Preface

The change-point problem has attracted many statistical researchers and practitioners during the last few decades. Here, we only concentrate on the sequential change-point problem. Starting from the Shewhart chart with applications to quality control [see Shewhart (1931)], several monitoring procedures have been developed for a quick detection of change. The three most studied monitoring procedures are the CUSUM procedure [Page (1954)], the EWMA procedure [Roberts (1959)] and the Shiryayev−Roberts procedure [Shiryayev (1963) and Roberts (1966)]. Extensive studies have been conducted on the performances of these monitoring procedures and comparisons in terms of the delay detection time. Lai (1995) made a review on the state of the art on these charts and proposed several possible generalizations in order to detect a change in the case of the unknown post-change parameter case. In particular, a window-limited version of the generalized likelihood ratio testing procedure studied by Siegmund and Venkatraman (1993) is proposed for a more practical treatment even when the observations are correlated.

In this work, our main emphasis is on the inference problem for the change-point and the post-change parameters after a signal of change is made. More specifically, due to its convenient form and statistical properties, most discussions are concentrated on the CUSUM procedure. Our goal is to provide some quantitative evaluations on the statistical properties of estimators for the change-point and the post-change parameters. It has to be stressed that there have been many studies on the inference problem for the change-point in the fixed sample size case under both parametric models and nonparametric models. The inference problem after sequential detection raises both theoretical and technical difficulties, as mentioned in a preliminary study by Hinkley (1971). From the theoretical point of view, due to the irregularity of the change-point problem, it will be interesting to see how the sequential sampling plan affects the statistical properties. Interestingly, we shall see that even in the normal case, the bias of the change-point estimator is different from the fixed sample size case even in the large sample situation. From the technical point of view, due to the nature of sequential sampling plan, the inference problem for the change-point and the post-change parameters raises more technical difficulties compared with other standard situations. The inference problem for the post-change parameters involves more careful treatments because of the uncertainty of the change-point estimator. From the application point of view, the change-point problem has been raised from a variety of areas where changes in structures of dynamic systems are the interest such as online quality control, financial market and economic systems, global warming and DNA sequence analysis, to mention a few. Motivated by these reasons, the author feels that it may attract researchers' and practitioners' interest from the results and techniques developed in this work as well as from several case studies.

The notes are organized as follows. In Chapter 1, we first introduce the regular CUSUM procedure and give a simple approximation for the average run lengths for design purposes. Then we explain the strong renewal theorem in the exponential family case. As a demonstration, we give a derivation for the approximation for the in-control average run length in the normal case. The estimators for the change-point and the post-change parameter are proposed.

In Chapter 2, we consider the bias and the absolute bias for the estimator of the change-point conditioning on a change that is detected. By assuming both the change-point and the threshold approach infinity, the asymptotic quasi-stationary bias is derived. In order to study the local properties, we further assume both the reference value and the post-change mean approach zero at the same order and obtain the second-order approximation for the bias. It is shown that in contrast to the fixed sample size case as considered in Wu (1999), the asymptotic bias is not zero even in the normal distribution case.

In Chapter 3, we propose a method of constructing a lower confidence limit for the change-point. This may be of interest from a quality inspection point of view, where we need to estimate the number of items to be inspected after a change is detected.

In Chapter 4, we concentrate on the inference problem for the post-change mean in the normal case in order to develop the technique. The bias and a corrected normal pivot based on the post-change mean estimator are studied. The techniques developed here provide new methods of studying the change-point problem.

In Chapter 5, we study the behavior of the post-change mean estimator when the signal is false. This serves two purposes. First, from theoretical aspects, the post-change mean estimator under a false signal can be treated as the sample mean for a conditional random walk with negative drift by staying positive. The results extend the available asymptotic theory in the sense that we obtain the second-order correction for the normal approximation. Second, by studying the behavior of the post-change mean under a false signal, it provides a method of separating the regular clusters from sparse changing segments.

In Chapter 6, we extend the results to a specific problem in the normal case when the variance (nuisance parameter) is subject to possible changes as well. In other words, we want to study the inference problem for the change-point and the post-change mean when the post-change variance needs to be estimated. This provides a robust analysis for the CUSUM procedure for monitoring changes in mean. We find that more technical difficulties are raised, particularly for constructing the confidence interval for the post-change mean.

In Chapter 7, we consider a sequential classification and segmentation procedure for alternatively changing means as an application of the results in Chapter 2. By using the dam process, we can classify alternatively changing means sequentially and also locate the segments after correction in the offline case. The method provides an alternative to the Bayesian sequential filtering procedure.

In Chapter 8, in order to deal with more complex post-change parameters such as linear post-change means in the normal case, we propose an adaptive CUSUM procedure. The adaptive CUSUM procedure uses a sequence of adaptive sequential tests with a two-sided stopping boundary. The advantage for this adaptive CUSUM procedure is its convenience of locating the change-point and estimating the post-change parameters. Theoretical properties of the procedure are studied under the simple change model. The application to global warming detection is used for an illustration.

In Chapter 9, we generalize the methods to a correlated data case. In particular, we consider the model-based CUSUM procedure when the observations follow an AR(1) model with mean subject to change. We study the effect of correlation on the biases of change-point estimator and post-change mean estimator. Simulation study for the effect of correlation on the classical CUSUM procedure is also carried out.

In Chapter 10, we consider the estimator for the change-point under the Shiryayev−Roberts procedure. A comparison with the CUSUM procedure is conducted. Some concluding remarks on the generalizations to other models including the multi-variate observation case and multidimensional parameter case are made with several possible open problems posted.

Readers with basic statistical training in sequential analysis with an elementary probability background should be able to understand most results. Wald's likelihood ratio identity [Wald (1947)] is used extensively. A convenient reference is Siegmund (1985). For those readers who need to review the basic properties about the CUSUM procedure, we refer to van Dobben De Bruyn (1968) and Hawkins and Olwell (1998). Most results are given under the simple change-point model; that means, the observations are independent. In fact, most discussions are concentrated on the normal distribution case, which, we hope, will attract most researchers and practitioners who are interested in change-point problems. The techniques and methods are nevertheless quite general and can be extended directly to the more general exponential family case and dependent observation models such as Markovian structures. Also to make the work more attractive to practitioners, several case studies are added at the end of each major chapter; these case studies show the detailed procedures of applying the methods and results to the real data sets. Original data sets are listed to form a relatively self-contained package.

The majority of the results in the notes were derived when the author was visiting the Department of Statistics at University of Michigan which has an encouraging and stimulating environment. Frequent discussions with the faculty there, particularly Professors M. Woodroofe and R. Keener, were very helpful. The writing was finished when the author was currently visiting the Department of Mathematics at the University of the Pacific, and their hospitality is greatly appreciated. Editorial assistance from Dr. J. Kimmel, editor of the series, A. Orrantia from the production department, and many comments from the series editors and two reviewers helped me to provide a much improved revision. The long-time support from my family makes the writing and research possible. This book is dedicated to my late father.

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September, 2004

Springer
http://www.springer.com/978-0-387-22927-0

Inference for Change Point and Post Change Means
After a CUSUM Test
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2005, XIII, 158 p., Softcover
ISBN: 978-0-387-22927-0