

## 2 Measuring Instruments and Their Properties

### 2.1. Types of Measuring Instruments

Measuring instruments are the technical objects that are specially developed for the purpose of measuring specific physical quantities. A general property of measuring instruments is that their accuracy is standardized.

Measuring instruments are divided into material measures, measuring transducers, indicating instruments, measuring setups, and measuring systems.

A *material measure* is a measuring instrument that reproduces one or more known values of a given physical quantity. Examples of measures are balance weights, measuring resistors, and measuring capacitors.

Single-valued measures, multiple-valued measures, and collections of measures are distinguished. Examples of multiple-valued measures are graduated rulers, measuring tapes, resistance boxes, and so on.

In addition to multiple-valued measures, which reproduce discrete values of quantities, multiple-valued measures exist that continuously reproduce quantities in some range, for example, a measuring capacitor with variable capacitance. Continuous measures are usually less accurate than discrete measures.

When measures are used to perform measurements, the measurands are compared with the known quantities reproduced by the measures. The comparison is made by different methods, but so-called comparators are a specific means that are used to compare quantities. The simplest comparator is the standard equal-armed pan balance.

A *comparator* is a measuring device that makes it possible to compare similar physical quantities and has a known sensitivity.

In some cases, quantities are compared without comparators, by experimenters, with the help of their viewing or listening perceptions.

Thus, when measuring the length of a body with the help of a ruler, the ruler is placed on the body and the observer fixes visually the graduations of the ruler (or fractions of a graduation) at the corresponding points of the body.

A *measuring transducer* is a measuring instrument that converts the measurement signals into a form suitable for transmission, processing, or storage. The

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measurement information at the output of a measuring transducer cannot, as a rule, be directly observed by the observer.

It is necessary to distinguish measuring transducers and the transforming elements of a complicated instrument. The former are measuring instruments, and as such, they have standard metrological properties (see below). The latter, on the other hand, do not have an independent metrological significance and are not used separately from the instrument of which they are a part.

Measuring transducers are diverse. Examples are thermocouples, resistance thermometers, measuring shunts, the measuring electrodes of  $pH$  meters, and so on. Measuring current or voltage transformers and measuring amplifiers are also measuring transducers. But this group of transducers is characterized by the fact that the signals at their inputs and outputs are a physical quantity of the same form, and only the dimension of the quantity changes. For this reason, these measuring transducers are called scaling measuring transducers.

Measuring transducers that convert an analog quantity at the input (the measurand) into a discrete signal at the output are called analog-to-digital converters. Such converters are manufactured in the form of autonomous, i.e., independent measuring instruments, and in the form of units built into other instruments, in particular, in the form of integrated microcircuits. Analog-to-digital converters are a necessary component of digital devices, but they are also employed in monitoring, regulating, and control systems.

An *indicating instrument* is a measuring instrument that is used to convert measurement signals into a form that can be directly perceived by the observer.

Based on the design of the input circuits, indicating instruments are just as diverse as measuring transducers, and it is difficult to survey all of them. Moreover, such a review and even classification are more important for designing instruments than for describing their general properties.

A common feature of all indicating instruments is that they all have readout devices. If these devices are implemented in the form of a scale and an indicating needle, then the indications of the instrument are a continuous function of the measurand. Such instruments are called analog instruments. If the indications of instruments are in a digital form, then such instruments are called digital instruments.

The definition of digital instruments presented above formally includes both automatic digital voltmeters, bridges, and similar instruments and induction meters for measuring electrical energy. In these instruments, however, the measuring transformations are performed in a discrete form, and in the case of induction meters, all measuring transformations of signals occur in an analog form and only the output signal assumes a discrete form. The conversions of measurement information into a discrete form have several specific features. Therefore, only instruments in which the measurement conversions occur in a discrete form are usually considered to be digital instruments.

The indications of digital instruments are easily recorded and are convenient for entering into a computer. In addition, their design usually makes it possible to obtain significantly higher accuracy than analog instruments. Moreover, when

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digital instruments are employed, no reading error occurs. However, when analog instruments are used, it is easier to judge trends in the variation of the measurands.

In addition to analog and digital instruments, analog-discrete measuring instruments also exist. In these instruments, the measuring conversions are performed in an analog form, but the readout device is a discrete unit. The readout device has a scale and a glowing strip, whose length changes discretely, which plays the role of the indicator. Sometimes the indicator is a glowing dot that moves along a scale.

Analog-discrete instruments combine the advantages of both analog and digital instruments. Induction meters for measuring electric energy are examples of such hybrid instruments.

In many, cases, measuring instruments are designed so that their indications are recorded. Such instruments are said to be *recording instruments*. Data can be recorded in the form of a continuous record of the variation of the measurand in time or in the form of a series of points of this dependence.

Instruments of the first type are called automatic-plotting instruments, and instruments of the second type are called printing instruments. Printing instruments can record the values of a measurand in digital form. Printing instruments give a discrete series of values of the measurand in some interval of time. The continuous record provided by automatic-plotting instruments can be regarded as an infinite series of values of the measurand.

Sometimes measuring instruments are equipped with induction, photooptical, or contact devices and relays for purposes of control or regulation. Such instruments are called regulating instruments. Designers strive to design regulating units so as not to reduce the accuracy of the measuring instrument. However, this is rarely possible.

Measuring instruments also customarily include comparators, mentioned above, for comparing measures, and null indicators, for example, galvanometers. The reason is that a comparator with a collection of measures becomes a comparison measuring instrument, whereas a galvanometer can be used as a highly sensitive indicating instrument.

A *measurement setup* is a collection of functionally and structurally integrated measuring instruments and auxiliary devices that provides efficient organization of the measurements. An example is the potentiometric setup for electric measuring instrument calibration.

A *measuring system* is a collection of functionally unified measuring, computing, and auxiliary means for obtaining measurement information and for converting and processing it to provide the user with information in the required form, introducing it into the control system, or performing logical functions automatically. Modern measuring systems include microprocessors and even entire computers, and apart from processing and providing output of the measurement information, they can control the measurement process.

Finally, systems whose units must, in accordance with the purpose of the system, operate under the same conditions can be distinguished from systems whose units operate under different conditions. We shall call the former *uniform measuring systems* and the latter *nonuniform measuring systems*. This classification makes it

easier to study questions concerning the metrological support of measuring systems and the calculation of the errors of such systems.

