Preface to the Second Edition

During the nine years since the publication of the first edition of this book, there has been substantial progress on the treatment of well-set problems of nonlinear solid mechanics. The main purposes of this second edition are to update the first edition by giving a coherent account of some of the new developments, to correct errors, and to refine the exposition. Much of the text has been rewritten, reorganized, and extended.

The philosophy underlying my approach is exactly that given in the following (slightly modified) Preface to the First Edition. In particular, I continue to adhere to my policy of eschewing discussions relying on technical aspects of theories of nonlinear partial differential equations (although I give extensive references to pertinent work employing such methods). Thus I intend that this edition, like the first, be accessible to a wide circle of readers having the traditional prerequisites given in Sec. **1**.2.

I welcome corrections and comments, which can be sent to my electronic mail address: ssa@math.umd.edu. In due time, corrections will be placed on my web page: http://www.ipst.umd.edu/Faculty/antman.htm.

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Preface to the First Edition

The scientists of the seventeenth and eighteenth centuries, led by Jas. Bernoulli and Euler, created a coherent theory of the mechanics of strings and rods undergoing planar deformations. They introduced the basic concepts of strain, both extensional and flexural, of contact force with its components of tension and shear force, and of contact couple. They extended Newton's Law of Motion for a mass point to a law valid for any deformable body. Euler formulated its independent and much subtler complement, the Angular Momentum Principle. (Euler also gave effective variational characterizations of the governing equations.) These scientists breathed life into the theory by proposing, formulating, and solving the problems of the suspension bridge, the catenary, the velaria, the elastica, and the small transverse vibrations of an elastic string. (The level of difficulty of some of these problems is such that even today their descriptions are seldom vouchsafed to undergraduates. The realization that such profound and beautiful results could be deduced by mathematical reasoning from fundamental physical principles furnished a significant contribution to the intellectual climate of the Age of Reason.) At first, those who solved these problems did not distinguish between linear and nonlinear equations, and so were not intimidated by the latter.

By the middle of the nineteenth century, Cauchy had constructed the basic framework of 3-dimensional continuum mechanics on the foundations built by his eighteenth-century predecessors. The dominant influence on the direction of further work on elasticity (and on every other field of classical physics) up through the middle of the twentieth century was the development of effective practical tools for solving linear partial differential equations on suitably shaped domains. So thoroughly did the concept of linearity pervade scientific thought during this period that mathematical physics was virtually identified with the study of differential equations containing the Laplacian. In this environment, the respect of the scientists of the eighteenth century for a (typically nonlinear) model of a physical process based upon fundamental physical and geometrical principles was lost.

The return to a serious consideration of nonlinear problems (other than those admitting closed-form solutions in terms of elliptic functions) was led by Poincaré and Lyapunov in their development of qualitative methods for the study of ordinary differential equations (of discrete mechanics) at the end of the nineteenth century and at the beginning of the twentieth century. Methods for handling nonlinear boundary-value problems were slowly developed by a handful of mathematicians in the first half of the twentieth century. The greatest progress in this area was attained in the study of direct methods of the calculus of variations (which are very useful in nonlinear elasticity).

A rebirth of interest in nonlinear elasticity occurred in Italy in the 1930's under the leadership of Signorini. A major impetus was given to the subject in the years following the Second World War by the work of Rivlin. For special, precisely formulated problems he exhibited concrete and elegant solutions valid for arbitrary nonlinearly elastic materials. In the early 1950's, Truesdell began a critical examination of the foundations of continuum thermomechanics in which the roles of geometry, fundamental physical laws, and constitutive hypotheses were clarified and separated from the unsystematic approximation then and still prevalent in parts of the subject. In consequence of the work of Rivlin and Truesdell, and of work inspired by them, continuum mechanics now possesses a clean, logical, and simple formulation and a body of illuminating solutions.

The development after the Second World War of high-speed computers and of powerful numerical techniques to exploit them has liberated scientists from dependence on methods of linear analysis and has stimulated growing interest in the proper formulation of nonlinear theories of physics. During the same time, there has been an accelerating development of methods for studying nonlinear equations. While nonlinear analysis is not yet capable of a comprehensive treatment of nonlinear problems of continuum mechanics, it offers exciting prospects for certain specific areas. (The level of generality in the treatment of large classes of operators in nonlinear analysis exactly corresponds to that in the treatment of large classes of constitutive equations in nonlinear continuum mechanics.) Thus, after two hundred years we are finally in a position to resume the program of analyzing illuminating, well-formulated, specific nonlinear problems of continuum mechanics.

