7. How to Reason about Velocity Relationships

In constrained linkage mechanism analysis, apart from knowing configurations and trajectories, it is desirable to know the velocities of various links as well as their interrelationships given certain kinematic constraints. This chapter introduces a method for deriving instantaneous velocity relationships among constrained bodies of a mechanism. The method utilizes the qualitative kinematic properties (i.e., *instantaneous rotation center*) of mechanisms, and permits computationally efficient solution to the problem of deriving velocity relationships.

Liu [132] previously proposed a qualitative approach to velocity analysis based on instantaneous rotation centers. His approach relied on a set of naive spatial inference rules and generated spatial envisionments which were sometimes considered too ambiguous to be practically useful. The current work, however, derives instantaneous centers by means of the qualitative-quantitative configuration analysis as presented in Chapter 6. Since the qualitative configuration modeling step is based on a complete set of trigonometric rules and of reasonable precision, the results of spatial analysis will contain less uncertainty. Furthermore, since quantitative spatial relationships are generated for each instantaneous configuration, successive qualitative modeling of instantaneous rotation centers will avoid the problem of combinatorial complexity caused by the ambiguous envisionments.

7.1 Instantaneous Rotation Center

Before describing the qualitative approach to velocity analysis, it will be useful to recall one of the properties of an *instantaneous rotation center* (instantaneous center) [91], that is,

> *The instantaneous linear velocities of points on a given link are perpendicular to the lines joining these points with an instantaneous center.*

Based on the **V-direction axiom**, the instantaneous center of an individual link in a linkage can be located. Consider the four-bar linkage, as shown in Figure 7.1. It can be readily realized that point D is a point on both link L_3 and link L_4. As a point on L_4, D moves with respect to L_1 about the center A and thus in a direction normal to L_4 itself. The motion of D on L_3 with respect to L_1 is in a direction perpendicular to L_4. Hence, by definition, the instantaneous center will lie on the line through D and the direction of L_4. Similar reasoning can be applied to infer that the instantaneous center of L_3 must also lie on the line through point C normal to L_2. Hence, P_{13} is at the intersection of two lines extended from L_2 and L_4.

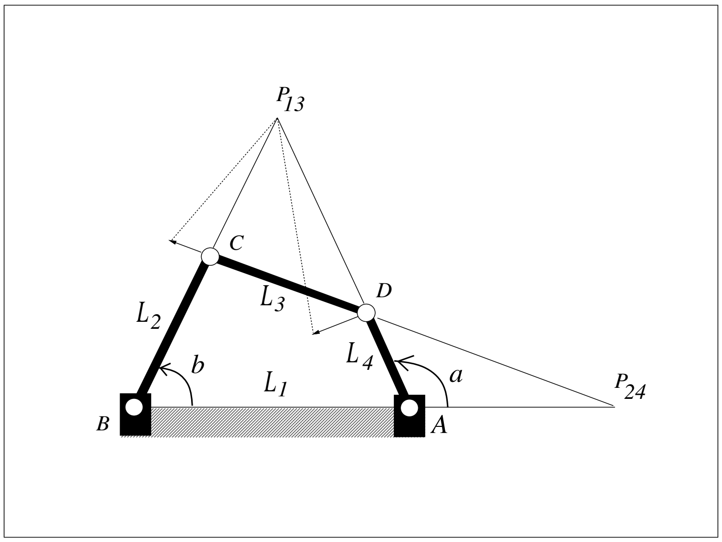

Fig. 7.1. An illustration of *instantaneous rotation centers.* By means of building instantaneous centers, the problem of qualitatively describing velocity relationships can be reduced to that of qualitative spatial analysis.

The concept of instantaneous centers is essential to motion analysis. It provides a geometric method in determining the relationship between two linear velocities of the same mechanism. The relative instantaneous velocities have the following geometric property:

> *The instantaneous linear velocity of a point on a given link is proportional to its radius of instantaneous rotation.*

Based on the **V-magnitude axiom**, it is possible to determine the *velocity distribution* on each link, and to infer the *motion transferred* from one link to another.

It is important to point out that the notion of instantaneous rotation can be applied not only to derive the linear velocity relationships, but also to infer angular velocity relationships. As an example, shown in Figure 7.1, the instantaneous center for the relative motion of links L_2 and L_4, namely P_{24}, is found at the intersection of two lines extended from L_1 and L_3. The two links behave instantaneously as though they were *spur gears* having internal contact at P_{24}. Hence, their angular velocity ratio is given by

$$t_{24} = \frac{\omega_a}{\omega_b} = \frac{BP_{24}}{AP_{24}} \tag{7.1}$$

which provides a simple and quick method of finding the instantaneous angular velocity relationships.

The idea of instantaneous centers has in fact been employed in classical kinematics for analyzing velocities. However, such an analysis is mainly based on the graphical construction of all the centers and the visual determination of the distances from a given center to certain points on the corresponding link. In order to locate the centers, it usually requires the use of Kennedy's Theorem [91], which states that *the centers of any three planar bodies lie on a straight line.*

In the method presented below, the same concept of instantaneous centers is used. Instead of graphically analyzing a linkage and constructing its centers, this method applies the qualitative-quantitative configuration analysis algorithm, as presented in Section 6.1, to infer the approximate locations of the centers. The location of an instantaneous center is determined according to axioms of instantaneous velocity direction and magnitude (i.e., **V-direction** and **V-magnitude**, respectively). The distances from a given center to other points of interest are first *qualitatively* inferred using algorithm `QUALITATIVE_CONFIG`, and then *quantitatively* located using algorithm `ANNEAL` (the simulated annealing). Thereafter, the analysis of instantaneous velocity relationships in the linkage is carried out.