## 2.2. The Concept of an Ideal Instrument: Metrological Characteristics of Measuring Instruments

Any technical object can be described by a collection of characteristics. Measuring instruments are not an exception in this respect. We shall divide all characteristics of measuring instruments into two groups: metrological, which is necessary for using a measuring instrument in the manner intended, and secondary. We shall include in the latter such characteristics as mass, dimensions, and degree of protection from moisture and dust. We shall not discuss characteristics of the secondary group, although sometimes they determine the selection and application of an instrument, because they are not directly related with the measurement accuracy.

By metrological characteristics of a measuring instrument, we mean the characteristics that make it possible to judge the suitability of the instrument for performing measurements in a known range with known accuracy, to obtain a value of the measurand, and to estimate its inaccuracy.

To sort the metrological characteristics, it is helpful to introduce the concept of an ideal instrument. The ideal of any technical object is its design, i.e., its model. For measuring instruments, this is not sufficient, because such a device must contain also an “impression” of the corresponding unit of measurement. The impression of the unit cannot be prepared; it must be obtained from a standard. We shall give several examples.

*Gauge block.* The ideal is a completely regular parallelepiped, one edge of which is determined exactly for the established units of length.

*Measure of constant voltage.* The ideal is a source of constant voltage with a value that is known exactly and that is free of any noise at the output.

*Measuring transformer.* The ideal is a voltage or current transformer with a conversion factor that is known exactly and that has no losses and parasitic noise in the input and output circuits.

*Integrating analog-to-digital converter of voltage.* The ideal is an instrument with an output voltage  $U_0$  that is related to the input voltage  $U_x$  by the dependence

$$U_0 = K_1 \int_{t_1}^{t_2} U_x dt,$$

where  $K_1 = \text{const}$  and  $\Delta t = t_2 - t_1 = \text{const}$ .

As a result, we obtain  $U_0 = K_2 U_x$ . The voltage  $U_0$  can be quantized, and a code, reflecting the voltage at the input without any distortions (with the exception of the quantization error, which can be made to be small), is obtained at the output of the converter.

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*Moving-coil ammeter.* In this instrument, a constant current flows through a moving coil and forms with the help of a permanent magnet a mechanical moment, which twists a spring to a point of balance. In the process, an indicating needle is deflected by an amount along a scale that is proportional to the current strength. Each value of the current strength corresponds to a definite indication, which is fixed by calibrating the instrument.

Thus, the ideal instrument performs (theoretically) a series of single-valued transformations and after calibration acquires a precise scale in the units of the measurand.

An ideal representation of each type of measuring instruments is formed for a specific model of the objects—carriers of the corresponding physical quantity.

We shall call the metrological characteristics established for ideal measuring instruments of a specific type the *nominal metrological characteristics*. An example of such a characteristic is the nominal value of the measure (10  $\Omega$ , 1 kG, etc.), the measurement range of the instrument (0–300 V, 0–1200  $^{\circ}\text{C}$ , etc.), the conversion range of the transducer, the value of the scale factor of the instrument scale, and so on.

The relation between the input and the output signals of instruments and transducers is determined by the transfer function. For instruments, it is fixed by the scale, whereas for measuring transducers, it is determined by a graph or an equation. Either the graph or the equation represents the nominal metrological characteristic if the graph or equation was determined (indicated) before these measuring instruments were developed.

The real characteristics of measuring instruments differ from the nominal characteristics because of fabrication errors and changes occurring in the corresponding properties in time.

An ideal measuring instrument (transducer) would react only to the measured physical quantity or to the parameter of the input signal of interest, and its indication would not depend on the external conditions, the power supply regime, and so on. For a real measuring transducer, as for other types of measuring instruments, these undesirable phenomena occur.

The quantities characterizing the external conditions are called influence quantities.

For some types of measuring instruments, the dependence of the output signal, the indications, or the error from one or another influence quantity can be represented as a functional dependence, called the influence function. The influence function can be expressed in the form of an equation (for example, the temperature dependence of the emf of standard cells) or a graph. In the case of a linear dependence, it is sufficient to give the coefficient of proportionality between the output quantity and the influence quantity. We shall call this coefficient the influence coefficient.

Influence coefficients and functions make it possible to take into account the conditions under which measuring instruments are used by introducing the corresponding corrections. The imperfection of measuring instruments is also manifested because when one and the same quantity is measured repeatedly under

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identical conditions, the results can differ somewhat from one another. In this case, it is said that the indications are nonrepeatable.

The inaccuracy of a measuring instrument is usually characterized by its error. We shall explain this concept for the example of an indication measuring instrument. Let the true value of a quantity at the input of the instrument be  $A_t$ . The instrument indicates the value  $A_r$ . The absolute error of the instrument will be

$$\zeta = A_r - A_t.$$

The nonrepeatability of the indications of the instrument is manifested by the fact that when  $A_t$  is measured repeatedly, the indications of the instrument will be somewhat different. For this reason, one can talk about a random component of instrument error. This component is referred to as the repeatability error of a measuring instrument.

The random component of instrument error is normally caused by friction in the supports of a movable part of the instrument and hysteresis phenomena, and its limits are sharp. The limits can be found experimentally if the quantity measured by the instrument varies continuously. The strength of the electric current, the voltage, and other quantities can be varied continuously. Correspondingly, the indications of ammeters and voltmeters can vary continuously. The indications of weighing balances and several other instruments cannot be varied continuously.

For instruments whose indications can vary continuously, the limits of the random error are found by continuously driving the indicator of the instrument up to the same scale marker, first from below and then from above (or vice versa) a marker. We will call *the dead band* the absolute value of the difference of the values of the measurand that are obtained in such a test and that correspond to a given scale marker of the instrument.

The dead band is the length of the range of possible values of the random component of instrument error, and one half of this length is the limiting value of the random.

Figure 2.1 shows graphs of the “input-output” in the presence of (a) only friction, (b) only hysteresis, and (c) friction together with hysteresis. These examples of processes reveal dead bands.

