Introduction

Today more than ever, modeling and simulation are central to a mechanical engineer's activity. Increasingly complex models are being used routinely on a daily basis. This revolution, which has just begun, is the result of the extraordinary progress in computer technology in terms of both hardware and software.

In order to represent a real problem, one does not use just a single model, but a series of models. Starting from a first model, called the reference model, practical or economic considerations, along with the wish to take advantage of certain particular situations, often lead to the introduction of additional simplifying hypotheses, called condensation hypotheses, which result in a new, more manageable model. This, for example, is the case of hypotheses which, starting from a continuous model of a medium subjected to a given environment, lead to a "finite element" model involving parameters such as the size and type of the elements, the number of iterations, the duration of the time increments….

Of course, it is imperative not to alter the reference model completely. Therefore, controlling the additional simplifying hypotheses is an obvious and major issue. This has been a constant preoccupation on the industrial level as well as in research. The new situation is that over the last twenty years truly quantitative tools for assessing the quality of a model compared to another reference model have appeared.

This work deals with the control of the hypotheses leading from a mechanical model, usually coming from continuum mechanics, to a numerical model, i.e. the mastery of the mechanical computation process itself. Particular attention is given to structural analysis which, in this context, is the most advanced domain. The term "structure" designates the material envelope, which can consist of metallic materials, composite materials, biomaterials … in solid, fluid or gaseous environments. The models being studied are not necessarily linear and high degrees of nonlinearity may be present (plasticity, viscoplasticity, unilateral contact…). The objective of structural analysis is to simulate the behavior of a structure subject to various solicitations (prescribed displacements and forces) numerically; in particular, the aim is to evaluate the state of damage of the structure and compare it with one or several limit states. The final stage consists in optimizing the structural parameters. The practical problems concern the dimensioning, optimization, reliability and even the manufacturing process of the object being designed or built.

The basic problem consists in defining and evaluating a measure of the error due to the discretization performed, in this case, by the finite element method.

Two situations must be dealt with, depending on whether the error is evaluated before or after the finite element calculation has been performed.

Today, for the first situation corresponding to what one calls "a priori" errors, only coarse evaluations are available. The second situation is more favorable: the finite element solution constitutes an additional piece of information. It is in the corresponding field of "a posteriori" error evaluation that the first research works on linear problems were published about twenty years ago.

The numerous techniques proposed can be categorized into three approaches:

the first approach relies on the concept of error in constitutive relation and on related field construction techniques [LADEVEZE, 1975];

the second approach relies on the concept of error indicator associated with the satisfaction of the equilibrium equations [BABUSKA - RHEINBOLDT, 1978];

the third approach is based on the unevenness of the finite element solution [ZIENKIEWICZ - ZHU, 1987].

In the present work, after having described the various approaches, we focus on the first family of estimators because, on the one hand, it has the strongest mechanical meaning and, on the other hand, contrary to the other two families, it can be extended without much difficulty to nonlinear evolution problems. This approach is based on a partition of the equations of the reference problem into:

admissibility conditions (kinematic constraint equations, equilibrium equations, initial conditions);

the constitutive relation.

Indeed, the constitutive relation has a special status: in practice, this is often the least reliable equation. Therefore, it is natural to set this equation apart and seek an approximate displacement-stress solution over the time interval being considered which verifies the most reliable group of equations (i.e. the admissibility conditions) exactly. This solution verifies a constitutive relation which, in general, differs from the constitutive relation of the material; thus, the quality of the approximation can be assessed by comparing this constitutive relation to that of the material. Energy norms or other norms which have a deep physical meaning are used to quantify this error.

An a priori obstacle is that it is difficult to construct admissible approximate solutions, i.e. solutions which verify the admissibility conditions exactly. Indeed, the usual approximations – particularly the approximations resulting from the application of the finite element method – fail to verify these conditions exactly because the calculated stresses are not in equilibrium with the applied forces. One circumvents this difficulty by a very general technique which enables one to construct an admissible approximate solution explicitly, therefore very inexpensively, starting from the approximate solution obtained by the finite element method. It should be noted that this construction technique takes advantage of the properties of the finite element solution.

Of course, we also present the other error estimators proposed in the literature and compare them to the constitutive relation error estimators.

Most error estimators do not provide information on local errors such as errors in the stresses. The construction of local error estimators is one of today's key issues on the research level. Very few works have been dedicated to this problem. Here, we present a recent theory which is an extension of the approach which led to our constitutive relation error estimators.

A significant part of this work is dedicated to the application of error estimators – whatever these estimators may be – to the control of the various parameters involved in a calculation, beginning with the parameters related to the mesh. Some examples illustrate the current state of the art.

Many developments presented here for the first time stem from recent research by the authors. Let us mention, for example, the extension of the concept of error in constitutive relation to nonlinear evolution problems and to dynamic problems, the adaptive improvements to nonlinear calculations in mechanics, the evaluation of local errors….

This work is addressed to all – students, researchers, engineers – who are interested in mechanics, from the construction of models to their simulation for industrial purposes.

The first chapter describes the reference problems, the approximate models obtained by the finite element method and the main sources of discretization errors.

Chapter 2 presents the bases of the constitutive relation error method for linear problems and outlines the techniques of construction of admissible fields. The concept of error in constitutive relation is also used to establish the theorems known as "energy theorems" which, in fact, result directly from the global formulation of the constitutive relation using overpotentials.

The other two major error estimation methods for linear problems proposed in the literature (error estimators based on the equilibrium deficiency and error indicators based on the smoothing of the finite element stresses) are presented in detail in Chapter 3.

For linear problems, simple examples of the use of the constitutive relation error measure and some elements of comparison of the global effectiveness of the various estimators are given in Chapter 4.

Chapter 5 is dedicated to the various techniques of finite element mesh adaptation. Particular emphasis is put on the "*h*" method, which is the most widely used today. Using the error estimates obtained in a preliminary calculation, it is possible to predict the element sizes necessary to achieve a predetermined level of quality. Examples of mesh adaptation in 2D and in 3D are given.

Chapter 6 (for nonlinear problems) and Chapter 7 (for vibration and transient dynamics problems) show how the constitutive relation error method enables one to derive consistent error estimates in these difficult situations.

Chapter 8 details the techniques used in the construction of admissible fields, whose central aspect is the construction of force densities on the interfaces between elements. The method is first introduced for the simpler case of 2D thermal problems, then detailed for 2D or 3D elasticity, incompressible elasticity and elastic plate problems. In this chapter, we also describe in detail an improved method of constructing the densities which increases the effectiveness of the error estimators in difficult situations (e.g. for elements with very high aspect ratios).

Chapter 9 presents recent works on the evaluation of local quantities (stresses, displacements…). Access to such estimates is crucial for industrial applications.

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