7.2 Velocity Relationship Analysis

This section discusses how to reason about the transfer of motion between two links as well as the velocity distribution on a given link. The qualitative-quantitative analysis of velocities will be illustrated with linkage examples.

Definition 7.2.1 (Velocity distribution and motion transfer). *Given any instantaneous configuration of the linkage, the velocity distribution in a certain link is defined as the absolute linear velocities of a set of points on the link. The absolute linear velocity relationship between a driver link and a driven link is referred to as motion transfer.*

In general, there are two primary types of motion transfer problems to be considered, with respect to whether the desired velocity is on a *floating* link or on a *follower* link. They are denoted as $TransFlt$ and $TransFlw$ types, respectively. The only distinction between the follower link and the floating link is that the former has one fixed end-point (i.e., connected to a fixed link where the frame of reference is located), whereas the latter has no fixed end-point. Under each of these two types, there exist two situations to be distinguished, depending on whether the input and the desired velocities are located in a single four-bar (or equivalent) linkage, or in two different four-bar linkages. These situations will be denoted by subscripts "$within$" and "$between$", respectively. Hence, in total, the analysis method will deal with four specific types of problems, as illustrated in Figure 7.2.

The details of the method are given as follows in an algorithm for analyzing motion transfer (`C_VELOCITY`). It should be noted that when the velocities of two or more points on a given link are analyzed, the *velocity distribution* of this link is obtained. Hence, the velocity distribution problem can be viewed as a special case of the motion transfer problem.

Algorithm `C_VELOCITY`

Input: Link lengths and a driver joint angle in a linkage mechanism (qualitative or quantitative), linear velocity of a specific link, and point(s) whose relative velocity is of interest.

Output: Quantitative velocity relationships.

1. **Linkage decomposition:** Find a set of *independent* sub-linkages (equivalent to four-bar linkage mechanisms) from the given linkage.
2. **Velocity input:** If the input linear velocity is not located at an end-point, compare distances from the fixed end-point of the link to that location, and to another end-point.

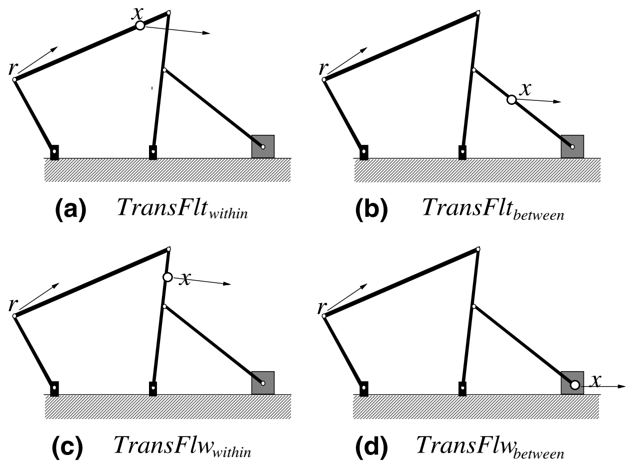

Fig. 7.2. Taking the linkage mechanism of Figure 6.4 for example, four types of qualitative velocity distribution problems can be identified, depending on whether or not the desired velocity (at point x) is on a *floating* link, and also whether or not the input velocity (at point r) and the desired velocity (at point x) are located in a single four-bar (or equivalent) linkage.

3. **Instantaneous center:** Start with the sub-linkage containing a given driver link. Locate the instantaneous center of its floating link by applying algorithm `I_CENTER` (see below).
4. If it is a $TransFlt_{within}$ problem, or a $Trans{*}{*}{*}_{between}$ problem and the linkage shares its floating link with another sub-linkage,
   a) Compare distances from the fixed end-point to the driver link and to the floating link.
   b) If it is a $TransFlt_{within}$ problem, based on obtained distance ordering, derive the velocity relationship between the known driver link and the desired point, and exit; else go to Step 6.
5. If it is a $TransFlw_{within}$ problem, or a $Trans{*}{*}{*}_{between}$ problem and the linkage shares its follower link with another sub-linkage,
   a) Compare distances from the fixed end-point to the driver link and to the follower link. If the desired point or the shared axis is not an end-point, further compare distances from the fixed end-point to the free end-point and to the desired point.

   b) If it is a $TransFlw_{within}$ problem, based on distance ordering obtained, derive the velocity relationship between the driver link and the desired point, and exit.

6. If the relative velocity at the *final* desired point is found, then exit; else find an associated sub-linkage and go to Step 2.

Symbol *** denotes either Flt or Flw.

The following algorithm provides details on how to locate an instantaneous axis for a floating link, as illustrated in Figure 7.3.

Algorithm `I_CENTER`

Input: Link lengths and a driver joint angle of a four-bar linkage (or equivalent) mechanism (qualitative or quantitative).

Output: The quantitative location of an instantaneous rotation center O of the floating link with respect to the fixed link L_1.