The random error of weighing scales is usually characterized by the standard deviation [12]. This characteristic of an instrument is calculated from the changes produced in the indications of the scales by a load with a known mass; the test is performed at several scale markers, including the limits of the measurement range. One method for performing the tests, and the computational formula for calculating the standard deviation of weighing scales, are presented in [12].

Measuring instruments are created to introduce certainty into the phenomena studied and to establish regular relations between the phenomena, and the uncertainty created by the non-single-valuedness of instrument indications interferes with using an instrument in the manner intended. For this reason, the first problem that must be solved when developing a new measuring device is to make its

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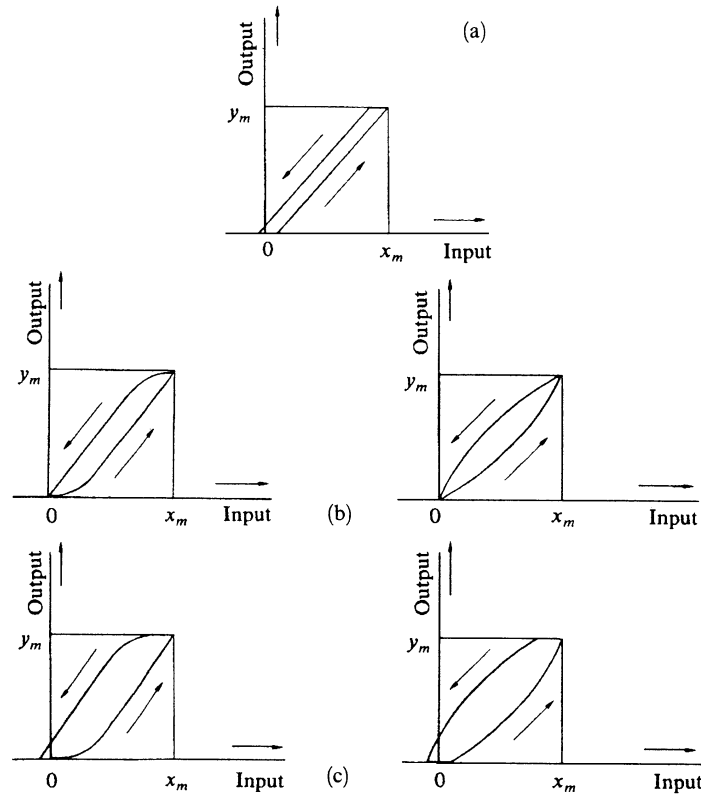


FIGURE 2.1. Dependences between the input and output of the instrument with a dead band. (a) In the presence of a friction, (b) in the presence of hysteresis (two types, for example), and (c) in the presence of a friction and hysteresis (two types mentioned above).

random error insignificant, i.e., either negligibly small compared with other errors or falling within prescribed limits as the limits of admissible errors for measuring devices of the given type.

If the random error is insignificant and the elements determining instrument accuracy are stable, then by calibration, the measuring device can always be “tied” to a corresponding standard and the potential accuracy of the instrument can be realized.

The value of a scale division or the value of a significant figure is the value of the measurand corresponding to the interval between two neighboring markers on the instrument scale or one figure of some digit of a digital readout device.

The *sensitivity* is the ratio of the change in the output value of the measuring instruments to the input value of the quantity that causes the output value to change. The sensitivity can be a nominal metrological characteristic and an actual characteristic of a real instrument.

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The *discrimination threshold* is the minimum change in the input signal that causes an appreciable change in the output signal.

The *resolution* is the smallest interval between two distinguishable neighboring discrete values of the output signal.

*Instability* (of a measuring instrument) is a general term that expresses the change in any property of the measuring instrument in time.

*Drift* is the change occurring in the output signal (always in the same direction) over a period of time that is significantly longer than the measurement time when using a given measuring instrument.

The drift and the instability do not depend on the input signal or the load, but they can depend on the external conditions. The drift is usually determined in the absence of a signal at the input.

The metrological characteristics of measuring instruments should also include their dynamic characteristics. These characteristics reflect the inertial properties of measuring instruments. It is necessary to know them to correctly choose and use many types of measuring instruments. The dynamical characteristics are examined below in Section 2.5.

The properties of measuring instruments can normally be described based on the characteristics enumerated above. For specific types of measuring instruments, however, additional characteristics are often required. Thus, for the gauge rods, the so-called flatness and polishability are important. For voltmeters, the input resistance is important. We shall not study such characteristics, because they refer only to individual types of measuring instruments.

### 2.3. Standardization of the Metrological Characteristics of Measuring Instruments

Measuring instruments can only be used as intended when their metrological properties are known. In principle, the metrological properties can be established by two methods. One method is to find the actual characteristics of a specific instrument. In the second method, the nominal metrological characteristics and the permissible deviations of the real characteristics from the nominal characteristics are given.

The first method is laborious, and for this reason, it is used primarily for the most accurate and stable measuring instruments. Thus, the second method is the main method. The nominal characteristics and the permissible deviations from them are given in the technical documentation when measuring instruments are designed, which predetermines the properties of measuring instruments and ensures that they are interchangeable.

In the process of using measuring instruments, checks are made to determine whether the real properties of the devices deviate from the established standards. If one real property deviates from its nominal value by an amount greater than demonstrated by the standards, then the measuring instrument is adjusted, remade, or discarded and no longer used.



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Thus, the choice of the nominal characteristics of measuring instruments and the designation of permissible deviations of the real characteristics from them—standardization of the metrological characteristics of measuring instruments—are of great importance for measurement practice. We shall examine the practice of standardization of the metrological characteristics of measuring instruments that has evolved.

Both the production of measuring instruments and the standardization of their characteristics initially arose spontaneously in each country. Later, rules that gave order to this standardization were developed in all countries in which instrument building was highly developed. The recommendations developed at this time by international organizations, primarily Publication 51 of the International Electrotechnical Commission (IEC), were of great importance for the preparation of national standards [8]. We should also mention the International Organization for Standardization (ISO) and the International Organization of Legal Metrology (OIML). The terminological documents are also of great value for this work [2], [4], [7].

We shall now return to the gist of the problem. The significance of nominal metrological characteristics, such as the upper limits of measurement ranges, the nominal values of the measures, the scale factors of instruments and so on, is chosen from a standardized series of values of these characteristics. There is nothing special here. Another task is to standardize the accuracy characteristics, errors, and stability.