The objective of this book is to carry out such studies for problems of nonlinear elasticity. It is here that the theory is most thoroughly established, the engineering tradition of treating specific problems is most highly developed, and the mathematical tools are the sharpest. (Actually, more general classes of solids are treated in our studies of dynamical problems; e.g., Chap. 15 is devoted to a presentation of a general theory of large-strain viscoplasticity.) This book is directed toward scientists, engineers, and mathematicians who wish to see careful treatments of uncompromised problems. My aim is to retain the orientation toward fascinating problems that characterizes the best engineering texts on structural stability while retaining the precision of modern continuum mechanics and employing powerful, but accessible, methods of nonlinear analysis.

My approach is to lay down a general theory for each kind of elastic body, carefully formulate specific problems, introduce the pertinent mathematical methods (in as unobtrusive a way as possible), and then conduct rigorous analyses of the problems. This program is successively carried out for strings, rods, shells, and 3-dimensional bodies. This ordering of topics essentially conforms to their historical development. (Indeed, we carefully study modern versions of problems treated by Huygens, Leibniz, and the Bernoullis in Chap. 3, and by Euler and Kirchhoff in Chaps. 4, 5, and 8.) This ordering is also the most natural from the viewpoint of pedagogy: Chaps. 2–6, 8–10 constitute what might be considered a modern course in nonlinear structural mechanics. From these chapters the novice in solid mechanics can obtain the requisite background in the common heritage of applied mechanics, while the experienced mechanician can gain an appreciation of the simplicity of geometrically exact, nonlinear (re)formulations of familiar problems of structural mechanics and an appreciation of the power of nonlinear analysis to treat them. At the same time, the novice in nonlinear analysis can see the application of this theory in simple, concrete situations.

The remainder of the book is devoted to a thorough formulation of the 3-dimensional continuum mechanics of solids, the formulation and analysis of 3-dimensional problems of nonlinear elasticity, an account of large-strain plasticity, a general treatment of theories of rods and shells on the basis of the 3-dimensional theory, and a treatment of nonlinear wave propagation and related questions in solid mechanics. The book concludes with a few self-contained appendices on analytic tools that are used throughout the text. The exposition beginning with Chap. 11 is logically independent of the preceding chapters. Most of the development of the mechanics is given a material formulation because it is physically more fundamental than the spatial formulation and because it leads to differential equations defined on fixed domains.

The theories of solid mechanics are each mathematical models of physical processes. Our basic theories, of rods, shells, and 3-dimensional bodies, differ in the dimensionality of the bodies. These theories may not be constructed haphazardly: Each must respect the laws of mechanics and of geometry. Thus, the only freedom we have in formulating models is essentially in the description of material response. Even here we are constrained to constitutive equations compatible with invariance restrictions imposed by the underlying mechanics. Thus, both the mechanics and mathematics in this book are focused on the formulation of suitable constitutive hypotheses and the study of their effects on solutions. I tacitly adopt the philosophical view that the study of a physical problem consists of three distinct steps: formulation, analysis, and interpretation, and that the analysis consists solely in the application of mathematical processes exempt from ad hoc physical simplifications.

The notion of solving a nonlinear problem differs markedly from that for linear problems: Consider boundary-value problems for the linear ordinary differential equation

$$\frac{d^2}{ds^2}\theta(s) + \lambda\theta(s) = 0, \tag{1}$$

which arises in the elementary theory for the buckling of a uniform column. Here λ is a positive constant. Explicit solutions of the boundary-value problems are immediately found in terms of trigonometric functions. For a nonuniform column (of positive thickness), (1) is replaced with

$$\frac{d}{ds}\left[B(s)\frac{d\theta}{ds}(s)\right] + \lambda\theta(s) = 0 \tag{2}$$

where B is a given positive-valued function. In general, (2) cannot be solved in closed form. Nevertheless, the Sturm-Liouville theory gives us information about solutions of boundary-value problems for (2) so detailed that for many practical purposes it is as useful as the closed-form solutions obtained for (1). This theory in fact tells us what is essential about solutions. Moreover, this information is not obscured by complicated formulas involving special functions. We accordingly regard this qualitative information as characterizing a solution.