1. Determine the joint angles from an intermediate link L_a (connecting L_1 and L_2) to L_2 and L_3.
2. **IF** $\texttt{MAX}(\theta_{2a}, \theta_{3a}) < \theta_{23}$ **THEN**
   a) **IF** $\texttt{SUM}(\theta_{12}, \theta_{14}) < \pi$ `OR` $\texttt{SUM}(\theta_{23}, \theta_{34}) > \pi$ **THEN**
   $OQ_1 = \texttt{SUM}(OP_1, L_2)$ and $OQ_2 = \texttt{SUM}(OP_2, L_4)$ (see Figure 7.3a), where OP_1 and OP_2 with respect to L_3 are computed from $\texttt{DIFF}(\pi, \theta_{23})$ and $\texttt{DIFF}(\pi, \theta_{34})$.
   b) **ELSE**
   $OQ_1 = \texttt{DIFF}(OP_1, L_2)$ and $OQ_2 = \texttt{DIFF}(OP_4, L_4)$ (see Figure 7.3b), where OP_1 and OP_2 with respect to L_3 are computed from θ_{23} and θ_{34}.
3. **IF** $\texttt{MAX}(\theta_{2a}, \theta_{3a}) > \theta_{23}$ **THEN**
   a) **IF** $\theta_{2a} > \theta_{3a}$ **THEN**
   $OQ_1 = \texttt{DIFF}(OP_1, L_2)$ and $OQ_2 = \texttt{SUM}(OP_2, L_4)$ (see Figure 7.3c), where OP_1 and OP_2 with respect to L_3 are computed from $\texttt{DIFF}(\pi, \theta_{34})$ and θ_{23}.
   b) **ELSE**
   $OQ_1 = \texttt{DIFF}(L_2, OP_1)$ and $OQ_2 = \texttt{DIFF}(L_4, OP_2)$ (see Figure 7.3d), where OP_1 and OP_2 with respect to L_3 are computed from θ_{34} and θ_{23}.

θ_{ij} denotes the joint angle between links L_i and L_j. The lengths and joint angles are determined using the `QUALITATIVE_CONFIG` and `ANNEAL` algorithms.

SUM(∗, ∗), DIFF(∗, ∗), MAX(∗, ∗), and MIN(∗, ∗) denote the sum, difference, maximum, and minimum of the two given parameters, respectively.

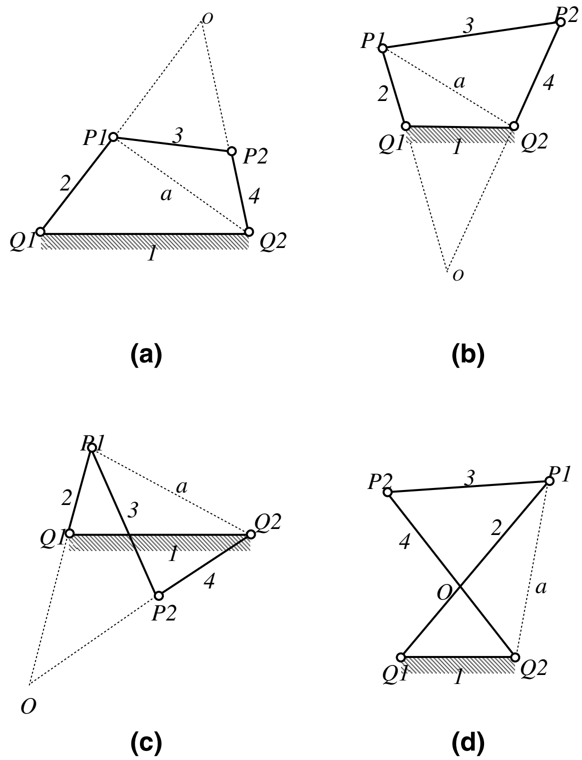

Fig. 7.3. Given a specific instantaneous configuration of a linkage, algorithm I_CENTER can generate spatial descriptions of the corresponding instantaneous center for a floating link.

Figure 7.4 presents a schematic review of the method for deriving instantaneous velocity relationships in a mechanism.

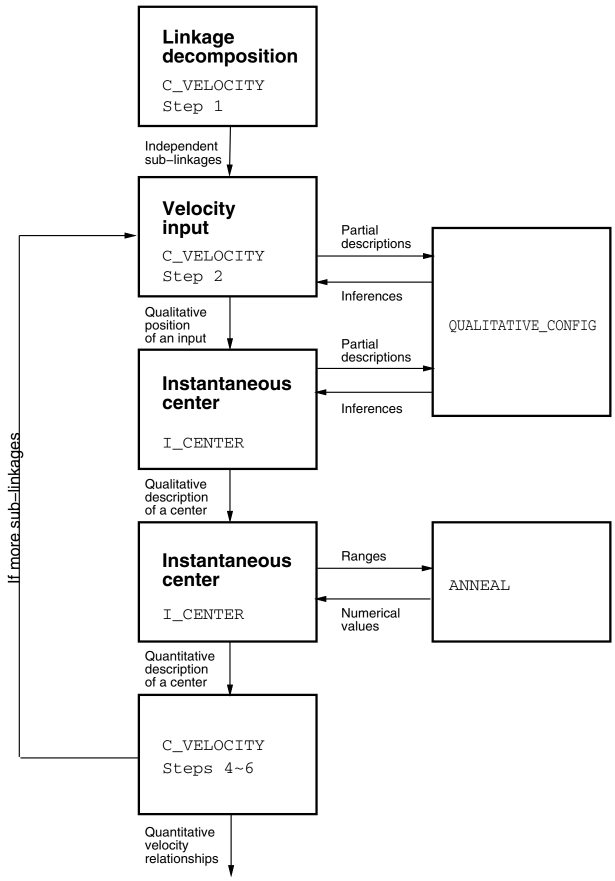

Fig. 7.4. A schematic review of the method for deriving instantaneous velocity relationships.

7.3 Examples

This section illustrates how algorithm `C_VELOCITY` can be applied to the qualitative velocity analysis, using the same linkage example shown in Figure 6.4.