Despite the efforts of designers, the real characteristics of measuring instruments depend to some extent on the external conditions. For this reason, some narrow ranges of values of all influence quantities are fixed first, and in this manner, the conditions under which measuring instruments are to be calibrated and checked are determined. These conditions are called reference conditions. The error of measuring instruments under reference conditions is called the *intrinsic error*.

In the standard in [7], this question is solved less formally: The conditions under which the characteristics of measuring instruments depend negligibly on the possible variations of influence quantities are called *reference* conditions. In other words, these are conditions under which the metrological characteristics are practically constant.

This definition of reference conditions seems attractive. I stated the identical idea in [44]. However, I also stated there my doubts in the possibility of implementing the idea. We shall return to this question in the next section.

Thus, the reference conditions of measuring instruments are prescribed and the intrinsic errors of measuring instruments are determined.

In addition to the reference conditions, the normal operating conditions of measuring instruments are also established, i.e., the conditions under which the characteristics of measuring instruments remain within certain limits and the measuring instruments can be employed as intended. Understandably, errors in the normal operating conditions are larger than errors in the reference conditions (i.e., these errors are larger than the intrinsic errors).

When any influence quantity exceeds the normal value or range for a reference condition, the error of the measuring instrument changes. This change is

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characterized and standardized by indicating the limit of the permissible additional error, by indicating the highest permissible value of the influence factor of the corresponding influence quantity, or by indicating the limit of the permissible error under the normal operating conditions.

The errors of measuring instruments are expressed not only in the form of absolute and relative errors, adopted for estimating measurement errors, but also in the form of *fiducial errors*. The fiducial error is the ratio of the absolute error of the measuring instrument to some standardizing value—fiducial value. The latter value is established by standards on separate types of measuring instruments. For indicating instruments, for example, the fiducial value is established depending on the characteristic features and character of the scale. The fiducial errors make it possible to compare the accuracy of measuring instruments that have different measurement limits. For example, the accuracy of an ammeter with a measurement limit of 1 A can be compared with that of an ammeter with a measurement limit of 100 A.

In addition, cases when the error of an indicating instrument is expressed in fractions of a graduation are also encountered.

For measuring transducers, the errors can be represented by the errors relative to the input or output.

Figure 2.2 shows the nominal and, let us assume, the real transfer functions of some transducer. The nominal dependence, as done in practice whenever possible, is assumed to be linear. We shall investigate the relationship between the errors of the transducer that are scaled to the input and the output.

We denote the input quantity by  $x$  and the output quantity by  $y$ . They are related by the relation

$$x = Ky,$$

where  $K$  is the nominal transduction constant.

At the point with true values of the quantities  $x_t$  and  $y_t$ , the true value of the transduction constant will be  $K_t = x_t/y_t$ . Calculations based on the nominal constant  $K$ , however, are given an error.

Let  $x_a = Ky_t$  and  $y_a = x_t/K$  be determined based on  $y_t$  and  $x_t$  (see Fig. 2.2). Then the absolute transducer error with respect to the input will be

$$\Delta x = x_a - x_t = (K - K_t)y_t.$$

The error with respect to the output is expressed analogously:

$$\Delta y = y_a - y_t = \left( \frac{1}{K} - \frac{1}{K_t} \right) x_t.$$

We note, first, that  $\Delta x$  and  $\Delta y$  always have different signs: If  $(K - K_t) > 0$ , then  $(1/K - 1/K_t) < 0$ .

But this is not the only difference. The quantities  $x$  and  $y$  can also have different dimensions; i.e., they can be physically different quantities, so that the absolute input and output errors are not comparable. For this reason, we shall study the

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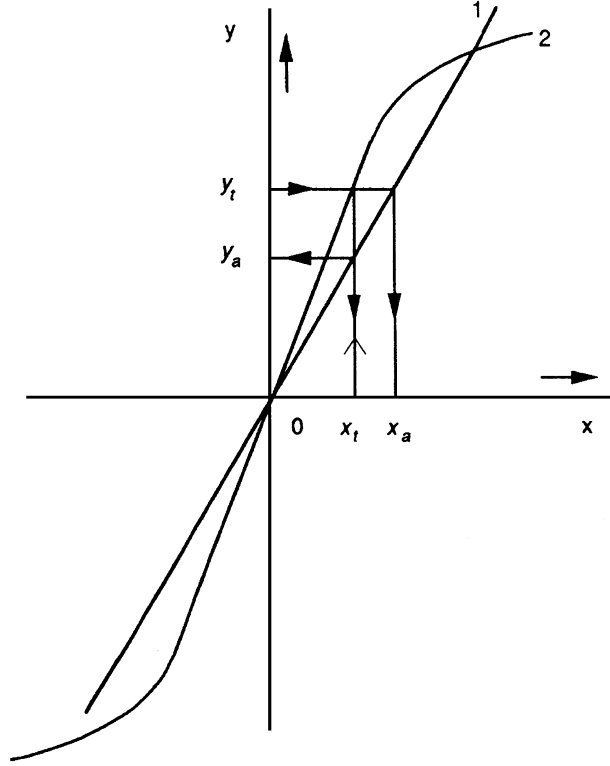


FIGURE 2.2. Nominal (curve 1) and real (curve 2) functions of a measuring transducer.

relative errors:

$$\varepsilon_x = \frac{\Delta x}{x_t} = (K - K_t) \frac{y_t}{x_t} = \frac{K - K_t}{K_t},$$

$$\varepsilon_y = \frac{\Delta y}{y_t} = \frac{(K_t - K) x_t}{K K_t} \frac{1}{y_t} = \frac{K_t - K}{K}.$$

As  $K_t \neq K$ , we have  $|\varepsilon_x| \neq |\varepsilon_y|$ .

We denote the relative error in the transduction constant at the point  $(x_t, y_t)$  as  $\varepsilon_k$ , where  $\varepsilon_k = (K - K_t)/K_t$ . Then

$$\frac{\varepsilon_x}{\varepsilon_y} = -(1 + \varepsilon_k).$$

However,  $\varepsilon_k \ll 1$ , and in practice relative errors with respect to the input and output can be regarded as equal in magnitude.

