
Preface

Research on noncommutative stationary processes leads to an interesting interplay between operator algebraic and probabilistic topics. Thus it is always an invitation to an exchange of ideas between different fields. We explore some new paths into this territory in this book. The presentation proceeds rather systematically and elaborates many connections to already known results as well as some applications. It should be accessible to anyone who has mastered the basics of operator algebras and noncommutative probability but, concentrating on new material, it is no substitute for the study of the older sources (mentioned in the text at appropriate places). For a quick orientation see the Summary on the following page and the Introduction. There are also additional introductions in the beginning of each chapter.

The text is a revised version of a manuscript entitled ‘Elements of a spatial theory for noncommutative stationary processes with discrete time index’, which has been written by the author as a habilitation thesis (Greifswald, 2002). It is impossible to give a complete picture of all the mathematical influences on me which shaped this work. I want to thank all who have been engaged in discussions with me. Additionally I want to point out that B. Kümmerer and his students C. Hertfelder and T. Lang, sharing some of their conceptions with me in an early stage, influenced the conception of this work. Getting involved with the research of C. Köstler, B.V.R. Bhat, U. Franz and M. Schürmann broadened my thinking about noncommutative probability. Special thanks to M. Schürmann for always supporting me in my struggle to find enough time to write. Thanks also to B. Kümmerer and to the referees of the original manuscript for many useful remarks and suggestions leading to improvements in the final version. The financial support by the DFG is also gratefully acknowledged.

Summary

In the first chapter we consider normal unital completely positive maps on von Neumann algebras respecting normal states and study the problem to find normal unital completely positive extensions acting on all bounded operators of the GNS-Hilbert spaces and respecting the corresponding cyclic vectors. We show that there exists a duality relating this problem to a dilation problem on the commutants. Some explicit examples are given.

In the second chapter we review different notions of noncommutative Markov processes, emphasizing the structure of a coupling representation. We derive related results on Cuntz algebra representations and on endomorphisms. In particular we prove a conjugacy result which turns out to be closely related to Kümmerer-Maassen-scattering theory. The extension theory of the first chapter applied to the transition operators of the Markov processes can be used in a new criterion for asymptotic completeness. We also give an interpretation in terms of entangled states.

In the third chapter we give an axiomatic approach to time evolutions of stationary processes which are non-Markovian in general but adapted to a given filtration. We call this an adapted endomorphism. In many cases it can be written as an infinite product of automorphisms which are localized with respect to the filtration. Again considering representations on GNS-Hilbert spaces we define adapted isometries and undertake a detailed study of them in the situation where the filtration can be factorized as a tensor product. Then it turns out that the same ergodic properties which have been used in the second chapter to determine asymptotic completeness now determine the asymptotics of nonlinear prediction errors for the implemented process and solve the problem of unitarity of an adapted isometry.

In the fourth chapter we give examples. In particular we show how commutative processes fit into the scheme and that by choosing suitable noncommutative filtrations and adapted endomorphisms our criteria give an answer to a question about subfactors in the theory of von Neumann algebras, namely when the range of the endomorphism is a proper subfactor.



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