Example 7.1: Velocity Relationships in a Four-Bar Linkage

Consider the mechanism shown in Figure 6.4, the motion transferred from an input crank (L_2) to a slider (L_6) is to be analyzed, as indicated in Figure 7.5. By applying graph searching, two independent four-bar linkages, A and B, can be found, and further, by definition, it is known that the problem is of $TransFlw_{between}$ type.

The velocity analysis starts with the linkage containing the driver link, i.e., linkage A. As the location of the linear velocity on the driver link is at its end-point, Step 2 of `C_VELOCITY` is bypassed. Next, the instantaneous rotation center of the floating link in linkage A is determined. In doing so, the above-mentioned `I_CENTER` algorithm is applied to obtain O_1Q_{11} and O_1Q_{12}, with respect to L_1, from θ_{12} and θ_{41}, and O_1P_{11} and O_1P_{12}, with respect to L_3, from θ_{23} and θ_{34}. From the results of O_1P_{11} and O_1P_{12}, the velocity relationship between $V_{P_{11}}$ and $V_{P_{12}}$ can be derived. Having computed the velocity at the joint of L_3 and L_4 with respect to the axis of L_4, it is possible to further analyze the velocity $V_{P_{21}}$ at the shared joint P_{21}.

Next, the second linkage, B, with the shared link as its driver link is considered, and the previous steps are repeated. Note that *the instantaneous center of the slider is located at infinity.* Thus, by applying `I_CENTER`, O_2P_{22} and O_2P_{21} can be derived, and consequently the relationship between $V_{P_{22}}$ and $V_{P_{21}}$. If all the velocity relationships obtained are combined, an approximate quantitative description of the motion transfer from L_2 to L_6, i.e., a relationship between $V_{P_{11}}$ and $V_{P_{22}}$, will be obtained.

Example 7.2: Velocity Constraints in an Ellipsograph Mechanism

Example 7.2 is concerned with the velocity relationships in an ellipsograph mechanism, as shown in Figure 7.6. Suppose that O is motionless with respect to a global fixed frame of reference. The constraint at point A allows the floating link ABP to translate along the y-axis and to rotate about the z-axis, while the constraint at point

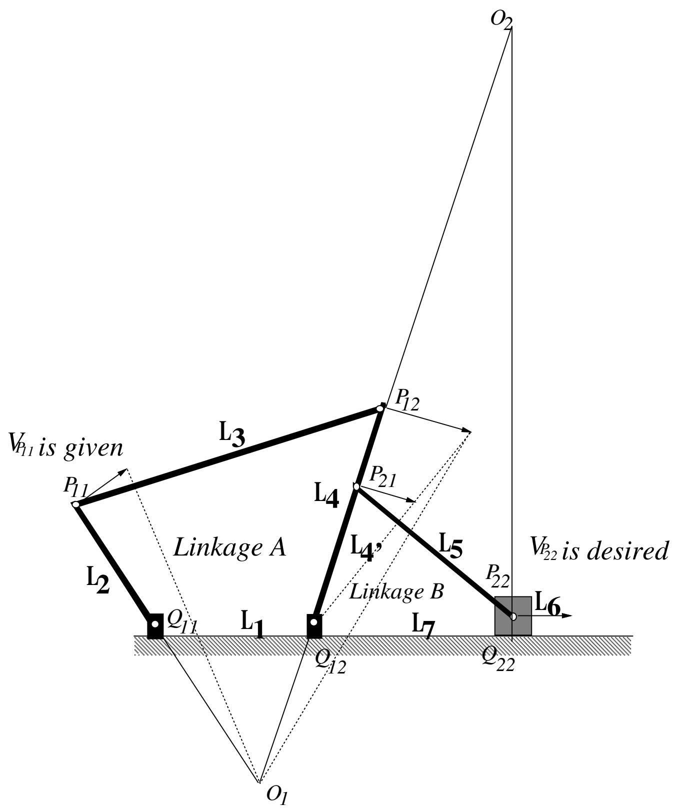

Fig. 7.5. An illustration of the velocity analysis with the same mechanism shown in Figure 6.4. There are two independent sub-linkages identified in this linkage. The velocity relationships are derived with algorithm C_VELOCITY. The instantaneous centers are computed with algorithm I_CENTER. In this mechanism, links L_1 and L_7 are both fixed with respect to a frame of reference. All the kinematic joints, except the joint between links L_6 and L_7 (a sliding joint), are revolute joints.

B allows ABP to translate along the x-axis and to rotate about the z-axis. The velocity at point P of the floating link relative to the velocity at A is desired.

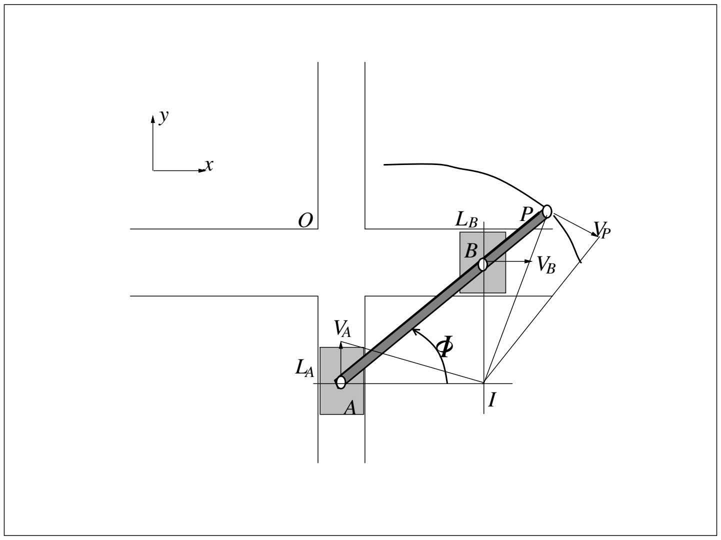

Fig. 7.6. Velocity analysis for an *ellipsograph* mechanism. In the mechanism, A translates along the y-axis and B translates along the x-axis. The velocity at point P, V_P, *relative* to the velocity at A, V_A, is desired.

