
Preface

Evolution equations describe time dependent processes as they occur in physics, biology, economy or other sciences. Mathematically, they appear in quite different forms, e.g., as parabolic or hyperbolic partial differential equations, as integrodifferential equations, as delay or difference differential equations or more general functional differential equations. While each class of equations has its own well established theory with specific and sophisticated methods, the need for a unifying view becomes more and more urgent.

To this purpose functional analytic methods have been applied in recent years with increasing success. In particular the concepts of *Abstract Cauchy Problems* and of *Operator Semigroups* on Banach spaces allow a systematic treatment of general evolution equations preparing the ground for a better theory even for special equations.

It is the aim of this volume to make this evident. Five contributions by leading experts present recent research on functional analytic aspects of evolution equations.

In the first contribution, **Giuseppe Da Prato** from the Scuola Normale Superiore in Pisa (Italy) gives an introduction to stochastic processes on infinite dimensional spaces. His approach is based on the concept of Markov semigroups and does not require familiarity with probability theory. The main emphasis is on important qualitative properties of these semigroups and the Ornstein-Uhlenbeck semigroup serves as his major example.

In the second contribution, which is by far the longest, **Peer Kunstmann** and **Lutz Weis** (both from the University of Karlsruhe in Germany) discuss the (maximal) regularity of the solutions of inhomogeneous parabolic Cauchy problems.

Regularity properties are fundamental for a theory of nonlinear parabolic equations. Since 1998 this theory has made enormous progress with spectacular breakthroughs based on new Fourier multiplier theorems with operator-valued functions and “square function estimates” for the holomorphic H^∞ -functional calculus (some of these results are due to the authors). This con-

tribution is a unifying and accessible presentation of this theory and some of its applications.

Control theoretic aspects of evolution equations in finite dimensions have been studied for a long time. Thanks to functional analytic tools there is now a well established infinite dimensional theory described in some recent monographs. However, this theory does not cover so-called boundary and point control problems. **Irena Lasiecka** from the University of Virginia (USA) developed (mostly with Roberto Triggiani) a systematic approach to these problems using a beautiful combination of abstract semigroups methods and sharp PDE estimates. In her contribution she explains this approach and discusses illustrating examples such as systems of coupled wave, plate and heat equations.

While in the previous contributions the focus is on the dynamics of the state variable, **Alessandra Lunardi** (University of Parma, Italy) studies moving boundary problems. In order to explain these highly nonlinear problems, she concentrates on the heat equation on a moving domain, first in one and then in higher dimensions. Her work is intended as an introduction to an important new field, to very recent results, and to interesting open problems.

Roland Schnaubelt (University of Halle, Germany) shows in the last contribution how nonautonomous linear evolution equations can be studied by a reduction to an autonomous problem to which semigroup methods apply. In particular, the well developed spectral theory for semigroups allows a systematic characterisation of, e.g., exponential dichotomy of the solutions. He then applies these results to obtain qualitative properties of the solutions to nonlinear equations.

These contributions were the basis of lectures given at the Autumn School on “Evolution Equations and Semigroups” at Levico Terme (Trento, Italy) from October 28 to November 2, 2001, within the program of the CIRM (Centro Internazionale per la Ricerca Matematica). We thank Professor Mario Miranda for the support provided to the School. Thanks are also due to the speakers for their cooperation and the permission to collect the expanded notes of their lectures. We hope that this volume will be valuable for beginners as well as for experts in evolution equations.

Trento and Tübingen,
March, 2004

Mimmo Iannelli
Rainer Nagel
Susanna Piazzera

Functional Analytic Methods for Evolution Equations

Da Prato, G.; Kunstmann, P.C.; Lasiecka, I.; Lunardi, A.;
Schnaubelt, R.; Weis, L. - Iannelli, M.; Nagel, R.; Piazzera,
S. (Eds.)

2004, CDLXXXIV, 474 p., Softcover

ISBN: 978-3-540-23030-4