
Preface

This book is aimed at postgraduate students in applied mathematics as well as at engineering and physics students with a firm background in mathematics. The first four chapters can be used as the material for a first course on inverse problems with a focus on computational and statistical aspects. On the other hand, Chapters 3 and 4, which discuss statistical and nonstationary inversion methods, can be used by students already having knowledge of classical inversion methods.

There is rich literature, including numerous textbooks, on the classical aspects of inverse problems. From the numerical point of view, these books concentrate on problems in which the measurement errors are either very small or in which the error properties are known exactly. In real-world problems, however, the errors are seldom very small and their properties in the deterministic sense are not well known. For example, in classical literature the error norm is usually assumed to be a known real number. In reality, the error norm is a random variable whose mean might be known.

Furthermore, the classical literature usually assumes that the operator equations that describe the observations are exactly known. Again, usually when computational solutions based on real-world measurements are required, one should take into account that the mathematical models are themselves only approximations of real-world phenomena. Moreover, for computational treatment of the problem, the models must be discretized, and this introduces additional errors. Thus, the discrepancy between the measurements and the predictions by the observation model are not only due to the “noise that has been added to the measurements.” One of the central topics in this book is the statistical analysis of errors generated by modelling.

There is rich literature also in statistics, especially concerning Bayesian statistics, that is fully relevant in inverse problems. This literature has been fairly little known to the inverse problems community, and thus the main aim of this book is to introduce the statistical concepts to this community. As for statisticians, the book contains probably little new information regarding, for example, sampling methods. However, the development of realistic observation

models based, for example, on partial differential equations and the analysis of the associated modelling errors might be useful.

As for citations, in Chapters 1–6 we mainly refer to books for further reading and do not discuss historical development of the topics. Chapter 7, which discusses our previous and some new research topics, also does not contain reviews of the applications. Here we refer mainly to the original publications as well as to sources that contain modifications and extensions which serve to illustrate the potential of the statistical approach.

Chapters 5–7, which form the second part of the book, focus on problems for which the models for measurement errors, errorless observations and the unknown are really taken as *models*, which themselves may contain uncertainties. For example, several observation models are based on partial differential equations and boundary value problems. It might be that part of the boundary value data are inherently unknown. We would then attempt to model these boundary data as random variables that could either be treated as secondary unknowns or taken as a further source of uncertainty and compute its contribution to the discrepancy between the observation model and the predictions given by the observation model.

In the examples, especially in Chapter 7 that discusses nontrivial problems, we concentrate on research that we have carried out earlier. However, we also treat topics that either have not yet been published or are discussed here with more rigor than in the original publications.

We have tried to enhance the readability of the book by avoiding citations in the main text. Every chapter has a section called “Notes and Comments” where the citations and further reading, as well as brief comments on more advanced topics, are given.

We are grateful to our colleague and friend, Markku Lehtinen, who has advocated the statistical approach to inverse problems for decades and brought this topic to our attention. Much of the results in Chapter 7 have been done in collaboration with our present and former graduate students - as well as other scientists. We have been privileged to work with them and thank them all. We mention here only the people who have contributed directly to this book by making modifications to their computational implementations or otherwise: Dr. Ville Kolehmainen for Sections 7.2 and 7.9, Dr. Arto Voutilainen for Section 7.4, Mr. Aku Seppänen for Sections 7.5 and 7.7 and Ms. Jenni Heino for Section 7.8. We are also much obliged to Daniela Calvetti for carefully reading and commenting the whole manuscript and to the above-mentioned people for reading some parts of the book. For possible errors that remain we assume full responsibility.

This work was financially supported by the Academy of Finland and the Finnish Academy of Science and Letters (JPK) to whom thanks are due. Thanks are also due to the inverse problems group at the University of Kuopio and to vice head of the Applied Physics department, Dr. Ari Laaksonen, who saw to the other author’s duties during his leave. Thanks are also due to Dr. Geoff Nicholls and Dr. Colin Fox from the University of Auckland, NZ, where

much of the novel material in this book was conceived during the authors' visits there.

Helsinki and Kuopio
June 2004

Jari P. Kaipio
Erkki Somersalo

Statistical and Computational Inverse Problems

Kaipio, J.; Somersalo, E.

2005, XVI, 340 p., Hardcover

ISBN: 978-0-387-22073-4