

# Table of Contents

|  |     |
|--|-----|
| <b>Preface</b> .....   | 15  |
| <b>Acknowledgments</b> .....   | 17  |
| <b>1 ASYMPTOTIC APPROXIMATIONS</b> .....   | 19  |
| 1.1 Asymptotic series .....  | 19  |
| 1.2 Fundamental concepts of asymptotics [129] .....                                    | 21  |
| 1.3 Transformations of asymptotical series [129] .....                                 | 23  |
| 1.4 Nonuniform expansions [129] .....  | 25  |
| 1.5 Non-dimensionalization .....   | 27  |
| 1.6 Asymptotics of integrals [129] .....   | 29  |
| <b>2 REGULAR PERTURBATIONS OF PARAMETERS</b> .....                                     | 47  |
| 2.1 Eigenvalue problems .....  | 47  |
| 2.2 Stability of oval cylindrical shell uniformly loaded by external<br>pressure ..... | 58  |
| 2.3 Stability of the cantilever beam .....   | 60  |
| 2.4 Adjoint operators method .....   | 62  |
| 2.5 Transformation of coordinates and variables .....                                  | 65  |
| 2.6 Asymptotic and real error .....  | 71  |
| 2.7 Numerical verification of asymptotic solution .....                                | 75  |
| 2.8 Removal of nonuniformities .....   | 77  |
| 2.9 Nonlinear vibrations of a stringer shell .....                                     | 81  |
| 2.10 Non-quasilinear asymptotics of nonlinear system .....                             | 84  |
| 2.11 Artificial small parameters .....   | 88  |
| 2.12 Method of small $\delta$ .....  | 91  |
| 2.13 Method of large $\delta$ .....  | 94  |
| 2.14 Choice of zero approximation .....  | 98  |
| 2.15 Lyapunov–Schmidt procedure .....  | 101 |
| 2.16 Nonlinear periodical vibrations of continuous structures .....                    | 103 |
| <b>3 SINGULAR PERTURBATION PROBLEMS</b> .....  | 117 |
| 3.1 The method of Gol’denveizer-Vishik-Lyusternik<br>[313, 645, 672, 673, 674] .....   | 117 |
| 3.2 Multiscale method .....  | 122 |

|          |  |            |
|----------|--|------------|
| 3.3      | Newton polygon and asymptotic integration parameters .....           | 125        |
| 3.4      | Stretched plate bending .....  | 131        |
| 3.5      | Simplification of the static equations of a cylindrical shell .....  | 133        |
| 3.6      | Boundary layer: Papkovitch approach .....                            | 136        |
| 3.7      | Edge boundary layer .....  | 138        |
| 3.8      | Incorporating of the singular part of solution .....                 | 140        |
| 3.9      | Plane theory of elasticity .....                                     | 140        |
| 3.10     | Asymptotic foundation model .....                                    | 144        |
| 3.11     | Vibrations of reinforced conical shells .....                        | 149        |
| <b>4</b> | <b>BOUNDARY VALUE PROBLEMS OF ISOTROPIC CYLINDRICAL SHELLS .....</b> | <b>157</b> |
| 4.1      | Governing relations .....  | 157        |
| 4.2      | Operator method .....  | 159        |
| 4.3      | Simplified boundary value problems .....                             | 160        |
| <b>5</b> | <b>BOUNDARY VALUE PROBLEMS – ORTHOTROPIC SHELLS ...</b>              | <b>169</b> |
| 5.1      | Governing relations .....  | 169        |
| 5.2      | Statical problems .....  | 173        |
| 5.3      | Non-linear dynamical problems .....                                  | 182        |
| 5.4      | Stability problems .....   | 189        |
| 5.5      | Error estimation using Newton's method .....                         | 197        |
| <b>6</b> | <b>COMPOSITE BOUNDARY VALUE PROBLEMS – ISOTROPIC SHELLS .....</b>    | <b>201</b> |
| 6.1      | Statical problems .....  | 202        |
| 6.2      | Equations of higher order approximations .....                       | 206        |
| 6.3      | Error estimation .....   | 208        |
| 6.4      | Dynamical problems .....   | 211        |
| 6.5      | Non-linear dynamical problems .....                                  | 217        |
| <b>7</b> | <b>COMPOSITE BOUNDARY VALUE PROBLEMS – ORTHOTROPIC SHELLS .....</b>  | <b>223</b> |
| 7.1      | Statical problems .....  | 223        |
| 7.2      | Dynamical problems .....   | 234        |
| 7.3      | Non-linear dynamical problems .....                                  | 235        |
| 7.4      | Stability problems .....   | 239        |
| <b>8</b> | <b>AVERAGING .....</b>   | <b>241</b> |
| 8.1      | Two-scales approach .....  | 241        |
| 8.2      | Visco-elastic problems and 'freezing' method .....                   | 244        |
| 8.3      | The successive change of variables .....                             | 246        |
| 8.4      | Application of the Lie groups .....                                  | 249        |
| 8.5      | Whitham method (non-linear WKB approach) .....                       | 256        |

