Chapter 2

A SURVEY ON LOCATION PROBLEMS WITH APPLICATIONS IN TELECOMMUNICATIONS

The aim of this survey is to gather some significant examples of location problems that arise in the design of telecommunications networks and review the polyhedral properties of these problems. The survey in Gourdin et al. [37] gives a more detailed analysis of the models and solution methods for most of the problems presented here. For earlier surveys, one can also refer to Boffey [10] and Klincewicz [46]. There is a very recent survey by Campbell et al. [13] on hub location problems. Here, we review only the hub location models for the single assignment version.

The basic questions related with the design of a telecommunications network are where should the concentrators be located, what type of machinery should be used, how should the terminals be connected (assigned) to the concentrators and how should the concentrators be connected among themselves or to some central node, how should the traffic be routed and what type of links and how many of these links should be installed on the edges of the network. It is quite hard to come up with answers to all these questions simultaneously. So, in most methods, the design is done in an iterative manner. Initially, the concentrator locations and assignments of terminals to the concentrators are determined. Then the design of the access networks and the design of the backbone network become independent and can be handled separately. Thus the location of concentrators is a crucial issue.

2.1 Facility Location Problems

The facility location models for telecommunications networks are concerned with the location of concentrators and the assignment of terminals to these concentrators.

The basic problems in most of these models are the Uncapacitated Facility Location Problem (UFLP), the Capacitated Facility Location Problem (CFLP) and the Capacitated Facility Location Problem with Single Assignment (CFLPS).

2.1.1 Uncapacitated Facility Location Problem

In this section, we formulate the UFLP and present some of its variants that have applications in telecommunications. The UFLP is as follows. Given the set of clients N and the set of possible locations for the facilities M, determine the location of facilities and assign the clients to these facilities in order to minimize the sum of the cost of installing facilities and the cost of serving clients via the installed facilities.

Let y_j be 1 if a facility is installed at location $j \in M$ and 0 otherwise. Define x_{ij} to be the fraction of demand of client $i \in N$ that is served by the facility at location $j \in M$.

Using these two sets of variables, the UFLP can be formulated as follows:

$$\min \sum_{i \in N} \sum_{j \in M} C_{ij} x_{ij} + \sum_{j \in M} F_j y_j \tag{2.1}$$

$$\text{s.t.} \sum_{j \in M} x_{ij} = 1 \qquad \forall i \in N \tag{2.2}$$

$$x_{ij} \leq y_j \qquad \forall i \in N, j \in M \tag{2.3}$$

$$x_{ij} \geq 0 \qquad \forall i \in N, j \in M \tag{2.4}$$

$$y_j \in \{0, 1\} \qquad \forall j \in M \tag{2.5}$$

where C_{ij} denotes the cost of serving client $i \in N$ by facility at location $j \in M$ and F_j denotes the cost of installing a facility at location $j \in M$.

By constraints (2.2) and (2.4), the demand of each client is served, and by constraints (2.3), it is served by a facility only if this facility is installed. The cost function (2.1) is the sum of the cost of serving clients and the cost of installing facilities.

For more information about the UFLP, one may refer to, e.g., Cornuejols et al. [22], Krarup and Pruzan [48], and Labbé et al. [49].

In the context of telecommunications, clients are terminals and facilities are concentrators. The UFLP can be solved to decide about the locations of concentrators and the assignments of terminals. Then the design of the backbone and access networks can be done separately. It can also be solved to design a network where both the backbone and access networks are stars (see Chapter4).

For polyhedral properties of UFLP, see, e.g., Canovas et al. [14], Cho et al. [18, 19], Cornuejols and Thizy [24], and Guignard [40].

Next, we present examples of network design problems where UFLP is either generalized or appears as a subproblem. These examples are about the design of networks where at least one component is a star or a tree.

