Preface

Toss a symmetric coin twice. What is the probability that both tosses will yield a head?

This is a well-known problem that anyone can solve. Namely, the probability of a head in each toss is $1/2$, so the probability of two consecutive heads is $1/2 \cdot 1/2 = 1/4$.

BUT! What did we do? What is involved in the solution? What are the arguments behind our computations? Why did we multiply the two halves connected with each toss?

This is reminiscent of the centipede[1] who was asked by another animal how he walks; he who has so many legs, in which order does he move them as he is walking? The centipede contemplated the question for a while, but found no answer. However, from that moment on he could no longer walk.

This book is written with the hope that we are not centipedes.

There exist two kinds of probabilists. One of them is the mathematician who views probability theory as a purely mathematical discipline, like algebra, topology, differential equations, and so on. The other kind views probability theory as the *mathematical modeling of random phenomena*, that is with a view toward applications, and as a companion to statistics, which aims at finding methods, principles and criteria in order to analyze data emanating from experiments involving random phenomena and other observations from the real world, with the ultimate goal of making wise decisions. I would like to think of myself as both.

What kind of a random process describes the arrival of claims at an insurance company? Is it one process or should one rather think of different processes, such as one for claims concerning stolen bikes and one for houses that have burnt down? How well should the DNA sequences of an accused offender and a piece of evidence match each other in order for a conviction? A

[1] Cent is 100, so it means an animal with 100 legs. In Swedish the name of the animal is *tusenfoting*, where “tusen” means 1000 and “fot” is foot; thus an animal with 1000 legs or feet.

milder version is how to order different species in a phylogenetic tree. What are the arrival rates of customers to a grocery store? How long are the service times? How do the clapidemia cells split? Will they create a new epidemic or can we expect them to die out? A classical application has been the arrivals of telephone calls to a switchboard and the duration of calls. Recent research and model testing concerning the Internet traffic has shown that the classical models break down completely and new thinking has become necessary. And, last but (not?) least, there are many games and lotteries.

The aim of this book is to provide the reader with a fairly thorough treatment of the main body of basic and classical probability theory, preceded by an introduction to the mathematics which is necessary for a solid treatment of the material. This means that we begin with basics from measure theory, such as σ-algebras, set theory, measurability (random variables) and Lebesgue integration (expectation), after which we turn to the Borel-Cantelli lemmas, inequalities, transforms and the three classical limit theorems: the law of large numbers, the central limit theorem and the law of the iterated logarithm. A final chapter on martingales – one of the most efficient, important, and useful tools in probability theory – is preceded by a chapter on topics that could have been included with the hope that the reader will be tempted to look further into the literature. The reason that these topics did not get a chapter of their own is that beyond a certain number of pages a book becomes deterring rather than tempting (or, as somebody said with respect to an earlier book of mine: "It is a nice format for bedside reading").

One thing that is *not* included in this book is a philosophical discussion of whether or not chance exist, whether or not randomness exists. On the other hand, probabilistic modeling is a wonderful, realistic, and efficient way to model phenomena containing uncertainties and ambiguities, regardless of whether or not the answer to the philosophical question is yes or no.

I remember having read somewhere a sentence like "There exist already so many textbooks [of the current kind], so, why do I write another one?" This sentence could equally well serve as an opening for the present book.

Luckily, I can provide an answer to that question. The answer is the short version of the story of the mathematician who was asked how one realizes that the fact he presented in his lecture (because this was really a he) was trivial. After 2 minutes of complete silence he mumbled

I know it's trivial, but I have forgotten why.

I strongly dislike the arrogance and snobbism that encompasses mathematics and many mathematicians. Books and papers are filled with expressions such as "it is easily seen", "it is trivial", "routine computations yield", and so on. The last example is sometimes modified into "routine, but tedious, computations yield". And we all know that behind things that are easily seen there may be years of thinking and/or huge piles of scrap notes that lead nowhere, and one sheet where everything finally worked out nicely.

Clearly, things become routine after many years. Clearly, facts become, at least *intuitively*, obvious after some decades. But in writing papers and books we try to help those who do not know yet, those who want to learn. We wish to attract people to this fascinating part of the world. Unfortunately though, phrases like the above ones are repellent, rather than being attractive. If a reader understands immediately that's fine. However, it is more likely that he or she starts off with something that either results in a pile of scrap notes or in frustration. Or both. And nobody is made happier, certainly not the reader. I have therefore avoided, or, at least, tried to avoid, expressions like the above unless they are adequate.

The main aim of a book is to be helpful to the reader, to help her or him to understand, to inform, to educate, and to attract (and not for the author to prove himself to the world). It is therefore essential to keep the flow, not only in the writing, but also in the reading. In the writing it is therefore of great importance to be rather extensive and not to leave too much to the (interested) reader.

A related aspect concerns the style of writing. Most textbooks introduce the reader to a number of topics in such a way that further insights are gained through exercises and problems, some of which are not at all easy to solve, let alone trivial. We take a somewhat different approach in that several such "would have been" exercises are given, together with their solutions as part of the ordinary text – which, as a side effect, reduces the number of exercises and problems at the end of each chapter. We also provide, at times, results for which the proofs consist of variations of earlier ones, and therefore are left as an exercise, with the motivation that doing almost the same thing as somebody else has done provides a much better understanding than reading, nodding and agreeing. I also hope that this approach creates the atmosphere of a dialogue rather than of the more traditional monologue (or sermon).

The ultimate dream is, of course, that this book contains no errors, no slips, no misprints. Henrik Wanntorp has gone over a substantial part of the manuscript with a magnifying glass, thereby contributing immensely to making that dream come true. My heartfelt thanks, Henrik. I also wish to thank Raimundas Gaigalas for several perspicacious remarks and suggestions concerning his favorite sections, and a number of reviewers for their helpful comments and valuable advice. As always, I owe a lot to Svante Janson for being available for any question at all times, and, more particularly, for always providing me with an answer. John Kimmel of Springer-Verlag has seen me through the process with a unique combination of professionalism, efficiency, enthusiasm and care, for which I am most grateful.

Finally, my hope is that the reader who has digested this book is ready and capable to attack any other text, for which a solid probabilistic foundation is necessary or, at least, desirable.

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