

Contents

Part I Optical phase space, Hamiltonian systems and Lie algebras

Introduction	3
1 Two fundamental postulates	5
1.1 Geometric postulate	5
1.2 Dynamic postulate	6
1.3 Conservation laws at discontinuities	7
1.4 Descartes' sphere and the Ibn Sahl law of refraction	8
1.5 Geometric optics and classical mechanics	11
2 Optical phase space	15
2.1 Ray coordinates and their manifold	15
2.2 Hamilton equations on the screen	18
2.3 Guides and their index profile	21
2.4 Paraxial optics and mechanics	23
3 Canonical transformations	25
3.1 Beams and the conservation of light	25
3.2 Conservation of the Hamiltonian structure	28
3.3 Hamiltonian evolution with Poisson brackets	33
3.4 One-parameter Lie groups	34
3.5 Hamiltonian flow of phase space	37
3.6 Some aberrations and their Fourier conjugates	38
3.6.1 Spherical aberration and pocus	39
3.6.2 Distorsion and coma	40
3.7 Multiparameter Lie algebras and groups	42
4 The roots of refraction and reflection	47
4.1 Refraction equations in screen coordinates	48
4.2 Factorization of refraction	49
4.3 Canonicity of the root transformation	51
4.4 Aberration series expansion for the root transformation	53
4.5 Refraction between inhomogeneous media	54
4.6 Factorization of reflection	55

A	Some historical comments	57
A.1	Antiquity	57
A.2	Age of Reason	58
A.3	Fermat's principle and the Lagrangian	59
A.4	Geometric optics in the nineteenth century	60
A.5	Hamiltonian formulations	61
A.6	The evolution of Sophus Lie	61
A.7	Symmetries and dynamics in the twentieth century	63

Part II Symmetry and dynamics of optical systems

	Introduction	67
5	Euclidean and Lorentzian maps	69
5.1	Presentations and realizations of symmetry	69
5.2	Translations in 3-space	72
5.3	Rotations of 3-space	74
5.4	Rotations of the screen	77
5.5	Euclidean and semidirect product groups	79
5.6	Lorentz boost of light-like vectors	81
5.7	Relativistic aberration of images	84
5.8	The Lorentz Lie algebra and group	87
5.9	Other global optical transformations	88
6	Conformal optics – Maxwell fish-eyes	91
6.1	On the eyes of fish and point rotors	91
6.2	Phase space and rotations	94
6.3	Restriction to conics	96
6.4	Stereographic map of phase space	97
6.5	Hidden symmetry and the Hamiltonian	103
6.6	Dynamical Lie algebra of the fish-eye	106
6.7	Conformal Lie algebra	108
6.8	The Kepler system and its hidden rotor	110
7	Axial symmetry reduction	113
7.1	Symmetry-adapted coordinates of phase space	113
7.2	Hamiltonian knife cuts hyperbolic onion	116
7.3	Stability of trajectories and critical rays	117
7.4	The reduced phase space of axis-symmetric systems	120
7.5	Hamiltonian structure on reduced phase space	122
7.6	Reconstruction of the ignored coordinate	123

8	Anisotropic optical media	127
8.1	Direction and momentum of rays	127
8.2	Hamilton equations for anisotropic media	128
8.3	Angular dependences of the refractive index	129
8.4	Comparison with Maxwellian anisotropy	131
B	Euclidean optical models	135
B.1	Manifolds of rays, planes and frames	135
B.2	Coset spaces for geometric and wave models	138
B.3	Conservation of volume and structure	141
B.4	Signal and Helmholtz models	144
B.5	Hilbert space of Helmholtz wavefields	145
B.6	Euclidean algebra in Helmholtz optics	148
B.7	The recipe for wavization	149
<hr/>		
	Part III The paraxial régime	
	<hr/>	
	Introduction	155
9	Optical elements of the symplectic group	157
9.1	Free spaces, thin lenses, and action on phase space	157
9.2	Linear canonical maps and symplectic matrices	162
9.3	Orthogonal and unitary matrices	165
9.4	Bargmann parameters and group covers	168
9.5	The Iwasawa and other decompositions	173
10	Construction of optical systems	179
10.1	Plane optical systems	179
10.2	Astigmatic lenses and magnifiers	184
10.2.1	Lenses	184
10.2.2	Magnifiers	185
10.2.3	Reflectors and rotators	187
10.3	U(2) fractional Fourier transformers	188
10.3.1	Central Fourier transforms	189
10.3.2	Separable Fourier transforms	189
10.3.3	SU(2)-Fourier transforms	190
10.4	Systems <i>cum</i> reflection	191
10.5	Minimal lens arrangements	196
10.5.1	The <i>abc</i> -parameters	196
10.5.2	One-lens <i>DL</i> D configurations	197
10.5.3	Two-lens configurations	199
10.5.4	Three-lens configurations	202

11 Classical Lie algebras	205
11.1 Lie algebras of the linear groups	205
11.2 The classical Cartan algebras	211
11.3 The Weyl trick for symplectic algebras	215
11.4 Phase space functions, operators and matrices	221
11.5 Roots and multiplets of the symplectic algebras	224
11.6 Roots and multiplets of the unitary algebras	232
11.7 Roots of the orthogonal algebras	237
12 Hamiltonian orbits	249
12.1 Orbits in $\mathfrak{sp}(2, \mathbb{R})$ for plane systems	249
12.2 Trajectories in $\mathbf{Sp}(2, \mathbb{R})$	252
12.3 $\mathfrak{sp}(4, \mathbb{R})$ Hamiltonians and eigenvalues	254
12.4 Hamiltonians in the $\mathfrak{so}(3, 2)$ basis	258
12.5 Equivalence under Fourier transformers and magnifiers	262
12.6 Separable, Lorentzian and Euclidean Hamiltonians	264
12.7 Evolution along $\mathfrak{sp}(4, \mathbb{R})$ -guides	267
12.8 Inhomogeneous Hamiltonians	270
C Canonical Fourier optics	273
C.1 The Royal Road to Fourier optics	273
C.2 Linear canonical transforms	277
C.3 Hyperdifferential forms	282
C.4 D -dim and radial canonical transforms	286

Part IV Hamilton-Lie aberrations

Introduction	291
13 Polynomials and aberrations in one dimension	293
13.1 Monomials in multiplets	293
13.2 Rank- K aberration algebras	296
13.3 Rank- K aberration groups	298
13.4 Aberrations of phase space	302
14 Axis-symmetric aberrations	309
14.1 Axis-symmetric aberrations in the Cartesian basis	309
14.2 The harmonic basis of aberrations	315
14.3 Harmonic aberration family features	321
14.4 Concatenation of aberrating systems	329
14.5 Aberration coefficients for optical elements	333

15 Parametric correction of fractional Fourier transformers . .	341
15.1 The lens arrangement	342
15.2 The warped-face guide arrangement	347
15.3 The cat's eye arrangement	349
15.4 Afterword	353
References	355
Index	365



<http://www.springer.com/978-3-540-22039-8>

Geometric Optics on Phase Space

Wolf, K.B.

2004, XV, 376 p., Hardcover

ISBN: 978-3-540-22039-8