Helme and Magnanti [43] consider the design of a satellite communications network where each terminal is directly assigned to a concentrator and concentrators are directly assigned to a central unit. Thus the backbone and access networks are stars. There is an operating cost for the concentrators, which is the cost of using the capacity of concentrators. To compute this cost, it is necessary to differentiate the traffic between terminals assigned to the same concentrator and the traffic between terminals assigned to different concentrators. This results in a quadratic cost function. The authors present a linearization for the problem and a branch and bound algorithm to solve it.

Chardaire et al. [17] consider the design of a two level network where backbone and access networks are stars. Each terminal is connected to a first level concentrator which is connected to a second level concentrator. All second level concentrators are connected to a central unit. They present two integer programming formulations, a simulated annealing algorithm and a family of cuts.

Chung et al. [21] consider the design of a network where the backbone is fully connected and the access networks are stars. The authors develop a formulation to find such a network that minimizes the cost of installing concentrators, cost of assigning terminals to concentrators and the cost of interconnecting concentrators. The total cost function is quadratic due to the last component. They linearize the formulation and present a dual-based solution procedure.

Mateus et al. [60] design a network, where the backbone and access networks are trees, in three phases. In the location phase, they solve a UFLP where the number of concentrators to be installed is bounded from above. Then, for each concentrator and the set of terminals assigned to it, they find a minimum cost tree network with the concentrator as the root. In the third phase, the backbone network is designed.

Current and Pirkul [26] consider the problem where the concentrators are connected to each other by a path of primary arcs and the terminals are connected to the concentrators by paths of secondary arcs. Two heuristics based on Lagrangian Relaxation are given.

Pirkul and Nagarajan [70] and Lee et al. [55] consider the design of a network where the backbone is a tree and access networks are stars. Pirkul and Nagarajan (1992) use a two-phase algorithm where in the first phase, they divide the set of nodes into regions and in the second phase, they determine, for each region, a path from the furthest node of the region to the central node such that the nodes of this path are concentrators. Lee et al. [55] present a formulation and apply Lagrangian Relaxation.

Gavish [35] formulates the problem of designing a network where the backbone is a star and access networks are trees. The objective function involves the cost of establishing the links and installing the concentrators. The model chooses among different types of links with different costs and capacities.

Gavish [36] discusses the evolution of the network topologies and the network design process and gives a model to design a network with no a priori topology. The author presents a Lagrangian Relaxation based solution procedure.

2.1.2 Capacitated Facility Location Problems

Capacitated location problems in telecommunications, relax the assumption that a concentrator can serve all terminals. These models are mostly variants of CFLP and CFLPS.

The CFLP is defined as follows. Given a set of clients N with known demands d_i for each client $i \in N$, and a set of possible locations for facilities M with a given capacity Q_j for each $j \in M$, the aim is to install facilities and serve the demands of the clients respecting the capacities of the facilities and to minimize the total cost of installing facilities and the cost of serving clients.

The CFLP can be formulated as follows:

$$\begin{aligned}
&\min \sum_{i \in N} \sum_{j \in M} C_{ij} x_{ij} + \sum_{j \in M} F_j y_j \\
&\text{s.t. } (2.2), (2.4), \text{ and } (2.5) \\
&\quad \sum_{i \in N} d_i x_{ij} \leq Q_j y_j \qquad\qquad \forall j \in M. \qquad (2.6)
\end{aligned}$$

Constraints (2.6) ensure that client i is served by facility j only if a facility is installed at location j and if the capacity of that facility is not exceeded.

Sridharan [77] gives a survey of the CFLP. Cornuejols et al. [23] compare relaxations and heuristics. Aardal [1] gives a branch and cut algorithm.

The CFLPS (also known as the CFLP with single sourcing or capacitated concentrator location problem) has an additional constraint that asks the demand of each client to be satisfied by exactly one facility.