In order to derive the velocity at point P, the intersection of L_A and L_B (i.e., the instantaneous center of the link ABP) is first located, which is denoted by I. Then, from the given position, $\angle\Phi$, of the link, the relationships between lengths AI and BI and between V_A and V_B are inferred. Similarly, the relationship between V_P and V_A can be obtained. Furthermore, the direction of V_P is known from $\angle AIP$. If V_P is expressed by a motion vector, then the approximate values of the velocity vector components in the x and y directions can be derived.

As the link moves, it may be desirable to know how V_P should change correspondingly in order to maintain V_A. Based on the above reasoning, the change can be readily analyzed. More specifically, with respect to the triangle $\triangle AIP$, if the angle $\angle PAI$ increases by a certain value, $\angle AIP$ and the distances AI and IP will change accordingly. As a result, a new velocity V_P can be determined.

7.4 Notes on the Application of *Velocity Analysis*

As a robotic application, the velocity analysis method illustrated in Example 7.2 can be used to *identify* external velocity constraints in a robot's task environment. Such velocity constraints are taken into account in programming the robot's compliant motions for certain manipulation tasks (such as turning the crank of the mechanism, as shown in Figure 7.6) [94, 189].

Mason [147] has proposed a method of planning robot compliant motions. This method requires that velocity constraints of individual links be propagated to a common point at the robot's effector by means of linear translational and rotational transformations. The new constraints obtained can then be translated into specific control strategies. This method outputs *accurate* velocity constraint information with respect to given task configuration and trajectory, provided that the information on the task geometry as well as appropriate linear transformations are given.

In real-world robot manipulation tasks, it may not be possible to obtain *exact geometric information* about a mechanism. Furthermore, the linear transformations for performing velocity propagations are usually difficult to formulate. In such cases, the method described in Sections 7.2 and 7.3 becomes handy to use.

7.5 Relative Motion Method of Analyzing Velocities

In the preceding sections, we have shown a qualitative geometric reasoning method formulated especially for the analysis of motion relationships in linkage mechanisms. Although the method may further be modified to handle more general CSI mechanisms, the resulting algorithm will conceivablely be complicated and dependent on the mechanisms being analyzed.

In this section, we will discuss a more general approach to deriving the qualitative description of linear velocities in CSI mechanisms based on individual bodies' *relative motions*. Typically, the information required is a description of the mechanism's configuration specifying qualitative positions (e.g., angular positions in the case of four-bar linkages) with respect to a set of *local reference frames* (i.e., relative coordinate systems as given in Definition 3.1.5).

From the definition of relative motion, we know that an absolute velocity may be expressed in terms of a sequence of velocities relative to an absolute velocity. In such a case, we say that the absolute velocity satisfies a *velocity* *constraint equation.* The fundamental idea of qualitative analysis with the relative velocity method is that, since we can find a set of motion vectors (see Figure 6.21) which qualitatively indicates the direction of relative motion, and an actual velocity vector is in fact proportional to its corresponding motion vector, we can write a velocity constraint equation describing the motion of a kinematic chain in terms of relative motion vectors. Furthermore, by evaluating and selecting sets of *vector modifiers* in the equation, we will be able to qualitatively determine both relative and absolute linear velocities.

7.5.1 Axioms and Theorems in Revolute or Prismatic-Pairing Body Motion

Of the various methods for transmitting motions, revolute and prismatic pairing methods are of the most interest in qualitative kinematics. Examples of mechanisms using these methods are linkages. In linkages, the motion of one link relative to another satisfies a certain constraint imposed by their intermediate pairs, and the velocity can be determined given the link's relative instantaneous position. In other words, it is possible to describe the constrained motion of a mechanism composed of such links in terms of the sum of individual links' relative motions.

Before we can qualitatively analyze the motions of a CSI mechanism using the relative motion approach, we will first formulate some fundamental axioms, theorems, and constructions concerning the motion of CSI mechanism components.

Revolute-Pairing Bodies. Suppose that body A is connected to body B by means of a revolute pair. In this case, the motion of A relative to B may be described in terms of the motion of A with respect to a reference frame on B originated at the rotational axis. In the foregoing discussion, we will use Cartesian coordinate systems as the relative reference frames. The relative instantaneous angular position of a given point on body A is defined as the smallest non-negative angle formed by the x-axis and the line segment passing through the point and its rotational axis. Hence, no matter in which quadrant the line segment lies, its relative angular position (θ) is always within the range of $[0, \frac{\pi}{2}]$.

Axiom 7.5.1 *Let a point on the body A be in rotation with respect to a reference frame and let l be the line segment passing through the point and its rotating axis. If the rotation is counterclockwise, then when l is in the first quadrant, the motion vectors corresponding to the set of qualitative angles can be described as in Table 7.1.*

θ_I =	va^-	va^+	a^-	a^+
(m_x, m_y)	(-s, vl)	(-m, l)	(-l, m)	(-vl, s)

Table 7.1. Relative motion vectors of a rotating point in quadrant I.

