

Preface

Considerable progress in the study of nonlinear boundary value problems for partial differential equations has been analyzed by extensive use of nonlinear functional analysis methods. These methods found their application in various fields of mathematics. It is natural to reduce these problems to nonlinear operator or differential-operator equations in functional spaces. Such approach, which was introduced by F. Browder (as corollaries of operator systems), provides results for specific systems. Similar approach is effective in optimization and control of systems problems which are described by nonlinear partial differential equations.

In this monograph, a new approach is given for the study of extremal problems in nonlinear operator and differential operator equations, as well as the variational inequalities in Banach spaces.

Presented is an axiomatic study of classes of nonlinear mappings (including the multivalued) in Cartesian product spaces. In their terms the functional-topological properties of the resolving operators of the systems which contain both operator and differential-operator equations are established. Defined are the existence conditions and properties of optimization problems solutions. Their weak expansions and necessary optimality conditions are constructed. Regularization methods are developed and the approximation schemes are proven.

The first part of the monograph “Methods of Nonlinear Analysis” presents the results obtained for nonlinear analysis problems which arise in the control and optimization theory of nonlinear infinite dimensional systems.

Chapter 1 “Preliminary Results” considers classes of monotone type nonlinear mappings in product of spaces. Their relationships are identified and the relating examples are provided. The maximal semimonotone mappings which act from a reflexive Banach space into the conjugate multivalued mappings are introduced and studied.

Chapter 2 “Functionals and Forms” deals with some classes of extremal problems for functionals which are the superposition of a real function and multivalued mappings in Banach spaces. The results obtained here are original and for the first time are stated in monographic literature. The main results here are theorems 2.1.1–2.1.5 on the existence of solution of the cor-

responding extremal problems. Elements of functionals differential analysis on “nonlinear sets” are also developed.

Chapter 3 “Nonlinear Operator Equations, Inclusions and Variational Inequalities” considers issues connected with the existence of solutions of nonlinear operator equations, operator inclusions and variational inequalities in Banach spaces. In addition to the known facts some new results are introduced here. In particular, for the first time, the notion of Λ -weak solution for an operator equation with densely defined mapping is introduced which is principal in theory of extremal problems expansion for operator equations. Theorems on existence and properties of Λ -weak solutions of operator equations and operator inclusions and also variational inequalities are proved. For variational inequalities with multivalued mappings “penalty mappings methods” are introduced and shown for the first time.

Chapter 4 “Differential-Operator Equations and Inclusions”. In this chapter similar questions are raised for evolutionary problems. This ends Part 1 of the monograph.

In the second part “Control Problems for the Distributed Parameters Objects”, by making a considerable use of the results obtained in the preceding chapters, methods of attack of control problems for objects described by nonlinear operator, differential-operator equations, inclusions and variational inequalities in Banach spaces with restrictions on control and phase variables are developed. Until recently it has been considered that mathematical models of the controlled systems had the property of uniqueness, i.e. each admissible set of controls corresponded to a unique state. At the same time numerous applied problems result in mathematical models, when the corresponding boundary problem may have many solutions with the fixed control function. The approach suggested in the monograph gives the possibility of embracing this class of problems.

Chapter 5 “Extremal Problems for the Nonlinear Operator Equations and Variational Inequalities” considers the class of optimizational problems for the objects described by nonlinear operator equations with inclusions and variational inequalities and constraints on inclusions and operator inequalities. Essential here is the fact that the corresponding operator equation has a multivalued resolving operator for which a number of extremal problems is established. Theorems on existence of the corresponding optimization problems solutions are proved, non-coercive extremal problems in the sense of V. Petryshyn are considered. Applying the notion of Λ -weak solution introduced in the previous section, considered here is the basis of the extremal problems weak expansions theory for operator equations and variational inequalities. Up to now this class of expansions has not been considered in the literature.

An original approach is also suggested by the authors to the regularization and construction of optimization problems solutions. With the corresponding differentiability requirements the necessary conditions of optimality in the form of variational inequalities are given. Here the quasi-differential calculus

of V. Pshenychnyi is extensively used and eventually the finite dimensional approximations of optimization problems and necessary optimality conditions are proposed.

In Chapter 6 “Optimal Control for Differential Operator Equations and Inclusions” similar range of questions is considered for evolutionary problems with the corresponding appropriate modifications. Here for the first time systems appear which contain operator and differential-operator equations. To the best of my knowledge, the question of existence of solutions has not been raised (also for extremal problems). Also, for the first time, is presented the systemic analysis of the optimal control for modified equations of hydrodynamic type, in particular for the Navier-Stokes’ equations.

In Chapter 7 “Some Problems of Synthesis in Distributed Parameters Systems” a rather novel idea is proposed regarding the synthesis of attracting sets for infinite dimensional systems. The results presented here are not only of theoretical, but also of practical value.

Chapter 8 “Control of Heat Transfer and Diffusion Processes”. Total methodology presented in the monograph is demonstrated on the example of nonlinear heat transfer. Mathematical model of the process has been studied, theorems on existence of the generalized solutions (generally speaking, without uniqueness) have been proved, optimization problems have been set and the existence of their solutions has been proved, the necessary conditions of optimality have been identified and finite dimensional approximations have been constructed.

In the following two chapters the experience of practical application of mathematical methods for control of distributed parameters processes is described. Considerable attention is given to the numerical modeling and the development and implementation of optimization algorithms.

This monograph provides novel and up-to-date research on the theory of control of nonlinear infinite dimensional systems which has been carried out in Ukraine during past 20 years. Such material appears for the first time in monographic literature. It is for the first time that this research is presented and it is of considerable interest to specialists in the field of nonlinear analysis and its applications, control theory and dynamics systems theory, as well as optimization methods.

This monograph because of its scientific level, general interest in the field, character and coverage of the material is recommended for publication by the Springer Publishing House.

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