The CFLPS can be formulated as follows:

$$\begin{aligned}
&\min \sum_{i \in N} \sum_{j \in M} C_{ij} x_{ij} + \sum_{j \in M} F_j y_j \\
&\text{s.t. } (2.2), (2.5), \text{ and } (2.6) \\
&\quad x_{ij} \in \{0, 1\} \qquad\qquad \forall i \in N, j \in M. \qquad (2.7)
\end{aligned}$$

Neebe and Rao [64] formulate the CFLPS as a set partitioning problem and solve it by a branch and price algorithm. The pricing problem decomposes into $|M|$ knapsack problems.

Heuristics based on Lagrangian relaxation can be found in, e.g., Beasley [9], Cortinhal and Captivo [25], Darby-Dowman and Lewis [27], Klincewicz and Luss [47], Mirzaian [63], Pirkul [69], Sridharan [76].

Variants of CFLP and CFLPS exist. In the first variant, multitype facilities are available. Each type of facility has a different cost and capacity. For this problem, Lee [54] proposes an algorithm based on cross decomposition, which combines Benders Decomposition and Lagrangian Relaxation and presents computational results. Amiri [3] studies a variant with delay costs. He models the system as a M/M/1 queuing system and gives two Lagrangian Relaxation based heuristics to solve this problem.

If reliability is an important issue, it is possible to serve one client by several facilities. Tang et al. [78] consider the problem where it is necessary to connect client i to exactly β_i facilities. This variation is called CFLP with multiple homing. Pirkul et al. [71] study the case where every client is served by two facilities, once for the primary coverage and then for the secondary or backup coverage.

In the remainder of this section, we review the polyhedral results on CFLP and CFLPS.

The CFLP is a relaxation of CFLPS. So, the valid inequalities for the CFLP are also valid for the CFLPS. Moreover, under some conditions, the facet defining inequalities of the polytope associated to CFLP are also facet defining for the polytope associated to CFLPS. We first survey the valid and facet defining inequalities for the CFLP polytope.

CFLP: Valid Inequalities and Facets

Leung and Magnanti [56] study the valid inequalities and facets of the polytope associated to the CFLP. They consider the case where all facilities have the same capacity Q. Let F be the set of $(x, y) \in \mathbb{R}_+^{|N||M|} \times \{0,1\}^{|M|}$ such that (x, y) satisfies constraints (2.6) and

$$\sum_{j \in M} x_{ij} \leq 1 \quad \forall i \in N$$

and let $PF = conv(F)$. Leung and Magnanti [56] prove that PF is full-dimensional and give facet defining inequalities.

For $N' \subseteq N$, define $D(N') = \sum_{i \in N'} d_i$. Let $N' \subset N$, $M' \subset M$ and $r = D(N') (\bmod Q)$ with $r = Q$ if $D(N')$ is a multiple of Q. If $\lceil D(N')/Q \rceil - 1$ facilities serve the demands of clients in N' with full capacity, then the residual demand to be satisfied by the last facility is r.

THEOREM 2.1 *(Leung and Magnanti [56]) Residual capacity inequality*

$$\sum_{i \in N'} \sum_{j \in M'} d_i x_{ij} - r \sum_{j \in M'} y_j \leq D(N') - r \lceil D(N')/Q \rceil \tag{2.8}$$

is valid and defines a facet of PF when $1 \leq r \leq Q - 1$ and $|M'| \geq \lceil D(N')/Q \rceil$.

Aardal et al. [2] study the valid inequalities and facets of the polytope associated to the CFLP. Define v_{ij} to be the flow between client i and facility j. In other words, $v_{ij} = d_i x_{ij}$. Consider the set X defined by the following constraints:

$$\sum_{j \in M} v_{ij} = d_i \qquad \forall i \in N$$
$$\sum_{i \in N} v_{ij} \leq Q_j y_j \qquad \forall j \in M$$
$$v_{ij} \geq 0 \qquad \forall i \in N, j \in M$$
$$y_j \in \{0, 1\} \qquad \forall j \in M.$$

Let $PX = conv(X)$. The authors assume that $\sum_{k \in M} Q_k - Q_j \geq D(N)$ for all $j \in M$. Then $dim(PX) = |M||N| + |M| - |N|$. Below we review results due to Aardal et al. [2].