We must stop to consider how the error of measures is determined: The error of measures is the difference between the nominal value of the measure and the true value of the quantity reproduced by the measure. Indeed, in the case of indicating

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instruments, the nominal value of measures is the analog of the indication of the instrument, and the definition given becomes obvious.

It is also interesting that measures that reproduce passive quantities, for example, mass, electric resistance, and so on, have only systematic errors. The error of measures of active quantities (electric voltage, electric current, etc.) can have both systematic and random components. Multiple-valued measures of passive quantities can have random errors from switching elements.

So, when the errors of measuring instruments are standardized, the permissible limits of the intrinsic and all additional errors are prescribed. At the same time, the reference and normal operating conditions are indicated.

Of all forms enumerated above for expressing the errors of measuring instruments, the best is the relative error, because in this case, the indication of the permissible limit of error gives the best idea of the level of measurement accuracy that can be achieved with the given measuring instrument. The relative error, however, usually changes significantly over the measurement range of the instrument, and for this reason, it is difficult to use for standardization.

The absolute error is frequently more convenient than the relative error. In the case of an instrument with a scale, the limit of the permissible absolute error can be standardized with the same numerical value for the entire scale of the instrument. But then it is difficult to compare the accuracies of instruments having different measurement ranges. This difficulty disappears when the fiducial value of errors are standardized.

In the following discussion, we shall follow primarily [8] and [9].

The limit of the permissible absolute error  $\Delta$  can be expressed by a single value (neglecting the sign)

$$\Delta = \pm a,$$

in the form of the linear dependence

$$\Delta = \pm(a + bx), \quad (2.1)$$

where  $x$  is the nominal value of the measure, the indication of a measuring instrument, or the signal at the input of a measuring transducer, and  $a$  and  $b$  are constants, or by a different equation,

$$\Delta = f(x).$$

When the latter dependence is complicated, it is given in the form of a table or graph.

The fiducial error  $\gamma$  (in percent) is defined by the formula

$$\gamma = 100\Delta/x_N,$$

where  $x_N$  is the fiducial value.

The fiducial value is assumed to be equal to the following:

- (i) The value at the end of the instrument scale, if the zero marker falls on the edge or off the scale.

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- (ii) The span that is a sum of the end values of the instrument scale (neglecting the signs), if the zero marker falls within the scale.
- (iii) The nominal value of the measurand, if it has been established.
- (iv) The length of the scale, if the scale graduations narrow sharply toward the end of the scale. In this case, the error and the length of the scale are expressed in the same units.

For instruments having a scale that is calibrated in units of a quantity for which a scale with a conventional zero is adopted (for example, in degrees Celsius) the *fiducial value* is assumed to be equal to the difference of the final and starting values of the scale (the measurement range or span).

According to Recommendation 34 of OIML [9], for measuring instruments with a zero marker within the scale, the fiducial value is taken to be equal to the larger (neglecting the sign) of the end values of the indication range of the instrument. According to Publication 51 of IEC [8], for electrical measuring instruments, it can be set equal to the sum of the end values of the scale.

A progressive and correct solution is the one recommended by OIML. Indeed, consider, for example, an ammeter with a scale 100–0–100 A and with a permissible absolute error of 1 A. In this case, the fiducial error of the instrument will be 1% according to OIML and 0.5% according to IEC. But when using this instrument, the possibility of performing a measurement with an error of up to 0.5% cannot be guaranteed for any point of the scale. An error not exceeding 1%, however, can be guaranteed when measuring a current of 100 A under reference conditions. The tendency to choose a fiducial value such that the fiducial error would be close to the relative error of the instrument was observed in the process of improving IEC Publication 51. Thus, in the previous edition of this publication, the fiducial value for instruments without a zeromarker on the scale was taken to be equal to the difference of the end values of the range of the scale, and now it is taken to be equal to the larger of these values (neglecting the sign). Consider, for example, a frequency meter with a scale 45–50–55 Hz and a limit of permissible absolute error of 0.1 Hz. Previously, the fiducial error of the frequency meter was assumed to be equal to 1%, and now it is equal to 0.2%. But when measuring a 50-Hz frequency, its relative error indeed will not exceed 0.2% (under reference conditions), and the 1% error has no relation to the error of frequency measurement with this meter, so that the new edition is more correct.

The next step in this direction was made in Recommendation 34 of OIML. One must hope that in the future IEC will take into account the recommendation of OIML, and the stipulation mentioned regarding electrical measurement instruments in the recommendation of OIML will disappear.

The fiducial error is expressed in percent, but it is not a relative error. As its limit is equal to that of the permissible relative error, the limit of the permissible relative error for each value of the measurand must be calculated according to the formula

$$\delta = \gamma \frac{x_N}{x}.$$

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The limit of permissible relative error  $\delta$  is usually expressed in percent according to the formula

$$\delta = \frac{100\Delta}{x} = \pm c.$$

If the limit of the absolute error  $\Delta$  is determined by formula (2.1), then the last expression is possible for  $a \approx 0$ .

For digital instruments, the errors are often standardized in the conventional form  $\pm(b + q)$ , where  $b$  is the relative error in percent and  $q$  is some number of figures of the least significant digit of the digital readout device. For example, an instrument with a measurement range of 0–300 mV is assigned the limits of permissible error  $\pm(0.5\% + 2)$ . The indicator of the instrument has four digits, so that the figure 2 of the least significant digit corresponds to 0.2 mV. Now the limit of the relative error of the instrument when measuring, for example, a voltage of 300 mV can be calculated as follows:

$$\delta = \pm \left( 0.5 + \frac{0.2 \times 100}{300} \right) = \pm 0.57\%.$$

Thus, to estimate the limit of permissible error of an instruments, some calculations must be performed. For this reason, although the conventional form gives a clear representation of the components of instrument error, it is inconvenient to use.

A more convenient form is given in Recommendation 34 of OIML: The limit of permissible relative error is expressed by the formula

$$\delta = \pm \left[ c + d \left( \frac{x_e}{x} - 1 \right) \right], \quad (2.2)$$

where  $x_e$  is the end value of the measurement range of the instrument or the input signal of a transducer and  $c$  and  $d$  are relative quantities.

With the adopted form of formula (2.2), the first term on the right-hand side is the relative error of the instrument at  $x = x_e$ . The second term in this expression characterizes the increase of the relative error as the indications of the instrument decrease.

