Preface to the First Edition

This book grew out of lecture notes I used in a course on difference equations that I have taught at Trinity University for the past five years. The classes were largely populated by juniors and seniors majoring in mathematics, engineering, chemistry, computer science, and physics.

This book is intended to be used as a textbook for a course on difference equations at both the advanced undergraduate and beginning graduate levels. It may also be used as a supplement for engineering courses on discrete systems and control theory.

The main prerequisites for most of the material in this book are calculus and linear algebra. However, some topics in later chapters may require some rudiments of advanced calculus and complex analysis. Since many of the chapters in the book are independent, the instructor has great flexibility in choosing topics for a one-semester course.

This book presents the current state of affairs in many areas such as stability, Z-transform, asymptoticity, oscillations, and control theory. However, this book is by no means encyclopedic and does not contain many important topics, such as numerical analysis, combinatorics, special functions and orthogonal polynomials, boundary value problems, partial difference equations, chaos theory, and fractals. The nonselection of these topics is dictated not only by the limitations imposed by the elementary nature of this book, but also by the research interest (or lack thereof) of the author.

Great efforts were made to present even the most difficult material in an elementary format and to write in a style that makes the book accessible to students with varying backgrounds and interests. One of the main features of the book is the inclusion of a great number of applications in economics, social sciences, biology, physics, engineering, neural networks, etc. Moreover, this book contains a very extensive and carefully selected set of exercises at the end of each section. The exercises form an integral part of the text. They range from routine problems designed to build basic skills to more challenging problems that produce deeper understanding and build technique. The asterisked problems are the most challenging, and the instructor may assign them as long-term projects. Another important feature of the book is that it encourages students to make mathematical discoveries through calculator/computer experimentation.

Chapter 1 deals with first-order difference equations, or one-dimensional maps on the real line. It includes a thorough and complete analysis of stability for many popular maps (equations) such as the logistic map, the tent map, and the Baker map. The rudiments of bifurcation and chaos theory are also included in Section 1.6. This section raises more questions and gives few answers. It is intended to arouse the reader's interest in this exciting field.

In Chapter 2 we give solution methods for linear difference equations of any order. Then we apply the obtained results to investigate the stability and the oscillatory behavior of second-order difference equations. At the end of the chapter we give four applications: the propagation of annual plants, the gambler's ruin, the national income, and the transmission of information.

Chapter 3 extends the study in Chapter 2 to systems of difference equations. We introduce two methods to evaluate A^n for any matrix A. In Section 3.1 we introduce the Putzer algorithm, and in Section 3.3 the method of the Jordan form is given. Many applications are then given in Section 3.5, which include Markov chains, trade models, and the heat equation.

Chapter 4 investigates the question of stability for both scalar equations and systems. Stability of nonlinear equations is studied via linearization (Section 4.5) and by the famous method of Liapunov (Section 4.6). Our exposition here is restricted to autonomous (time-invariant) systems. I believe that the extension of the theory to nonautonomous (time-variant) systems, though technically involved, will not add much more understanding to the subject matter.

Chapter 5 delves deeply into Z-transform theory and techniques (Sections 5.1, 5.2). Then the results are applied to study the stability of Volterra difference scalar equations (Sections 5.3, 5.4) and systems (Sections 5.5, 5.6). For readers familiar with differential equations, Section 5.7 provides a comparison between the Z-transform and the Laplace transform. Most of the results on Volterra difference equations appear here for the first time in a book.

Chapter 6 takes us to the realm of control theory. Here, we cover most of the basic concepts including controllability, observability, observers, and stabilizability by feedback. Again, we restrict the presentation to autonomous (time-invariant) systems, since this is just an introduction to this vast and growing discipline. Moreover, most practitioners deal mainly with time-invariant systems.

In Chapter 7 we give a comprehensive and accessible study of asymptotic methods for difference equations. Starting from the Poincaré Theorem, the chapter covers most of the recent development in the subject. Section 7.4 (asymptotically diagonal systems) presents an extension of Levinson's Theorem to difference equations, while in Section 7.5 we carry our study to nonlinear difference equations. Several open problems are given that would serve as topics for research projects.

Finally, Chapter 8 presents a brief introduction to oscillation theory. In Section 8.1, the basic results on oscillation for three-term linear difference equations are introduced. Extension of these results to nonlinear difference equations is presented in Section 8.2. Another approach to oscillation theory, for self-adjoint equations, is presented in Section 8.3. Here we also introduce a discrete version of Sturm's Separation Theorem.

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