Let $M' \subset M$ be such that $\sum_{j \in M'} Q_j > \sum_{j \in M} Q_j - D(N)$. Set M' is called a cover with respect to N and M and a minimal cover if, in addition, for all $S \subset M'$, we have $\sum_{j \in S} Q_j \leq \sum_{j \in M} Q_j - D(N)$.

As the demand of all clients cannot be satisfied through facilities in $M \backslash M'$, at least one of the facilities in M' has to be installed. So $\sum_{j \in M'} y_j \geq 1$ is a valid inequality.

THEOREM 2.2 *(Aardal et al. [2]) If M' is a minimal cover with respect to M and N, $Q_{min} = \min_{j \in M'} Q_j$ and $\sum_{j \in M \backslash M'} Q_j + Q_{min} > D(N)$, then $\sum_{j \in M'} y_j \geq 1$ defines a facet of $conv(X \cap \{y \in \{0,1\}^{|M|} : y_j = 1 \;\; \forall j \in M \backslash M'\})$.*

A subset $M' \subset M$ such that $\lambda = \sum_{j \in M'} Q_j - D(N) > 0$ is called a flow cover set with respect to M and N. If facility at node $j \in M'$ is closed, then the maximum amount of flow between facilities in M' and clients does not change if $Q_j \leq \lambda$ and decreases by $Q_j - \lambda$ otherwise.

THEOREM 2.3 *(Aardal et al. [2]) Let M' be a flow cover with respect to M and N such that $max_{j \in M'} Q_j > \lambda$ and $\sum_{j \in M} Q_j > D(N) + Q_k$ for all $k \in M'$. Then the flow cover inequality*

$$\sum_{i \in N} \sum_{j \in M'} v_{ij} + \sum_{j \in M'} (Q_j - \lambda)^+ (1 - y_j) \leq D(N)$$

defines a facet of PX.

Flow cover inequalities can be generalized as follows. Let $M' \subseteq M$ and $N' \subseteq N$. For each $j \in M'$, let $N_j' \subseteq N'$. Define $\overline{Q}_j = min\{Q_j, D(N_j')\}$. Set M' is a flow cover with respect to M and N' if $\lambda = \sum_{j \in M'} \overline{Q}_j - D(N') > 0$.

THEOREM 2.4 *(Aardal et al. [2]) Let $M' \subset M$ be a flow cover with respect to M and $N' \subseteq N$. Let $K \subset M'$ be the set of facilities for which $\overline{Q}_j < Q_j$. Assume $\sum_{k \in M} Q_k > Q_j + D(N)$ for all $j \in M'$. The effective capacity inequality*

$$\sum_{j \in M'} \sum_{i \in N_j'} v_{ij} + \sum_{j \in M'} (\overline{Q}_j - \lambda)^+ (1 - y_j) \le D(N')$$

defines a facet of PX if and only if $N_{j_1}' \cap N_{j_2}' = \emptyset$ for all $j_1, j_2 \in K$, $N_j' = N'$ for all $j \in M' \backslash K$, $\cup_{j \in K} N_j' \subset N'$, $\overline{Q}_j > \lambda$ for all $j \in K$ and if $|K| \le 1$ then there exists $j \in M' \backslash K$ with $Q_j > \lambda$.

Another family of valid inequalities by Aardal et al. [2] is the family of submodular inequalities. A set function f on $N = \{1, .., n\}$ is submodular if $f(A) + f(B) \ge f(A \cup B) + f(A \cap B)$ for all $A, B \subseteq N$. Let $N' \subseteq N$, $M' \subseteq M$ and $N_j' \subseteq N'$ for all $j \in M'$. The function

$$\begin{aligned}
f(M') = \max & \sum_{j \in M'} \sum_{i \in N_j'} v_{ij} & \\
\text{s.t.} & \sum_{i \in N_j'} v_{ij} \le \overline{Q}_j & \forall j \in M' \\
& \sum_{j \in M' : i \in N_j'} v_{ij} \le d_i & \forall i \in N' \\
& v_{ij} \ge 0 & \forall i \in N', j \in M'
\end{aligned}$$

is submodular on M.