Formula (2.2) can be obtained from  $\pm(b + q)$  as follows. To the figure  $q$ , there corresponds the measurand  $qD$ , where  $D$  is the value of one figure in the same digit as the figure  $q$ , in units of the measurand. In the relative form, it is equal to  $qD/x$ . Now the sum of the terms  $b$  and  $qD/x$  has the following physical meaning: It is the limit of permissible relative error of the instrument.

So

$$\delta = \left( b + \frac{qD}{x} \right).$$

With the help of identity transformation, we obtain

$$\delta = b + \frac{qD}{x} + \frac{qD}{x_e} - \frac{qD}{x_e} = \left( b + \frac{qD}{x_e} \right) + \frac{qD}{x_e} \left( \frac{x_e}{x} - 1 \right).$$

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Writing

$$c = b + \frac{qD}{x_e}, \quad d = \frac{qD}{x_e},$$

we obtain formula (2.2).

In application to the example of a digital millivoltmeter studied above, we have

$$\delta = \pm \left[ 0.57 + 0.07 \left( \frac{x_e}{x} - 1 \right) \right].$$

It is clear that the last expression is more convenient to use, and in general, it is more informative than the conventional expression.

Note that for standardization, the error limits are established for the total instrument error and not for the separate components. If, however, the instrument has an appreciable random component, then a permissible limit is established separately for it. For example, aside from the limit of the permissible intrinsic error, the limit of the permissible dead band or hysteresis is also established. Sometimes, however, the limits are nonetheless set separately for the systematic and random components. For example, the error of reference standards is customarily given in this manner in Russia.

Additional errors of measuring instruments are standardized by prescribing the limits for each additional error separately. The intervals of variation of the corresponding influence quantities are indicated simultaneously with the limits of the additional errors. The collection of ranges provided for all influence quantities determines the normal operating conditions of the measuring instrument. The limit of permissible additional error is often represented in proportion to the value of the influence quantity or its deviation from the limits of the interval determining the standard values of these quantities. In this case, the corresponding coefficients are standardized. We shall call it the *influence coefficient*.

In the case of measuring instruments the term *variation of indications* is used as well as the term *additional error*. The term variation of indications is used, in particular, for electric measuring instruments [8].

The additional errors arising when the influence quantities are fixed are systematic errors. For different instruments of the same type, however, they can have different values and, what is more, different signs. For this reason, in the overwhelming majority of standards, the limits of additional errors are set both positive and negative with equal numerical values. For example, the change in the indications of an electric measuring instrument caused by a change in the temperature of the surrounding medium should not exceed the limits  $\pm 0.5\%$  for each  $10^\circ\text{C}$  change in temperature under normal operating conditions (the numbers here are arbitrary).

If, however, the properties of a measuring device are sufficiently uniform, it is best to standardize the influence function, i.e., to indicate the dependence of the indications of the instruments or output signals of the transducers on the influence quantities and the limits of permissible deviations from each such dependence. If the influence function can be standardized, then it is possible to introduce

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corrections to the indications of the instruments and thereby to use the capabilities of the instruments more fully.

It should be emphasized that the properties of only the measuring instruments are standardized with the help of the norms of the additional errors. The actual additional error that can arise in a measurement will depend not only on the properties of the measuring instrument but also on the value of the corresponding influence quantity.

Often a measuring instrument has an electrical signal on its input. This input signal can be characterized by several parameters. One of them reflects the value of a measurand. This parameter is called the *informative parameter*: By measuring its value, we can find the value of the measurand. All other parameters do not have direct connections with the value of the measurand, and they are called *noninformative parameters*.

Measuring instruments are constructed to make them insensitive to all noninformative parameters of the input signal. This result, however, cannot be achieved completely, and in the general case, the effect of the noninformative parameters is only decreased. Furthermore, for all noninformative parameters, it is possible to determine limits such that when the noninformative parameters vary within these limits, the total error of the measuring instrument will change insignificantly, which makes it possible to establish the reference ranges of the values of the noninformative parameters.

If some noninformative parameter falls outside the reference limits, then the error arising is regarded as an *additional error*. The effect of each non-informative parameter is standardized separately, as for influence quantities.

Standardization of the effect of the noninformative parameters and estimation of the errors arising from them are performed based on the same assumptions as those used for taking into account the additional errors caused by the external influence quantities.

The errors introduced by changes in the noninformative parameters of the input signals are occasionally called *dynamic errors*. In the case of several parameters, however, little information is provided. It is more informative to give each error a characteristic name, as is usually done in electric and radio measurements. For example, the change produced in the indications of an ac voltmeter by changes in the frequency of the input signal is called the frequency error. In the case of a voltmeter, for measurements of the peak variable voltages, apart from the frequency errors, the errors caused by changes in the widths of the pulse edges, the decay of the flat part of the pulse, and so on, are taken into account.

The errors caused by deviations of the noninformative parameters of the input signal from the standard values should also be included among the additional errors of a measuring instrument. For example, for a voltmeter in an electromagnetic system, the frequency of the alternating current is one noninformative parameter of the signal. According to Section 1.3, these errors are because one or more parameters of the model do not correspond to the properties of the real object. As these errors are characteristic for the measuring instruments in which they are observed, they are usually given the name of the corresponding parameter of the