The elastica theory of the Bernoullis and Euler, which is a geometrically exact generalization of (1), is governed by the semilinear equation

$$\frac{d^2}{ds^2}\theta(s) + \lambda\sin\theta(s) = 0. \tag{3}$$

It happens that boundary-value problems for (3) can be solved explicitly in terms of elliptic functions, and we again obtain solutions in the traditional sense. On the other hand, for nonuniform columns, (3) must be replaced by

$$\frac{d}{ds}\left[B(s)\frac{d\theta}{ds}(s)\right] + \lambda \sin\theta(s) = 0, \tag{4}$$

for which no such solutions are available. In Chap. 5 we develop a nonlinear analog of the Sturm-Liouville theory that gives detailed qualitative information on solutions of boundary-value problems for (4). The theory has the virtues that it captures all the qualitative information about solutions of (3) available from the closed-form solutions and that it does so with far less labor than is required to obtain the closed-form solutions. We shall not be especially concerned with models like (4), but rather with its generalizations in the form

$$\frac{d}{ds}\left[\hat{M}\left(\frac{d\theta}{ds}(s), s\right)\right] + \lambda \sin\theta(s) = 0. \tag{5}$$

Here $\hat{M}$ is a given constitutive function that characterizes the ability of the column to resist flexure. When we carry out an analysis of equations like (5), we want to determine how the properties of $\hat{M}$ affect the properties of solutions. In many cases, we shall discover that different kinds of physically reasonable constitutive functions give rise to qualitatively different kinds of solutions and that the distinction between the kinds of solutions has great physical import. We regard such analyses as constituting solutions.

The prerequisites for reading this book, spelled out in Sec. **1**.2, are a sound understanding of Newtonian mechanics, advanced calculus, and linear algebra, and some elements of the theories of ordinary differential equations and linear partial differential equations. More advanced mathematical topics are introduced when needed. I do not subscribe to the doctrine that the mathematical theory must be fully developed before it is applied. Indeed, I feel that seeing an effective application of a theorem is often the best motivation for learning its proof. Thus, for example, the basic results of global bifurcation theory are explained in Chap. 5 and immediately applied there and in Chaps. 6, 9, and 10 to a variety of buckling problems. A self-contained treatment of degree theory leading to global bifurcation theory is given in the Appendix (Chap. 21).

A limited repertoire of mathematical tools is developed and broadly applied. These include methods of global bifurcation theory, continuation methods, and perturbation methods, the latter justified whenever possible by implicit-function theorems. Direct methods of the calculus of variations are the object of only Chap. 7. The theory is developed here only insofar as it can easily lead to illuminating insights into concrete problems; no effort is made to push the subject to its modern limits. Special techniques for dynamical problems are mostly confined to Chap. 18 (although many dynamical problems are treated earlier).

This book encompasses a variety of recent research results, a number of unpublished results, and refinements of older material. I have chosen not to present any of the beautiful modern research on existence theories for 3-dimensional problems, because the theory demands a high level of technical expertise in modern analysis, because very active contemporary research, much inspired by the theory of phase transformations, might very strongly alter our views on this subject, and because there are very attractive accounts of earlier work in the books of Ciarlet (1988), Dacorogna (1989), Hanyga (1985), Marsden & Hughes (1983), and Valent (1988). My treatment of specific problems of 3-dimensional elasticity differs from the classical treatments of Green & Adkins (1970), Green & Zerna (1968), Ogden (1984), Truesdell & Noll (1965), and Wang & Truesdell (1973) in its emphasis on analytic questions associated with material response. In practice, many of the concrete problems treated in this book involve but one spatial variable, because it is these problems that lend themselves most naturally to detailed global analyses. The choice of topics naturally and strongly reflects my own research interests in the careful formulation of geometrically exact theories of rods, shells, and 3-dimensional bodies, and in the global analysis of well-set problems.

There is a wealth of exercises, which I have tried to make interesting, challenging, and tractable. They are designed to cause the reader to (i) complete developments outlined in the text, (ii) carry out formulations of problems with complete precision (which is the indispensable skill required of workers in mechanics), (iii) investigate new areas not covered in the text, and, most importantly, (iv) solve concrete problems. Problems, on the other hand, represent what I believe are short, tractable research projects on generalizing the extant theory to treat minor, open questions. They afford a natural entrée to bona fide research problems.

This book had its genesis in a series of lectures I gave at Brown University in 1978–1979 while I was holding a Guggenheim Fellowship. Its exposition has been progressively refined in courses I have subsequently given at the University of Maryland and elsewhere. I am particularly indebted to many students and colleagues who have caught errors and made useful suggestions. Among those who have made special contributions have been John M. Ball, Carlos Castillo-Chavez, Patrick M. Fitzpatrick, James M. Greenberg, Leon Greenberg, Timothy J. Healey, Massimo Lanza de Cristoforis, John Maddocks, Pablo Negrón-Marrero, Robert Rogers, Felix Santos, Friedemann Schuricht, and Li-Sheng Wang. I thank the National Science Foundation for its continued support, the Air Force Office of Scientific Research for its recent support, and the taxpayers who support these organizations.

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