Define $\phi_j(A) = f(A \cup \{j\}) - f(A)$ for $j \in N \backslash A$.

THEOREM 2.5 *(Aardal et al. [2]) Let $N' \subseteq N$, $M' \subseteq M$ and $N_j' \subseteq N'$ for all $j \in M'$. The submodular inequality*

$$\sum_{j \in M'} \sum_{i \in N_j'} v_{ij} + \sum_{j \in M'} \phi_j(M' \backslash \{j\})(1 - y_j) \le f(M')$$

is valid for PX.

Conditions under which submodular inequalities define facets of PX are given in Aardal et al. [2].

CFLPS: Valid Inequalities and Facets

Let $PF' = conv(F \cap \{x_{ij} \in \{0,1\} \; \forall i \in N, j \in M\})$. PF' is the polytope associated to the CFLPS. Assume that all facilities have the same capacity Q. As CFLP is a relaxation of the CFLPS, the residual capacity inequalities are also valid for this polytope. But they are not facet defining in general.

Consider the case where the demands of the clients are equal. Then we can assume that each client has unit demand and the capacity of a facility is in terms of the number of clients assigned to it.

THEOREM 2.6 *(Leung and Magnanti [56]) Assume that the demands of all clients are equal. Let Q be the maximum number of clients that can be served by a facility. Let* $N' \subset N$ *and* $M' \subset M$ *and define* $r' = |N'|(modQ)$ *with* $r' = Q$ *if* $|N'|$ *is a multiple of* Q*. The residual capacity inequality*

$$\sum_{i \in N'} \sum_{j \in M'} x_{ij} - r' \sum_{j \in M'} y_j \leq |N'| - r' \lceil |N'|/Q \rceil \tag{2.9}$$

is facet defining for PF' *when* $1 \leq r' \leq Q-1$ *and* $|M'| \geq \lceil \frac{|N'|}{Q} \rceil$.

Leung and Magnanti [56] develop an alternative formulation for the CFLPS with unit demands and different capacities. The capacity of a facility is viewed as a collection of unit capacity facilities. Define x_{ijk} to be 1 if client i is assigned to unit k of facility j and 0 otherwise for all $i \in N$, $j \in M$ and $k = 1, .., Q_j$ and define also $\overline{y}_j = 1 - y_j$ for all $j \in M$. Then the feasible set of CFLPS is defined by the following set of constraints:

$$\sum_{j \in M} \sum_{k=1}^{Q_j} x_{ijk} \leq 1 \qquad \forall i \in N \tag{2.10}$$

$$\sum_{i \in N} x_{ijk} + \overline{y}_j \leq 1 \qquad \forall j \in M, k = 1, .., Q_j \tag{2.11}$$

$$\sum_{k=1}^{Q_j} x_{ijk} + \overline{y}_j \leq 1 \qquad \forall i \in N, j \in M \tag{2.12}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in N, j \in M, k = 1, .., Q_j \tag{2.13}$$

$$\overline{y}_j \in \{0,1\} \qquad \forall j \in M. \tag{2.14}$$

Constraints (2.10) imply that a client can be assigned to at most one unit of capacity of a facility. Constraints (2.11) ensure that if a client is assigned to the kth unit of capacity of facility j, then there has to be a facility at node j and that no other client can be assigned to this capacity unit. If client i is served by some unit of capacity of facility j, then there has to be a facility at node j, by constraints (2.12).

