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## Preface

In 1999, a friend of mine, Kazuhiro Sakuma, kindly asked me to give a series of lectures in the Kwansai Seminar on Differential Analysis, held at the Kinki University, Japan. At that time, I was studying the global topology of differentiable maps of 4-dimensional manifolds into lower dimensional manifolds. Sakuma and I had obtained a lot of interesting results concerning the relationship between the singularities of such maps and the differentiable structures of 4-dimensional manifolds; however, our results were not based on a systematic theory and were not satisfactory in a certain sense. So I was trying to construct such a systematic theory when I was asked to give lectures.

I wondered what kind of objects can reflect the *global* properties of manifolds. “Singularity” of a differentiable map can be such an object, but it is *local* in nature. I already knew that the notion of the Stein factorization played an important role in the global study of such maps; for example, refer to the works of Burlet–de Rham [7] or Kushner–Levine–Porto [28, 30]. Stein factorization is constructed by considering the connected components of the fibers of a given map.

This inspired me to consider singular fibers of differentiable maps. I promptly started the classification of singular fibers of stable maps of orientable 4-manifolds into 3-manifolds. It was not a difficult task, though quite tedious. Then I obtained the modulo two Euler characteristic formula in terms of the number of a certain singular fiber, by using Szűcs’s formula [55], which Nuño Ballesteros and I had also obtained independently [36, 37]. The formula on the number of singular fibers was so beautiful that I was very happy to be able to present such a result in the Kwansai Seminar, in November 1999.

After attending my lectures, one of the participants, Toru Ohmoto, gave me a very important remark. He said “Your argument is closely related to Vassiliev’s universal complex of multi-singularities. You just increased the number of generators for each cochain complex using the topology of singular fibers”.

So I began to study Vassiliev’s work and at the same time began to elaborate my results. It took a long time to write down all the details. A preprint

version of the whole work was finished only in the middle of 2003, when I was staying in Strasbourg, France.

Now the acknowledgment follows. First of all, I would like to thank Kazuhiro Sakuma and the co-organizer Shuzo Izumi for kindly asking me to give a series of lectures at the Kwansai Seminar. I would like to thank Toru Ohmoto for his important remark at the seminar. Without these people, this work would have never appeared.

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In January 2004, the results in this book were presented in a mini-course given at the University of Tokyo, Japan. I would like to thank all the participants at the mini-course, who attended it with enthusiasm and posed a lot of questions. In particular, I would like to thank Mikio Furuta for his excellent questions with fantastic ideas: in fact, I included some of the results based on his ideas in this book. I would also like to thank Masamichi Takase and Keiichi Suzuoka for their invaluable comments on my mini-course. I would like to thank Yukio Matsumoto, my ex-supervisor, for inviting me to give such a mini-course.

I would like to thank Vincent Blanlœil for inviting me to Strasbourg in 2003, where I could finish the first draft of this work. I would also like to express my thanks to Rustam Sadykov for posing many interesting questions concerning the book.

Finally, I would like to thank all the members of my family, especially to Célia, for their patience and support during the preparation of the book.

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