

# Table of Contents

<b>1</b>	<b>Introduction</b>	1
1.1	The Frenkel-Kontorova Model	1
1.2	The Sine-Gordon Equation	5
<b>2</b>	<b>Physical Models</b>	9
2.1	General Approach	9
2.2	A Mechanical Model	10
2.3	Dislocation Dynamics	12
2.4	Surfaces and Adsorbed Atomic Layers	14
2.5	Incommensurate Phases in Dielectrics	18
2.6	Crowdions and Lattice Defects	20
2.7	Magnetic Chains	21
2.8	Josephson Junctions	23
2.9	Nonlinear Models of the DNA Dynamics	25
2.10	Hydrogen-Bonded Chains	27
2.11	Models of Interfacial Slip	29
<b>3</b>	<b>Kinks</b>	31
3.1	The Peierls-Nabarro Potential	31
3.2	Dynamics of Kinks	38
3.2.1	Effective Equation of Motion	38
3.2.2	Moving Kinks	40
3.2.3	Trapped Kinks	42
3.2.4	Multiple Kinks	44
3.3	Generalized On-Site Potential	47
3.3.1	Basic Properties	48
3.3.2	Kink Internal Modes	50
3.3.3	Nonsinusoidal On-Site Potential	54
3.3.4	Multiple-Well Potential	58
3.3.5	Multi-Barrier Potential	63
3.4	Disordered Substrates	66
3.4.1	Effective Equation of Motion	68
3.4.2	Point Defects	72
3.4.3	External Inhomogeneous Force	73

3.5	Anharmonic Interatomic Interaction .....	75
3.5.1	Short-Range Interaction .....	77
3.5.2	Nonconvex Interatomic Potentials .....	82
3.5.3	Kac-Baker Interaction .....	89
3.5.4	Long-Range Interaction .....	92
3.5.5	Compacton Kinks .....	96
<b>4</b>	<b>Breathers</b> .....	99
4.1	Perturbed Sine-Gordon Breathers .....	99
4.1.1	Large-Amplitude Breathers .....	99
4.1.2	Small-Amplitude Breathers .....	102
4.2	Breather Collisions .....	103
4.2.1	Many-Soliton Effects .....	105
4.2.2	Fractal Scattering .....	107
4.2.3	Soliton Cold Gas .....	109
4.3	Impurity Modes .....	111
4.3.1	Structure and Stability .....	111
4.3.2	Soliton Interactions with Impurities .....	116
4.4	Discrete Breathers .....	121
4.4.1	General Remarks .....	121
4.4.2	Existence and Stability .....	122
4.4.3	The Discrete NLS Equation .....	125
4.4.4	Dark Breathers .....	131
4.4.5	Rotobreathers .....	134
4.5	Two-Dimensional Breathers .....	136
4.6	Physical Systems and Applications .....	138
<b>5</b>	<b>Ground State</b> .....	141
5.1	Basic Properties .....	141
5.2	Fixed-Density Chain .....	149
5.2.1	Commensurate Configurations .....	149
5.2.2	Incommensurate Configurations .....	159
5.3	Free-End Chain .....	165
5.3.1	Frank-van-der-Merwe Transition .....	167
5.3.2	Devil's Staircase and Phase Diagram .....	171
5.4	Generalizations of the FK Model .....	174
5.4.1	On-Site Potential of a General Form .....	174
5.4.2	Anharmonic Interatomic Potential .....	177
5.4.3	Nonconvex Interaction .....	184
<b>6</b>	<b>Statistical Mechanics</b> .....	195
6.1	Introductory Remarks .....	195
6.2	General Formalism .....	197
6.3	Weak-Bond Limit: Glass-Like Properties .....	202
6.3.1	Ising-Like Model .....	202