Leung and Magnanti [56] use the set packing structure of the problem to derive facet defining inequalities. They show that the cliques of the conflict graph associated to this set packing give rise to families of facets which are constraints (2.10), (2.11) and (2.12). They also investigate the odd holes in the conflict graph and derive valid inequalities.

Deng and Simchi-Levi [28] study the general case where clients can have different demands and they give the following valid inequalities:

THEOREM 2.7 *(Deng and Simchi-Levi [28]) Let $N' \subseteq N$ and $M' \subseteq M$. Define $b(N')$ to be the minimum number of facilities needed to serve all clients in M'. The binpacking inequality*

$$\sum_{i \in N'} \sum_{j \in M'} x_{ij} - \sum_{j \in M'} y_j \leq |N'| - b(N')$$

is valid for PF'.

Under some conditions, these inequalities define facets of the convex hull of the feasible set when the values of some of the variables are fixed. Other valid and facet defining inequalities are presented by Deng and Simchi-Levi [28].

2.2 p-Median Problems

Given a set of demand points, the p-median problem asks to locate p facilities on a subset of these nodes to minimize the total distance from each demand point to the closest facility. In the p-median problem, different from the UFLP, the number of facilities to be located is given. So to obtain a formulation for the p-median problem, we add the constraint $\sum_{j \in M} y_j = p$ to the formulation of UFLP.

Refer to Bozkaya et al. [11], Labbé et al. [49] and Mirchandani [62] for surveys on the p-median problem.

Consider the case where $N = M$. Let d_{ij} be the distance between nodes i and j.

If a facility is located at node j then node j is assigned to itself as $d_{jj} = 0$. So $y_j = x_{jj}$ for all $j \in N$.

Avella and Sassano [4] eliminate variables x_{jj} by substituting $x_{jj} = 1 - \sum_{i \in N \setminus \{j\}} x_{ji}$. Then the p-median problem can be formulated as follows:

$$
\begin{aligned}
\min & \sum_{i \in N} \sum_{j \in N \setminus \{i\}} d_{ij} x_{ij} && && \\
\text{s.t.} & \sum_{j \in N \setminus \{i\}} x_{ij} + x_{mi} \leq 1 && \forall i \in N, m \in N \setminus \{i\} && (2.15) \\
& \sum_{i \in N} \sum_{j \in N \setminus \{i\}} x_{ij} = |N| - p && && (2.16) \\
& x_{ij} \in \{0,1\} && \forall i \in N, j \in N \setminus \{i\}. && (2.17)
\end{aligned}
$$

For $i \in N$, if node $m \in N \setminus \{i\}$ is assigned to node i, then by constraints (2.15), node i cannot be assigned to any other node and so there is a facility at node i. If no node is assigned to node i, then constraint (2.15) implies that node i can be assigned to at most one node.

As we require p facilities to be located, the number of nodes that are assigned to other nodes is $|N| - p$ and this is given in constraint (2.16).

If we remove constraint (2.16), we obtain a special stable set problem. This means that valid inequalities known for the stable set polytope are also valid for the p-median polytope. Let P_p be the convex hull of vectors x that satisfy (2.15)-(2.17).

Avella and Sassano [4] study the properties of P_p.

THEOREM 2.8 *(Avella and Sassano [4]) For $p \geq 2$, the polytope P_p has dimension $|N|(|N| - 1) - 1$.*

Avella and Sassano [4] present two families of valid inequalities for the p-median polytope P_p, namely $W - 2$ inequalities and I^*-cover inequalities. Here we review the results on the $W - 2$ inequalities.

Define $A = \{(i,j) : i \in N, j \in N \setminus \{i\}\}$.

