and therefore will be less accurate than some other methods. However, when such problems do occur, our FACE (fully adaptive CE) algorithm (see Chapter 5) should identify it reliably.

Chapter 1 starts with some background on the cross-entropy method. It provides a summary of mathematical definitions and concepts relevant for this book, including a short review of various terms and ideas in probability, statistics, information theory, and modern simulation. A good self-contained *entry point* to this book is the tutorial Chapter 2, which provides a gradual introduction to the CE method, and shows immediately the elegance and versatility of the CE algorithm. In Chapter 3 we discuss the state of the art in efficient simulation and adaptive importance sampling using the CE concept. Chapter 4 deals with CE optimization techniques, with particular emphasis on combinatorial optimization problems. In Chapter 5 we apply CE to continuous optimization problems, and give various modifications and enhancements of the basic CE algorithm. The contemporary subject of noisy (stochastic) optimization is discussed in Chapter 6. Due to its versatility, tractability, and simplicity, the CE method has great potential for a diverse range of new applications, for example in the fields of computational biology, graph theory, and scheduling. Various applications, including DNA sequence alignment, are given in Chapter 7. A connection between the CE method and machine learning — specifically with regard to optimization — is presented in Chapter 8. A wide range of exercises is provided at the end of each chapter. Difficult exercises are marked with a \* sign. Finally, example CE programs, written in Matlab, are given in the appendix.

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