6.3.2	Configurational Excitations .....	205
6.3.3	Two-Level Systems and Specific Heat .....	208
6.4	Strong-Bond Limit: Gas of Quasiparticles .....	211
6.4.1	Sharing of the Phase Space and Breathers .....	214
6.4.2	Kink-Phonon Interaction .....	215
6.4.3	Kink-Kink Interaction .....	218
6.4.4	Discreteness Effects .....	218
6.5	Statistical Mechanics of the FK Chain .....	220
6.5.1	Transfer-Integral Method .....	220
6.5.2	The Pseudo-Schrödinger Equation .....	225
6.5.3	Susceptibility .....	227
6.5.4	Hierarchy of Superkink Lattices .....	233
6.5.5	Equal-Time Correlation Functions .....	234
6.5.6	Generalized FK Models .....	239
<b>7</b>	<b>Thermalized Dynamics .....</b>	<b>243</b>
7.1	Basic Concepts and Formalism .....	243
7.1.1	Basic Formulas .....	245
7.1.2	Mori Technique .....	247
7.1.3	Diffusion Coefficients .....	249
7.1.4	Noninteracting Atoms .....	251
7.1.5	Interacting Atoms .....	253
7.2	Diffusion of a Single Kink .....	257
7.2.1	Langevin Equation .....	258
7.2.2	Intrinsic Viscosity .....	261
7.2.3	Anomalous Diffusion .....	263
7.2.4	Kink Diffusion Coefficient .....	265
7.3	Dynamic Correlation Functions .....	268
7.4	Mass Transport Problem .....	272
7.4.1	Diffusion in a Homogeneous Gas .....	273
7.4.2	Approximate Methods .....	276
7.4.3	Phenomenological Approach .....	281
7.4.4	Self-Diffusion Coefficient .....	284
7.4.5	Properties of the Diffusion Coefficients .....	286
<b>8</b>	<b>Driven Dynamics .....</b>	<b>291</b>
8.1	Introductory Remarks .....	291
8.2	Nonlinear Response of Noninteracting Atoms .....	292
8.2.1	Overdamped Case .....	293
8.2.2	Underdamped Case .....	294
8.3	Overdamped FK Model .....	300
8.4	Driven Kink .....	306
8.5	Instability of Fast Kinks .....	308
8.6	Supersonic and Multiple Kinks .....	316
8.7	Locked-to-Sliding Transition .....	323

8.7.1	Commensurate Ground States	323
8.7.2	Complex Ground States and Multistep Transition	323
8.8	Hysteresis	328
8.9	Traffic Jams	330
8.10	Periodic Forces: Dissipative Dynamics	334
8.11	Periodic Driving of Underdamped Systems	339
<b>9</b>	<b>Ratchets</b>	<b>343</b>
9.1	Preliminary Remarks	343
9.2	Different Types of Ratchets	345
9.2.1	Supersymmetry	345
9.2.2	Diffusional Ratchets	346
9.2.3	Inertial Ratchets	353
9.3	Solitonic Ratchets	356
9.3.1	Symmetry Conditions	357
9.3.2	Rocked Ratchets	357
9.3.3	Pulsating Ratchets	361
9.4	Experimental Realizations	363
<b>10</b>	<b>Finite-Length Chain</b>	<b>365</b>
10.1	General Remarks	365
10.2	Ground State and Excitation Spectrum	366
10.2.1	Stationary States	366
10.2.2	Continuum Approximation	369
10.2.3	Discrete Chains	370
10.2.4	Vibrational Spectrum	372
10.3	Dynamics of a Finite Chain	374
10.3.1	Caterpillar-Like Motion	374
10.3.2	Adiabatic Trajectories	375
10.3.3	Diffusion of Short Chains	379
10.3.4	Stimulated Diffusion	381
10.4	Nonconvex Potential	381
<b>11</b>	<b>Two-Dimensional Models</b>	<b>383</b>
11.1	Preliminary Remarks	383
11.2	Scalar Models	385
11.2.1	Statistical Mechanics	389
11.2.2	Dynamic Properties	391
11.3	Zigzag Model	392
11.3.1	Ground State	394
11.3.2	Aubry Transitions	397
11.3.3	Classification of Kinks	400
11.3.4	Zigzag Kinks	405
11.3.5	Applications	413
11.4	Spring-and-Ball Vector 2D Models	415

11.4.1 The Ground State .....	417
11.4.2 Excitation Spectrum .....	420
11.4.3 Dynamics .....	420
11.5 Vector 2D FK Model .....	422
11.5.1 Locked-to-Sliding Transition .....	423
11.5.2 “Fuse-Safety Device” on an Atomic Scale .....	429
<b>12 Conclusion .....</b>	<b>431</b>
<b>13 Historical Remarks .....</b>	<b>435</b>
<b>References .....</b>	<b>441</b>
<b>Index .....</b>	<b>465</b>



<http://www.springer.com/978-3-540-40771-3>

The Frenkel-Kontorova Model  
Concepts, Methods, and Applications  
Braun, O.M.; Kivshar, Y.  
2004, XVIII, 472 p., Hardcover  
ISBN: 978-3-540-40771-3