THEOREM 2.9 *(Avella and Sassano [4]) Let $W \subseteq N$ with $3 \leq |W| \leq |N| - p + 1$ and $H \subset A_W = \{(i,j) \in A : i \in W, j \in W\}$ such that for each $j \in W$ there exists exactly one $i \in W$ such that $(i,j) \in H$. Let $U = \{i \in W :$ there is no j such that $(i,j) \in H\}$. The $W - 2$ inequality*

$$\sum_{(i,j) \in (A_W \setminus H \cup \{(i,j): i \in U, j \in I \setminus W\})} x_{ij} \leq |W| - 2$$

is valid for P_p. The inequality defines a facet of the p-median polytope if and only if $|U| \leq \max(1, |W| - 3)$.

$W - 2$ inequalities include families of well known inequalities like clique, lifted odd hole, lifted odd anti-hole, etc.

Avella and Sassano [4] give separation algorithms for the two families of inequalities and present computational results.

The capacitated p-median problem is not studied much. Lorena and Senne [57] apply column generation and Maniezzo et al. [58] propose a heuristic method to solve this problem.

2.3 Hub Location Problems with Single Assignment

The hub location problems are different from the facility location problems in the sense that they consider the communication between pairs of nodes. There is a traffic between any pairs of nodes. The traffic from node i to node m goes from node i to the facility (hub) to which node i is assigned, then to the facility to which node m is assigned and finally to node m. There is a cost for routing this traffic in the network.

We review the models for hub location problems with single assignment.

2.3.1 Uncapacitated Hub Location Problem with Single Assignment

Let t_{im} denote the traffic from node i to node m. Let C_{ij} denote the cost of routing the traffic between demand point i and facility j and let B_{jl} denote the cost of routing the traffic between two facilities j and l. Then Uncapacitated Hub Location Problem with Single Assignment (UHLP) can be formulated as follows (O'Kelly [66]):

$$\begin{aligned} \min \sum_{i \in N} \sum_{j \in M} C_{ij} \sum_{m \in N} (t_{im} + t_{mi}) x_{ij} + \sum_{j \in M} F_j y_j \\ + \sum_{j \in M} \sum_{l \in M} B_{jl} \sum_{i \in N} \sum_{m \in N} t_{im} x_{ij} x_{ml} \end{aligned} \tag{2.18}$$

s.t. (2.2), (2.3), (2.5), and (2.7).

If constraints (2.7) are replaced by constraints (2.4), then one obtains a formulation of the Uncapacitated Hub Location Problem with Multiple Assignment. Even though for the UFLP, there exists always an optimal solution which satisfies (2.7), this is no longer true for the Uncapacitated Hub Location Problem with Multiple Assignment.

Notice that the objective function (2.18) is quadratic. Several linearizations exist in the literature (see Campbell [12], Ebery [29], Ernst and Krishnamoorthy [31], Labbé et al. [53], and Skorin-Kapov et al. [75]).

For i and m in N and j and l in M, define $X_{jl}^{im} = x_{ij}x_{ml}$. Skorin-Kapov et al. [75] give the following linear formulation:

$$\begin{aligned}
\min & \sum_{i \in N} \sum_{j \in M} C_{ij} \sum_{m \in N} (t_{im} + t_{mi}) x_{ij} + \sum_{j \in M} F_j y_j \\
& + \sum_{j \in M} \sum_{l \in M} B_{jl} \sum_{i \in N} \sum_{m \in N} t_{im} X_{jl}^{im} \\
\text{s.t. } & (2.2), (2.3), (2.5), \text{ and } (2.7) \\
& \sum_{l \in M} X_{jl}^{im} = x_{ij} \qquad \forall i \in N, m \in N, j \in M \\
& \sum_{j \in M} X_{jl}^{im} = x_{ml} \qquad \forall i \in N, m \in N, l \in M \\
& X_{jl}^{im} \geq 0 \qquad \forall i \in N, m \in N, j \in M, l \in M.
\end{aligned}$$

For the case where the routing costs satisfy the triangle inequality, Ernst and Krishnamoorthy [31] propose the following formulation. For $i \in N$, $j \in M$ and $l \in M \setminus \{j\}$, define f_{jl}^i to be the flow of traffic originating at node i and traveling on the direct link from j to l.