### 2.3. Standardization of the Metrological Characteristics 45

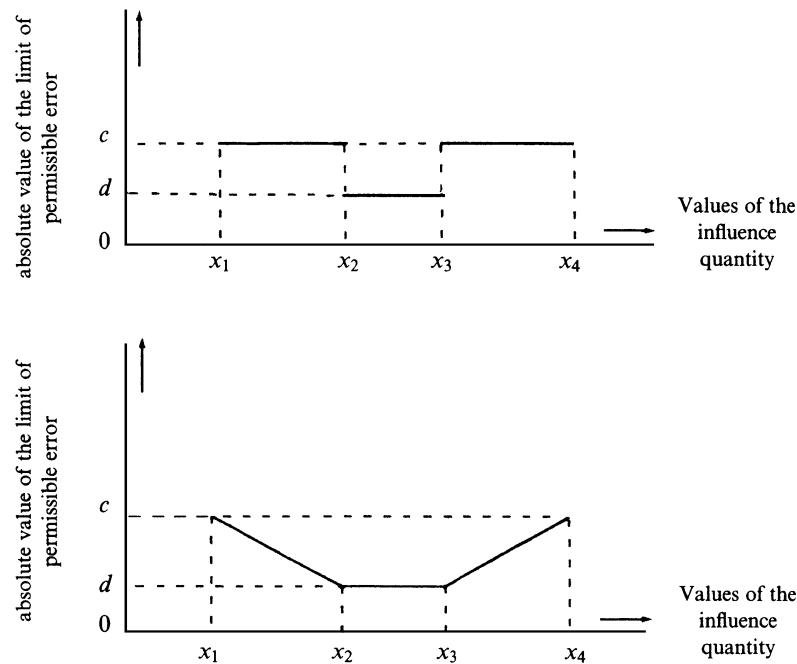


FIGURE 2.3. Variants of standardization of the limits of additional errors of measuring instruments. The interval  $(x_3 - x_2)$  corresponds to *reference conditions*; the interval  $(x_4 - x_1)$  corresponds to the *normal operating conditions*;  $d$  is the limit of permissible intrinsic error;  $c$  is the limit of permissible error in the *normal operating conditions*; and  $(c - d)$  is the limit of permissible additional error.

model. Thus, in the foregoing example, the model of the signal is a sinusoidal voltage with a fixed parameter (frequency). The corresponding error is called the frequency error.

The basic cases of standardization of additional errors are shown in Fig. 2.3.

*Stability of measuring instruments.* Stability, like accuracy, is a positive quality of a measuring instrument. Just as the accuracy is characterized by inaccuracy (error, uncertainty), stability is characterized by instability.

Instability is standardized by the value of the limits of permissible variations of the error over a definite period of time or by prescribing different error limits to different “lifetimes” of the instrument after it is calibrated. In addition, limits are sometimes prescribed for the drift of the indications of the instrument; these limits, naturally, are indicated together with the time. It is desirable to standardize drift for the automatic-plotting instruments. But standards for the drift are also helpful for other types of measurement instruments, because it makes it possible to judge how often the indications or the zeros of the instruments must be corrected.

To correct the indications of electric measuring instruments, standard cells or electronic voltage stabilizers are often built into them. For example, weak sources

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of stable radioactivity are built into meters for measuring the parameters of radioactive radiations.

It is significant that separate standardization of the drift does not change the standards for the instrumental error; i.e., it gives additional information about the properties of the instruments.

The second method of standardization of instability consists of indicating different standards for the error of the instrument for different periods of time after the instrument is calibrated. For example, a table with the following data is provided in the specifications of some digital instrument:

Time after calibration	24 hour	3 month	1 year	2 years
Temperature	$23 \pm 1^\circ\text{C}$	$23 \pm 5^\circ\text{C}$	$23 \pm 5^\circ\text{C}$	$23 \pm 5^\circ\text{C}$
Limit of error	$0.01\% + 1 \text{ digit}$	$0.015\% + 1 \text{ digit}$	$0.02\% + 1 \text{ digit}$	$0.03\% + 2 \text{ digit}$

The limits of error are presented here in the conventional form.

The first method for standardizing instability of instruments is widely used in Russia, and the second method is widely used in the United States. The second method reveals more fully the capabilities of instruments. For example, the limits of error of a digital instrument manufactured in Russia, with the parameters indicated in the table above, would have to be checked once per year  $\pm (0.02\% + 1 \text{ digit})$ . The maximum instrument accuracy that can be realized in a short period of time after calibration, although in a more restricted temperature regime, would remain unknown.

Standardization predetermines the properties of measuring instruments and is closely related with the concept of accuracy classes of measuring instruments.

Accuracy classes were initially introduced for indicating electric measuring instruments [8]. Later this concept was also extended to all other types of measuring instruments [9]. Unification of the accuracy requirements of measuring instruments, the methods for determining them, and the notation in general are certainly useful to both the manufacturers of measuring instruments and to users, because it makes it possible to limit, without harming the manufacturers or the users, the list of instruments, and it makes it easier to use and check the instruments. We shall discuss this concept in greater detail.

In [2], the following definition is given for the term accuracy class (the following definition is that close to that given in [8]): The accuracy class is a class of measuring instruments that meet certain metrological requirements that are intended to keep errors within specified limits.

Every accuracy class has conventional notation, established by agreement—the class index—that is presented in [8] and [9].

On the whole, the accuracy class is a generalized characteristic that determines the limits for all errors and standards for all other characteristics of measuring instruments that affect the accuracy of measurements performed with their help.

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For measuring instruments whose permissible limits of intrinsic error are expressed in the form of relative or fiducial errors, the following series of numbers, which determine the limits of permissible intrinsic errors and are used for denoting the accuracy classes, was established in [9]:

$$(1, 1.5, 1.6, 2, 2.5, 3, 4, 5, \text{ and } 6) \times 10^n,$$

where  $n = +1, 0, -1, -2, \dots$ ; the numbers 1.6 and 3 can be used, but they are not recommended. For any one value of  $n$ , not more than five numbers of this series are allowed. The limit of permissible intrinsic error for each type of measuring instrument is set equal to one number in the indicated series.

Conventional designations of accuracy classes, employed in documentation accompanying measuring instruments, as well as the designations imposed on them, have been developed with the numbers in the indicated series. Of course, this process refers to measuring instruments whose errors are standardized in the form of relative and fiducial errors. Table 2.1 gives examples of the adopted designations of accuracy classes of these measuring instruments.

In those cases when the limits of permissible errors are expressed in the form of absolute errors, the accuracy classes are designated by Latin capital letters or roman numerals.

If formula (2.2) is used to determine the limit of permissible error, then both numbers  $c$  and  $d$  are introduced into the designation of the accuracy class. These numbers are selected from the series presented above, and in calculating the limits of permissible error for a specific value of  $x$ , the result is rounded off so that it would be expressed by not more than two significant figures; the roundoff error should not exceed 5% of the computed value.

The limits of all additional errors and other metrological characteristics of measuring instruments must be related with their accuracy class. In general, it is impossible to establish these relations for all types of measuring instruments simultaneously—measuring instruments are too diverse. For this reason, these relations must be given in the specifications together with the characteristics of specific types of measuring instruments, which the designers formulate.

