

## Foreword

Due to the continuing progress of sensor technology, the availability of 3-D cameras is already foreseeable. These cameras are capable of generating a large set of measurement points within a very short time. There is a variety of 3-D camera applications in the fields of robotics, rapid product development and digital factories. In order to not only visualize the point cloud but also to recognize 3-D object models from the point cloud and then further process them in CAD systems, efficient and stable algorithms for 3-D information processing are required. For the automatic segmentation and recognition of such geometric primitives as plane, sphere, cylinder, cone and torus in a 3-D point cloud, efficient software has recently been developed at the Fraunhofer IPA by Sung Joon Ahn. This book describes in detail the complete set of 'best-fit' algorithms for general curves and surfaces in space which are employed in the Fraunhofer software.

Best-fit algorithms estimate curve/surface parameters by minimizing the shortest distances between the curve/surface and the measurement points. In many tasks of industrial image processing and optical 3-D measuring techniques, the fitting of curves and surfaces to a set of points plays an important role. Not only within the range of coordinate metrology and quality control but also for applications of automatic real object reconstruction, the fast and accurate fitting of geometric primitives will be of great benefit. By employing the best-fit algorithms in the Fraunhofer software, it is possible to reconstruct the geometric primitives in a way which is both CAD- and construction-friendly. The geometric primitives are obtained with all the necessary model parameters defining the form and orientation of the geometric primitives in space.

In principle, the software developed at the Fraunhofer IPA can be used everywhere where geometric primitives need to be quickly and automatically recognized from a given point cloud. Thus robotics represents a broad area of application of the Fraunhofer software, including the navigation of autonomous vehicles or the grasping of workpieces. A further application lies in factory digitization; this is concerned with the construction of a virtual 3-D production environment from an existing factory building which can be used, for example, as the starting point for simulating or planning the factory. Here, immense measurement data is produced and it is extremely time-consuming and therefore cost-intensive to process them manually. The automatic recognition of geometric primitives enables efficient factory digitization

because a factory building can be composed by extensively using such simple geometric elements as piping (torus, cylinder) and planes.

The Fraunhofer software is able to segment a 3-D point cloud automatically. Without further user interaction, the suitable geometric primitives are fitted to the point cloud and the associating subsets of the point cloud are extracted at the same time. In order to accomplish this segmentation, no pre-treatment of the point cloud is required by hand. Furthermore, the point cloud does not have to fulfil special conditions. Without any point order or point arrangement, it is possible to process a large point cloud. The real objects must thereby not be completely covered by the measurement points. Even the incomplete collection of object points, e.g. due to shadings caused by the limited accessibility of the measuring devices to the object surface – which occurs frequently with optical sensors – does not cause any difficulties. Also, if a point cloud has been contaminated by severe measurement errors, it is still possible to obtain very good results. With these characteristics, the Fraunhofer software is applicable to a wide range of engineering areas where fast and accurate processing of 3-D point clouds is carried out.

Sung Joon Ahn has been working as a research scientist for the Fraunhofer IPA in Stuttgart, Germany, since April 1990. For his doctoral thesis, Dr. Ahn was awarded his research doctorate with the highest academic distinction ‘summa cum laude’ from the University of Stuttgart on 15th January 2004. This book is a replica of his doctoral thesis. We look forward to continuing our cooperation with Dr. Ahn after his return to Korea as a professor of the Sungkyunkwan University, especially in the fields of computer vision, intelligent robots, digital factories and reverse engineering where we particularly need his professional knowledge.

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## Preface

*Least-squares* is a term which implies that the model parameters are determined by *minimizing the square sum* of some predefined error measures. If a data set is a set of measurement points in 2-D/3-D space, the most natural error measure is the shortest distance between the model feature and the measurement point. This is because the measurement point is a probable observation of the *nearest* object point to the measurement point. The least-squares model fitting with this error measure is called *orthogonal distance fitting* (ODF). The square sum of the error distances can be described in two ways: the distance-based cost function and the coordinate-based cost function. The numerical methods minimizing these two cost functions are respectively called the *distance-based algorithm* and the *coordinate-based algorithm*.

The ODF task has two sub-problems: not only do the model parameters have to be found but also the shortest distance points on a model feature from each given point. In general, the shortest distance point is by nature nonlinear to the model parameters. Thus, the ODF problem is inherently a *nonlinear minimization problem* solved through iteration. There are two minimization strategies for solving the ODF problems through iteration. The *total method* simultaneously finds both the model parameters and the shortest distance points, whilst the *variable-separation method* finds them alternately using a nested iteration scheme. Four combinations of the two numerical methods and the two minimization strategies can be applied. The combination of the distance-based algorithm with the total method results in an underdetermined linear equation system for iteration and is thus impractical for solving ODF problems. As a result, three algorithmic approaches for solving ODF problems are realistic:

- Algorithm I: combination of the coordinate-based algorithm with the total method
- Algorithm II: combination of the distance-based algorithm with the variable-separation method
- Algorithm III: combination of the coordinate-based algorithm with the variable-separation method.

This book presents the ODF Algorithms I–III for implicit and parametric curves/surfaces in 2-D/3-D space possessing the following algorithmic features:

- Estimation of the model parameters minimizing the square sum of the shortest distances between a model feature and the given points in 2-D/3-D space
- General application to parametric and implicit curves/surfaces

- Estimation of the model parameters in terms of form, position, and rotation
- General implementation of the distance-based and coordinate-based algorithms
- General implementation of the total and variable-separation methods
- Solving ODF problems with additional constraints
- Provision of parameter-testing possibilities
- Robust and fast convergence
- Low computing costs and memory space usage
- Low implementation costs for a new model feature
- Automatic determination of the initial parameter values for iteration.

Chapter 1 first reviews the mathematical descriptions of curves and surfaces in space. The ordinary least-squares fitting algorithm is compared with the ODF algorithm in general and the current ODF algorithms are reviewed in detail. Chapter 2 begins with the ODF of lines/planes which can be solved in closed form. The two numerical methods, the *distance-based algorithm* and the *coordinate-based algorithm*, are then presented in a very general manner without any assumption of the type of model feature to be fitted at this stage. Chapter 3 deals with Algorithms II and III for *implicit* curves/surfaces. To find the shortest distance point on an implicit feature from a given point, the generalized Newton method and the method of Lagrangian multipliers are described. Chapter 4 deals with Algorithms I–III for *parametric* curves/surfaces. To locate the shortest distance point on a parametric feature from a given point, the Newton method and the Levenberg-Marquardt algorithm are described. Chapter 5 gives two application examples of the ODF algorithms for *object reconstruction*. A semi-automatic procedure for segmentation, outlier elimination and model fitting in point clouds is proposed and examined. Chapter 6 summarizes the contents of this book. Appendices A–C provide practical and helpful information for applying the general ODF algorithms to fit specific model features.

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