Theorem 7.5.1 (Change of direction). *In Axiom 7.5.1, if the point rotates in an opposite direction, then the corresponding motion vectors will have the same magnitudes as before, but with opposite directions.*

Theorem 7.5.2 (Symmetrical property of a circular motion). *In Axiom 7.5.1, if l is in the second quadrant, then the direction of the y-coordinate components (m_y) in the corresponding motion vectors will change from positive to negative. If l is in the third quadrant, then the x and y components in the corresponding motion vectors will both change directions. If l is in the fourth quadrant, then the x components in the corresponding motion vectors will change direction.*

In general, it is always possible to determine the constrained motion of a mechanism from its reversed motion.

Theorem 7.5.3 (Inversion of a constrained motion). *Suppose that some constrained relative motion in a constrained closed-loop kinematic chain, A, is given. If one link of A moves over its entire range of motion, but with an opposite driving direction at each position, then the motions of all links in A reverse their directions.*

Prismatic-Pairing Bodies. The relative motion between two bodies A and B of a prismatic pair can be described in the same way as that of the revolute pair. A reference frame for the motion of A is fixed on B.

Axiom 7.5.2 *If the x-axis of the Cartesian system is parallel to a common tangent on the contact surface, then the motion of A relative to the frame can be described in terms of A's relative motion vector, $(\pm vl, 0)$.*

Axiom 7.5.3 *The prismatic motion of A relative to B is equivalent to the rotation of A relative to B with its center at infinity.*

Having understood the relative motion vectors of revolute-pairing and prismatic-pairing mechanism components, we can readily determine the constrained motion of an intermediately connected (e.g., linkage) mechanism, the details of which will be shown in Chapter 7.5.

7.5.2 Kinematic Modeling

In order to derive a velocity constraint equation, we will first find the relative motion vectors at the pairs of *links* (here the term link is used in a general sense). In general, corresponding to a specific chain progression in a derived mechanism graph, there exists a directed kinematic chain, $p_0 \rightarrow^{l_1} p_1 \rightarrow^{l_2} p_2 \rightarrow^{l_3} \ldots \rightarrow^{l_n} p_n$, where $p_{k-1} \rightarrow^{l_k} p_k$ denotes that link l_k is directed from lower pair p_{k-1} to p_k. In this chain, the local relative coordinate system for link l_k will be centered at the pairing contact p_{k-1} on l_{k-1}. In other words, the determination of positions for local reference coordinate systems in a mechanism will depend on the direction we choose for the chain progression. Given a set of relative reference frames, the derivations of relative motion vectors in relation to specific pairing contacts can be based on the axioms and theorems presented in Section 7.5.1.

As we know, an actual velocity vector has the same direction as its corresponding motion vector and their magnitudes are proportional to each other. Therefore, having obtained the motion vectors of a set of connected links, we can further find a constraint equation of the actual velocity vectors. In doing so, we may apply the following two theorems of constrained kinematic chains.

Theorem 7.5.4 (Loop postulate). *The algebraic sum of relative actual velocity vectors associated with the consecutive lower pairs of links in a simple closed-loop kinematic chain is zero.*

From Theorem 7.5.4, it is possible to further derive the following theorem:

Theorem 7.5.5 (Vertex postulate). *The actual velocity vectors of two links with respect to the same frame are equal at their lower-pairing contact.*

7.6 Qualitative Analysis of Relative Velocities

In this section, we discuss how to derive the qualitative description of motion of any specific link given the kinematic model of a CSI mechanism expressed in terms of an actual velocity constraint equation.

7.6.1 Solving Velocity Constraint Equations

The essence of qualitative reasoning about the motion of a CSI mechanism lies in the use of a heuristic search technique to modify the qualitative values of velocity vectors in a constraint equation initialized by motion vectors. The problem of heuristic search for appropriate velocity values can be stated as follows:

Given an initial representation of a velocity constraint equation, as expressed in terms of relative motion vectors, determine for each motion vector a sequence of modifiers such that the resulting vectors best satisfy the equation. This set of vectors is considered a qualitative solution of the velocity equation and therefore gives the absolute or relative velocities of links in the mechanism.

Here, the vectors that best satisfy the velocity equation are defined as those which, as compared to others resulting from *further* applying modifiers, yield the smallest error with respect to the equation. In order to obtain the overall best solution, the heuristic search is carried out in such a way that at each iteration all the possible modifiers are evaluated and those that can give a temporary best solution with respect to the previous vectors are selected. During each modification of velocity vectors, the derivations and evaluations of qualitative vectors are constructed from the inference rules of qualitative vector operation. The modified velocity vectors are termed *intermediate velocity vectors* or temporary velocity vectors.

7.6.2 An Algorithm for Determining Linear Velocities

An algorithm for determining linear velocities of a CSI mechanism utilizing the relative motion representation is given as follows:

Algorithm `LINEAR_VELOCITY`

Input: A representation of the mechanism's configuration in terms of the instantaneous position of each link with respect to some local reference frame at its lower pair.

Output: The desired velocity vector of a given link with respect to a fixed or moving link.

1. Derive a mechanism graph representation of the CSI mechanism.
2. Determine the independent loops in the graph which correspond to the constrained kinematic subchains in the mechanism.
3. Find the subchain which contains a link whose relative velocity at a certain pair is given.
4. Divide the subchain into two distinct chain progressions directed from the fixed link to the known pair.