$$\begin{aligned}
\min & \sum_{i \in N} \sum_{j \in M} C_{ij} \sum_{m \in N} (t_{im} + t_{mi}) x_{ij} + \sum_{j \in M} F_j y_j + \sum_{j \in M} \sum_{l \in M \setminus \{j\}} B_{jl} \sum_{i \in N} f_{jl}^i \\
\text{s.t. } & (2.2), (2.3), (2.5), \text{ and } (2.7) \\
& \sum_{l \in M \setminus \{j\}} f_{jl}^i - \sum_{l \in M \setminus \{j\}} f_{lj}^i = \sum_{m \in I} t_{im} (x_{ij} - x_{mj}) \qquad \forall i \in N, j \in M \\
& f_{jl}^i \geq 0 \qquad \forall i \in N, j \in M, l \in M \setminus \{j\}.
\end{aligned}$$

For $i \in N$ and $j \in M$, define $Q_{ij} = \sum_{l \in M \setminus \{j\}} B_{jl} \sum_{m \in N} t_{im} x_{ij} x_{ml}$. Ebery [29] derives the following formulation:

$$\begin{aligned}
\min & \sum_{i \in N} \sum_{j \in M} C_{ij} \sum_{m \in N} (t_{im} + t_{mi}) x_{ij} + \sum_{j \in M} F_j y_j + \sum_{i \in N} \sum_{j \in M} Q_{ij} \\
\text{s.t. } & (2.2), (2.3), (2.5), \text{ and } (2.7)
\end{aligned}$$

$$Q_{ij} \geq \sum_{l \in M \setminus \{j\}} B_{jl} \sum_{m \in N} t_{im}(x_{ij} + x_{ml} - 1) \qquad \forall i \in N, j \in M$$

$$Q_{ij} \geq 0 \qquad \forall i \in N, j \in M.$$

The formulation by Skorin-Kapov et al. [75] uses $O(|N|^2|M|^2)$ variables. Ernst and Krishnamoorthy [31] decrease the size of variables by an order of $|N|$ by modeling the traffic as multicommodity flow aggregated by origins. But the LP relaxation of this formulation is weaker than the one of the first formulation. The formulation by Ebery [29] uses $O(|N||M|)$ variables but the author mentions that solving this formulation takes longer than solving the formulation of Ernst and Krishnamoorthy [31].

A branch and bound algorithm and an exact method based on shortest paths for the case where the number of hubs is fixed can be found in Ernst and Krishnamoorthy [31] and [32], respectively. A Lagrangian Relaxation heuristic is given in Pirkul and Schilling [72].

2.3.2 Capacitated Hub Location Problem with Single Assignment

The Capacitated Hub Location Problem with Single Assignment (CHLP) generalizes the UHLP by introducing capacity constraints. Ernst and Krishnamoorthy [30] study the CHLP. They derive a formulation by adding capacity constraints to the formulation by Ernst and Krishnamoorthy [31]. They develop heuristics based on simulated annealing and random descent.

2.4 Conclusion

Having reviewed the literature on location problems for telecommunications networks, we did not encounter any work on the QCL-C. Chung et al. (1992) consider a simpler version where there are no capacity constraints and the cost of connecting two concentrator nodes is independent of the terminals assigned to them and thus is independent of the traffic.

Hub location problems are closer to QCL-C. But, the type of capacity in QCL-C is not considered in this literature.

Polyhedral approaches are rarely used to solve concentrator location problems. In fact, exact methods are rare in general. There are polyhedral results on location problems (UFLP, CFLP, CFLPS, and p-median) but not for versions where routing decisions are involved or where there is a cost associated to traffic.

This is also the case for hub location problems. Campbell et al. (2002) mention that little is known about the polyhedral structure of hub location problems.

Chapters 4, 5 and 6 of this book attempt to fill this gap by presenting polyhedral properties of the QCL-C and its variants.

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