Preface

This book is concerned with various topics that center around equivariant holomorphic maps of Hermitian symmetric domains and is intended for specialists in number theory and algebraic geometry. In particular, it contains a comprehensive exposition of mixed automorphic forms that has never appeared in book form.

The period map $\omega : \mathcal{H} \to \mathcal{H}$ of an elliptic surface E over a Riemann surface X is a holomorphic map of the Poincaré upper half plane $\mathcal{H}$ into itself that is equivariant with respect to the monodromy representation $\chi : \Gamma \to SL(2, \mathbb{R})$ of the discrete subgroup $\Gamma \subset SL(2, \mathbb{R})$ determined by X. If ω is the identity map and χ is the inclusion map, then holomorphic 2-forms on E can be considered as an automorphic form for Γ of weight three. In general, however, such holomorphic forms are mixed automorphic forms of type $(2, 1)$ that are defined by using the product of the usual weight two automorphy factor and a weight one automorphy factor involving ω and χ. Given a positive integer m, the elliptic variety E^m can be constructed by essentially taking the fiber product of m copies of E over X, and holomorphic $(m+1)$-forms on E^m may be regarded as mixed automorphic forms of type $(2, m)$. The generic fiber of E^m is the product of m elliptic curves and is therefore an abelian variety, or a complex torus. Thus the elliptic variety E^m is a complex torus bundle over the Riemann surface X.

An equivariant holomorphic map $\tau : \mathcal{D} \to \mathcal{D}'$ of more general Hermitian symmetric domains $\mathcal{D}$ and $\mathcal{D}'$ can be used to define mixed automorphic forms on $\mathcal{D}$. When $\mathcal{D}'$ is a Siegel upper half space, the map τ determines a complex torus bundle over a locally symmetric space $\Gamma \backslash \mathcal{D}$ for some discrete subgroup Γ of the semisimple Lie group G associated to $\mathcal{D}$. Such torus bundles are often families of polarized abelian varieties, and they are closely related to various topics in number theory and algebraic geometry. Holomorphic forms of the highest degree on such a torus bundle can be identified with mixed automorphic forms on $\mathcal{D}$ of certain type. Mixed automorphic forms can also be used to construct an embedding of the same torus bundle into a complex projective space. On the other hand, sections of a certain line bundle over this torus bundle can be regarded as Jacobi forms on the Hermitian symmetric domain $\mathcal{D}$.

The main goal of this book is to explore connections among complex torus bundles, mixed automorphic forms, and Jacobi forms of the type described above. Both number-theoretic and algebro-geometric aspects of such connections and related topics are discussed.

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