5. For each chain progression, according to the given direction, express the velocity at the known pair in terms of the relative velocities of consecutive links. Connect these two expressions into a velocity constraint equation (Theorem 7.5.5).
6. For each chain progression, find the relative motion vectors of links utilizing the axioms and theorems presented in Section 7.5.1.
7. Transform the actual velocity vector terms in the original velocity constraint equation into corresponding modified motion vectors. If a relative velocity term is the given velocity then write its qualitative value.
8. Modify the set of motion (or intermediate) vectors in the new equation by using vector modifiers until the resulting vectors yield the smallest qualitative error in the original constraint equation. In each step of modification, all combinations of possible modifiers are evaluated and the (temporarily) best one is selected and applied.
9. Let the set of resulting intermediate vectors be the qualitative solution of the original velocity constraint equation. If the desired relative velocity between two links is within the current loop, then find its qualitative vector value by adding or subtracting the consecutive relative velocities in the given progression direction; else find the absolute velocity value of the link shared by another independent loop and consider the subchain corresponding to the new loop back to Step 4.

It should be noted that the *temporarily best modifiers* for a set of intermediate velocity vectors, as mentioned in Step 8, are defined such that

$$max(|E_{xi}|, |E_{yi}|) = min\{max(|E_{xj}|, |E_{yj}|)\}$$

where E_{xj} and E_{yj} denote the x and y velocity-component errors of the constraint equation, respectively, resulting from applying one of the four modifiers, j. E_{xi} and E_{yi} denote the errors resulting from applying temporarily best modifier i.

7.7 An Example

In this section, we present an example of qualitative reasoning about instantaneous linear velocities of a linkage mechanism with the relative motion method.

Example 7.3: Relative Motions in a Quick-Return Mechanism

The linkage to be analyzed is a quick-return mechanism, as shown in Figure 7.7, where the velocity of point d is desired and V_{b_2} is given as a qualitative row vector (l, l). It can be noted that if the velocity at point b_4 which lies on the link l_4 is known, then V_d can be inferred by comparing the distances from the axis c to b_4 and to d. Therefore, the subgoal of the velocity analysis becomes the determination of linear velocity at b_4.

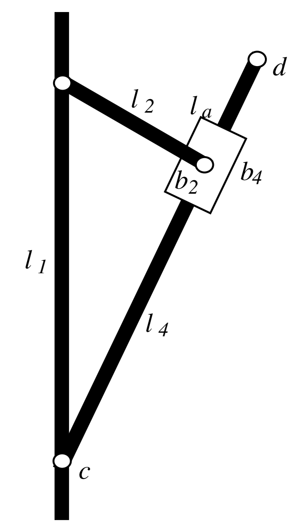

Fig. 7.7. A quick-return mechanism.

To begin the analysis, we represent the mechanism in an equivalent mechanism graph. Since component 3 is constrained to slide along link l_4, we can obtain an equivalent linkage mechanism by adding an imaginary link l_a between l_2 and l_4. The corresponding mechanism graph is given in Figure 7.8. It is obvious that the derived graph contains only one independent loop.

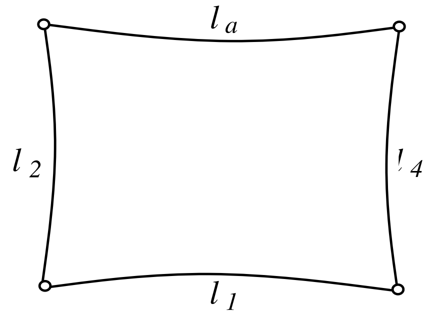

Fig. 7.8. The mechanism graph of a quick-return mechanism shown in Figure 7.7.

The next step is to construct a velocity constraint equation from the graph. We divide the loop into two distinct chains from one fixed joint to the known joint b_2 and, for each of the two chains, write V_{b_2} in terms of the sum of pairwise relative velocities. As the linear velocity at the endpoint of link l_2 relative to the fixed link l_1 is given, $V_{b_2 \leftarrow l_2}$, i.e., the velocity derived from the chain containing l_2, will be written in terms of the known qualitative row vector (l, l). Consequently, by Theorem 7.5.5 (vertex postulate), we can write a velocity constraint equation for this particular closed-loop mechanism as follows:

$$V_{b_2 \leftarrow l_a, l_4 \ldots} = V_{b_2 \leftarrow l_2} \tag{7.2}$$

or

$$V_{l_4/l_1} + V_{l_a/l_4} = (l, l) \tag{7.3}$$

where V_{l_4/l_1} and V_{l_a/l_4} denote the velocities of links l_4 and l_a relative to l_1 and l_4, respectively.

We know that the velocities V_{l_4/l_1} and V_{l_a/l_4} are proportional to their corresponding relative motion vectors m_{l_4/l_1} and m_{l_a/l_4}. Therefore, Eq. 7.3 can further be approximately rewritten as

$$\lambda_1 m_{l_4/l_1} + \lambda_2 m_{l_a/l_4} = (l, l) \tag{7.4}$$

where λ_1 and λ_2 denote the series of qualitative vector modifiers to be found. The relative motion vectors, corresponding to the given configuration, are shown in Figure 7.9. They are derived straightforwardly from Axioms 7.5.1, 7.5.2, and Theorem 7.5.1.

