
Contents

1	Introduction	1
1.1	A convex functional	1
1.2	A functional with supercritical growth	2
1.3	Construction of the transformations	5
2	Uniqueness of critical points (I)	9
2.1	One-parameter transformation groups	9
2.2	Variational sub-symmetries and uniqueness of critical points	10
2.3	Uniqueness results for critical points of constrained functionals	12
2.4	First order variational integrals	12
2.5	Classical uniqueness results	13
2.5.1	Convex functionals	13
2.5.2	Uniqueness of a saddle point	14
2.5.3	Strict variational sub-symmetry w.r.t. an affine subspace	17
2.5.4	Uniqueness of positive solutions for sublinear problems	21
2.5.5	Simplicity of the first eigenvalue	24
3	Uniqueness of critical points (II)	27
3.1	Riemannian manifolds	27
3.2	The total space $M \times \mathbb{R}^k$	30
3.3	One-parameter transformation groups on $M \times \mathbb{R}^k$	31
3.4	Action of transformation groups on functions	32
3.5	Rate of change of derivatives and volume-forms	35
3.6	Rate of change of first-order variational functionals	39
3.6.1	Partial derivatives of Lagrangians	39
3.6.2	The rate of change formula	41
3.6.3	Noether's formula and Pohožaev's identity	43
3.7	Admissible transformation groups	44
3.8	Rate of change formula for solutions	46
3.9	Variational sub-symmetries	48
3.10	Uniqueness of critical points	50

3.11	Uniqueness of critical points for constrained functionals	53
3.11.1	Functional constraints	53
3.11.2	Pointwise constraints	54
3.12	Differentiability of the group orbits	56
4	Variational problems on Riemannian manifolds	59
4.1	Example manifolds and their representations	59
4.2	Supercritical boundary value problems	61
4.2.1	A weak substitute for the vector-field \mathbf{x}	62
4.2.2	Critical points of a free functional	63
4.2.3	Critical points of constrained functionals	66
4.2.4	Applications	68
4.3	Harmonic maps	71
4.4	Supercritical boundary value problems: revisited	73
4.4.1	A better substitute for the vector-field \mathbf{x} ?	74
4.4.2	Conformal vector-fields and conformal maps	76
4.4.3	Yamabe's equation	79
4.4.4	Yamabe's equation with boundary terms	80
4.4.5	Conformal vector fields on conformally flat manifolds	80
4.4.6	The bifurcation problem on $\mathbb{R}^n, \mathbb{S}^n, \mathbb{H}^n$	83
4.4.7	The bifurcation problem on rotation surfaces	84
4.5	Harmonic maps into conformally flat manifolds	86
5	Scalar problems in Euclidean space	89
5.1	Extensions of Pohožaev's result to more general domains	89
5.1.1	Nonlinear Neumann boundary conditions	94
5.1.2	Extension to operators of q -Laplacian type	97
5.1.3	Extension to the mean-curvature operator	99
5.2	Uniqueness of non-zero solutions	100
5.3	The subcritical case	106
5.4	Perturbations of conformally contractible domains	107
5.5	Uniqueness in the presence of radial symmetry	110
5.5.1	Radially symmetric problems on $\mathbb{R}^n, \mathbb{S}^n, \mathbb{H}^n$	112
5.5.2	The radially symmetric q -Laplacian	119
5.5.3	Partial radial symmetry	121
5.6	Notes on further results	123
6	Vector problems in Euclidean space	127
6.1	The Emden-Fowler system	127
6.2	Boundary displacement problem in nonlinear elasticity	130
6.2.1	Uniqueness for the boundary displacement problem (compressible case)	132
6.2.2	Uniqueness for the boundary displacement problem (incompressible case)	134
6.3	A uniqueness result in dimension two	134

6.4	H. Wente's uniqueness result for closed surfaces of prescribed mean curvature	137
A	Fréchet-differentiability	139
B	Lipschitz-properties of g_ϵ and Ω_ϵ	141
	References	145
	Index	151

Uniqueness Theorems for Variational Problems by the
Method of Transformation Groups

Reichel, W.

2004, XIV, 158 p., Softcover

ISBN: 978-3-540-21839-5