Preface

This work is aimed at an audience with a sound mathematical background wishing to learn about the rapidly expanding field of mathematical finance. Its content is suitable particularly for graduate students in mathematics who have a background in measure theory and probability.

The emphasis throughout is on developing the mathematical concepts required for the theory within the context of their application. No attempt is made to cover the bewildering variety of novel (or 'exotic') financial instruments that now appear on the derivatives markets; the focus throughout remains on a rigorous development of the more basic options that lie at the heart of the remarkable range of current applications of martingale theory to financial markets.

The first five chapters present the theory in a discrete-time framework. Stochastic calculus is not required, and this material should be accessible to anyone familiar with elementary probability theory and linear algebra.

The basic idea of pricing by arbitrage (or, rather, by non-arbitrage) is presented in Chapter 1. The unique price for a European option in a single-period binomial model is given and then extended to multi-period binomial models. Chapter 2 introduces the idea of a martingale measure for price processes. Following a discussion of the use of self-financing trading strategies to hedge against trading risk, it is shown how options can be priced using an equivalent measure for which the discounted price process is a martingale. This is illustrated for the simple binomial Cox-Ross-Rubinstein pricing models, and the Black-Scholes formula is derived as the limit of the prices obtained for such models. Chapter 3 gives the 'fundamental theorem of asset pricing', which states that if the market does not contain arbitrage opportunities there is an equivalent martingale measure. Explicit constructions of such measures are given in the setting of finite market models. Completeness of markets is investigated in Chapter 4; in a complete market, every contingent claim can be generated by an admissible self-financing strategy (and the martingale measure is unique). Stopping times, martingale convergence results, and American options are discussed in a discrete-time framework in Chapter 5.

The second five chapters of the book give the theory in continuous time. This begins in Chapter 6 with a review of the stochastic calculus. Stopping times, Brownian motion, stochastic integrals, and the Itô differentiation rule are all defined and discussed, and properties of stochastic differential equations developed.

The continuous-time pricing of European options is developed in Chapter 7. Girsanov's theorem and martingale representation results are developed, and the Black-Scholes formula derived. Optimal stopping results are applied in Chapter 8 to a thorough study of the pricing of American options, particularly the American put option.

Chapter 9 considers selected results on term structure models, forward and future prices, and change of numéraire, while Chapter 10 presents the basic framework for the study of investment and consumption problems.

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