|           |  |     |
|-----------|--|-----|
| <b>9</b>  | <b>CONTINUALIZATION</b>  | 259 |
| <b>10</b> | <b>HOMOGENIZATION</b>  | 267 |
| 10.1      | ODEs with rapidly oscillating coefficients                                 | 267 |
| 10.2      | Axisymmetric bending of corrugated circle plate                            | 272 |
| 10.3      | Deformation of reinforced membrane   | 276 |
| 10.4      | Ribbed strip – two-scale and Fourier homogenization                        | 280 |
| 10.5      | Ribbed plate – direct homogenization                                       | 284 |
| 10.6      | Perforated membrane  | 292 |
| 10.7      | Composite with periodic cubic inclusions                                   | 302 |
| 10.8      | Torsion of bar with periodic parallelepiped inclusions                     | 307 |
| 10.9      | Solution of cell problem: perturbation of boundary form                    | 313 |
| 10.10     | Linear vibrations of a beam with concentrated masses and discrete supports | 319 |
| <b>11</b> | <b>INTERMEDIATE ASYMPTOTICS – DYNAMICAL EDGE EFFECT METHOD</b>             | 333 |
| 11.1      | Linear preliminaries   | 333 |
| 11.2      | Nonlinear beam vibrations  | 336 |
| 11.3      | Nonlinear rectangular plate vibrations                                     | 340 |
| 11.4      | Nonlinear shallow shell vibrations   | 345 |
| 11.5      | Rayleigh-Ritz-Bolotin approach   | 353 |
| 11.6      | Parallelogram plate vibrations   | 357 |
| 11.7      | Sectorial plate nonlinear vibrations                                       | 360 |
| <b>12</b> | <b>LOCALIZATION</b>  | 363 |
| 12.1      | Localization in linear chains  | 363 |
| 12.2      | Localization in nonlinear chain  | 370 |
| 12.3      | Localization of shell buckling   | 372 |
| 12.4      | Localization of vibration in plates and shells                             | 373 |
| <b>13</b> | <b>IMPROVEMENT OF PERTURBATION SERIES</b>                                  | 377 |
| 13.1      | Padé approximants (PA)   | 377 |
| 13.2      | The effect of autocorrection   | 379 |
| 13.3      | Extending of perturbation series   | 381 |
| 13.4      | Improvement iterational procedures convergence                             | 384 |
| 13.5      | Nonuniformities elimination  | 387 |
| 13.6      | Error estimation of asymptotic approaches                                  | 388 |
| 13.7      | Localized solutions and blow-up phenomenon                                 | 388 |
| 13.8      | Gibbs phenomena  | 389 |
| 13.9      | Boundary conditions perturbation method                                    | 392 |
| 13.10     | Bifurcation problem  | 401 |
| 13.11     | Borel summation and superasymptotics                                       | 402 |
| 13.12     | Domb–Sykes plot [340, 659]   | 404 |
| 13.13     | Extraction of singularities from perturbation series [340, 659]            | 406 |

|                                |  |            |
|--------------------------------|--|------------|
| 13.14                          | Analytical continuation [407] . . . . .                                  | 409        |
| <b>14</b>                      | <b>MATCHING OF LIMITING ASYMPTOTIC EXPANSIONS . . . . .</b>              | <b>417</b> |
| 14.1                           | Two-point Padé approximants . . . . .                                    | 417        |
| 14.2                           | Quasifractional approximants . . . . .                                   | 424        |
| 14.3                           | Post-buckling behaviour of shallow convex shell . . . . .                | 429        |
| <b>15</b>                      | <b>COMPLEX VARIABLES IN NONLINEAR DYNAMICS . . . . .</b>                 | <b>435</b> |
| 15.1                           | Nonlinear oscillator with cubic anharmonicity . . . . .                  | 436        |
| 15.2                           | System of two weakly coupled nonlinear oscillators . . . . .             | 447        |
| 15.3                           | Nonlinear dynamics of an infinite chain of coupled oscillators . . . . . | 453        |
| 15.4                           | Nonlinear dynamics of an infinite chain of coupled particles . . . . .   | 460        |
| <b>16</b>                      | <b>OTHER ASYMPTOTICAL APPROACHES . . . . .</b>                           | <b>463</b> |
| 16.1                           | Matched asymptotic procedure . . . . .                                   | 463        |
| 16.2                           | Hilbert transform . . . . .  | 465        |
| 16.3                           | Normal forms in non-linear problems . . . . .                            | 467        |
| 16.4                           | WKB - approach . . . . .   | 470        |
| 16.5                           | The WKB method and turning points . . . . .                              | 473        |
| 16.6                           | A distributional approach . . . . .                                      | 478        |
| <b>AFTERWORD . . . . .</b>     |  | <b>483</b> |
|                                | Asymptotics and Computers . . . . .                                      | 483        |
|                                | Are Asymptotic Methods a Panacea? . . . . .                              | 485        |
| <b>References . . . . .</b>    |  | <b>489</b> |
| <b>Subject index . . . . .</b> |  | <b>529</b> |

Asymptotical Mechanics of Thin-Walled Structures

Andrianov, I.V.; Awrejcewicz, J.; Manevitch, L.I.

2004, XX, 515 p., Hardcover

ISBN: 978-3-540-40876-5