Having obtained Eq. 7.4, the next step of velocity analysis is to evaluate the possible combinations of the predefined modifiers and assign the best suitable set to the equation. The criterion is that the

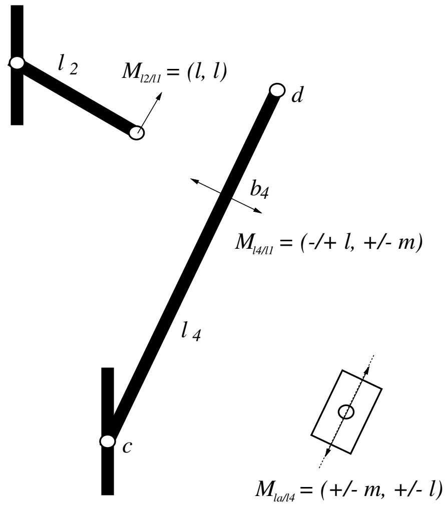

Fig. 7.9. The relative motion vectors of links in the mechanism of Figure 7.7.

intermediate vectors resulting from applying modifiers should yield the smallest error in Eq. 7.3. This step is repeated until the error cannot be further reduced. Table 7.2 shows such an iterative process and Table 7.3 gives the details of the qualitative modifier evaluations in Step 3. In the tables, the qualitative inferences involved are based on the rules given in Section 7.5.1 and the error of Step i is defined as follows:

$$E_i = {\lambda_1}^i v_{l_4/l_1}^{(i-1)} + {\lambda_2}^i v_{l_a/l_4}^{(i-1)} - V_{b-2 \leftarrow l_2} \tag{7.5}$$

where ${\lambda_1}^i$ and ${\lambda_2}^i$ denote the qualitative vector modifiers being applied in Step i, and $v_{l_4/l_1}^{(i-1)}$ and $v_{l_a/l_4}^{(i-1)}$ denote the intermediate vectors resulted from the iterative Step $i-1$. Note that in Step 3 the modifier *Inverse* is not evaluated. This is because the directions of velocities have been modified in Step 1 and therefore the remaining steps will deal only with magnitudes.

From Table 7.2 it may be noticed that since the error cannot be further reduced, the value of v_{l_4/l_1}, i.e., $(s, -vs)$, will be considered as the approximate value of V_{l_4/l_1}, the qualitative value of the linear velocity of link l_4 at the instantaneous pairing point b_4 relative to fixed link l_1. Therefore, by comparing the distances from the fixed axis c to b_4 and to d, we can determine the qualitative value of the linear velocity at d. As given in the instantaneous configuration of

Steps	$\lambda_1 m_{l_4/l_1}$	$\lambda_2 m_{l_a/l_4}$	$V_{b_2 \leftarrow l_2}$	Errors
0	$(-l, m)$	(m, l)	(l, l)	$(-vl, m)$
1	$Inverse(-l, m)$	$Identity(m, l)$	(l, l)	$(m, -m)$
	$\Rightarrow (l, -m)$	$\Rightarrow (m, l)$		
2	$Decrease(l, -m)$	$Identity(m, l)$	(l, l)	$(s, -s)$
	$\Rightarrow (m, -s)$	$\Rightarrow (m, l)$		
3	$Decrease(m, -s)$	$Identity(m, l)$	(l, l)	$(vs, -vs)$
	$\Rightarrow (s, -vs)$	$\Rightarrow (m, l)$		

Table 7.2. The modifications of motion (and intermediate velocity) vectors.

this problem, distance cd is almost twice as long as distance cb_4, which is to say that the magnitude of V_d is similarly twice as large as that of V_{b_4}. Hence, we can derive the qualitative value of V_{b_4} to be $(l, -s)$. This step is readily understood by following the discussion presented in Chapter 6.

Steps	Increase	Decrease	Identity	$v_{l_4/l_1} + v_{l_a/l_4}$	$V_{b_2 \leftarrow l_2}$	Errors
$3a$	${\lambda_1}^3$	${\lambda_2}^3$	-	$(l, -m) + (s, m)$	$(l, -l)$	$(s, -l)$
$3b$	${\lambda_1}^3$	-	${\lambda_2}^3$	$(l, -m) + (m, l)$	$(l, -l)$	$(m, -m)$
$3c$	-	${\lambda_1}^3$	${\lambda_2}^3$	$(s, -vs) + (m, l)$	$(l-, l)$	$*(vs, -vs)$
$3d$	${\lambda_2}^3$	${\lambda_1}^3$	-	$(l, vl) + (s, -vs)$	$(-l, l)$	$(s, 0)$
$3e$	${\lambda_2}^3$	-	${\lambda_1}^3$	$(l, vl) + (m, -s)$	$(l-, l)$	$(m, -vs)$
$3f$	-	${\lambda_2}^3$	${\lambda_1}^3$	$(m, l) + (m, -s)$	$(l, -l)$	$(s, -s)$
$3g$	${\lambda_2}^3, {\lambda_1}^3$	-		$(l, vl) + (l, -m)$	$(-l, l)$	$(l, -s)$
$3h$	-	${\lambda_2}^3, {\lambda_1}^3$		$(s, m) + (s, -vs)$	$(-l, l)$	$(0, -s)$

Table 7.3. The evaluations of qualitative modifiers in Step 3.

7.8 Summary

In this chapter, we have shown how to analyze the velocity relationships of a linkage mechanism given its dimensional specifications. We have developed an algorithm for the velocity analysis, which applies the kinematic concept of instantaneous axis. This approach, although computationally efficient, is to some extent limited to the analysis of linkage-like mechanisms.

Later in this chapter, we showed how to utilize the relative motion vector representation of mechanism components and to generate solutions by resolving qualitative motion constraint equations. Although this approach appears to be less computationally efficient than the qualitative trigonometric reasoning approach, it is more applicable in solving general CSI mechanism problems.

The algorithms described in this chapter are designed particularly for solving *instantaneous* velocity problems. It should be noted that these algorithms can also be extended to handle the kinematic state transitions of a moving CSI mechanism. In such a case, the analysis should be preceded by a step consisting of partitioning the value of an input displacement into a quantity space and computing the set of corresponding qualitative configurations, as mentioned in the previous chapters.

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