TABLE 2.1. Designations of accuracy classes.

Form of the expression for the error	Limit of permissible error (examples)	Designation of the accuracy class (for the given example)
Fiducial error, if the fiducial value is expressed in units of the measurand	$\gamma = \pm 1.5\%$	1.5
Fiducial error, if the fiducial value corresponds to the span	$\gamma = \pm 0.5\%$	0.5
Relative error, constant	$\delta = \pm 0.5\%$	$\textcircled{0.5}$
Relative error, increasing as the measurand decreases	$\delta = \pm \left[ 0.02 + 0.01 \left( \frac{x_e}{x} - 1 \right) \right] \%$	0.02/0.01

## 2.4. Some Suggestions for Changing Methods of Standardization of Errors of Measuring Instruments and Their Analysis

Standardization, i.e., establishment of standards, is basically a volitional act. For this reason, in principle, different suggestions can be made for solving this question, and in the last few years, several new methods for expressing the errors of measuring instruments and for standardizing them have indeed been proposed.

To evaluate these suggestions, it is necessary to determine how well they solve problems for whose sake the properties of measuring instruments are standardized. From what we have said above, it can be concluded that the purpose of standardization of errors of measuring instruments is to solve the following problems:

- (1) To ensure that the entire collection of measuring instruments of the same type have the required accuracy and to ensure that they are uniform and interchangeable.
- (2) To make sure that it is possible to evaluate the instrumental measurement errors according to established standards for metrological properties of measuring instruments.
- (3) To ensure that measuring instruments can be compared with one another according to accuracy.

The first problem is ultimately solved by monitoring new measuring instruments during the manufacturing process and checking periodically the units that are in use. As measuring instruments are employed individually, the standards must be established so that it is possible to check that each sample measuring instrument satisfies these standards.

To solve the second problem successfully, it is desirable to know accurately the properties of measuring instruments. For this reason, the established standards must be as close as possible to the real properties of the measuring instruments. The degree of detail with which the errors of measuring instruments can be described is limited by the instability of the instruments, by the change in their errors in time, as well as by the degree of nonuniformity of the measuring instruments introduced by their construction and manufacturing technology. In addition, the calibration process must be simple. Complicated methods for describing and standardizing the errors of measuring instruments, which lead to laborious and prolonged checks, are nonviable.

Having made these preliminary remarks, we shall now examine the most interesting suggestions.

- (1) The calculation of the errors of measuring instruments under real conditions involves summation of the errors and presents several difficulties. For this reason, it has been repeatedly suggested that the reference conditions be extended to absorb all possible values of the influence quantities. One would think that in so doing

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the additional errors of measuring instruments would vanish and only the intrinsic error would remain, and all difficulties would be simply resolved.

The actual properties of measuring instruments, however, do not depend on the method by which they are standardized, and they remain unchanged. Suppose that in the usual method of standardization, we have the following:

- $\Delta_0$ , the limit of permissible intrinsic error; and
- $\Delta_i$ , the limit of permissible additional error, caused by the change in the  $i$ th influence quantity from the standard value to the limit of the range of the given influence quantity ( $i = 1, \dots, n$ ) for normal operating conditions.

By transferring to a new method of standardization, the manufacturer of the instruments can adopt as the limit of permissible error of the measuring instrument only the arithmetic sum

$$\Delta = \Delta_0 + \sum_{i=1}^n \Delta_i.$$

The manufacturer cannot proceed otherwise, because he or she must guarantee that the errors of a given measuring instrument will be less than  $\Delta$  for any combination of limiting values of the influence quantities. What then can this suggestion give?

From the standpoint of evaluating the measurement errors, it can significantly simplify the procedure. But in exchange, the error is significantly overestimated because under the actual operating conditions of the measuring instrument in the overwhelming majority of the cases, the influence quantities do not all reach their limiting values simultaneously and in the most unfavorable combination. For this reason, even the arithmetic sum of the errors occurring in a specific measurement will be less than  $\Delta$  and closer to the real value.

With respect to uniformity and interchangeability of measuring instruments of the same type, the suggestion worsens the existing situation, because the same value of  $\Delta$  can be obtained for different values of the components. Thus, to adopt this suggestion means taking a step backward compared with the present situation.

The foregoing analysis also shows that the definition of reference conditions given in [7] gives the most complete disclosure of the properties of a measuring instrument. In the overwhelming majority of the cases, however, before a measuring instrument can be developed, it is necessary to establish the technical requirements that it must meet. In the process, the reference conditions and the permissible limits of the intrinsic error are determined. During the design process, the investigators and designers strive to satisfy these requirements within some margin. Normally this result is possible, which essentially means that the reference conditions established earlier can be defined more stringently. But if this path is followed, then the reference conditions would have to be redetermined after the sample measuring instruments have been built, and it would be found that they are diverse for different types of measuring instruments, which would

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create great difficulties for technical monitoring services and calibrating laboratories. For this reason, on the whole, the reference conditions are best determined by agreement between specialists, and these conditions should be unified as much as possible for different types of measuring instruments. When developing measuring instruments, however, it should be kept in mind that if the intrinsic error is appreciably correlated with one or another influence quantity, then the real properties of the measuring instruments are not completely disclosed by the prescribed standards.

(2) It has been suggested that the integral accuracy index  $I$ , calculated according to the formula

$$I = \sqrt{\sum_{i=0}^n \varepsilon_i^2},$$

where  $\varepsilon_i$  is the limit of additional error determined by the  $i$ th influence quantity and  $\varepsilon_0$  is the limit of intrinsic error, be standardized.

It is clear that one and the same value of  $I$  can be obtained for different values of the components. For example, one instrument can have a large temperature error and a small frequency error, whereas the opposite could be true for a different instrument. Ultimately, replacing one instrument by another (of the same type) results in a large error, and this error cannot be estimated beforehand. Therefore, it becomes more difficult to estimate the measurement errors. In addition, uniformity of measuring instruments is not achieved. The conclusion is obvious: The suggestion is not acceptable.

(3) Another suggestion was to characterize the accuracy of instruments by the weighted mean of the permissible relative error, determined according to the formula

$$\delta_c = \int_{x_i}^{x_f} [\varepsilon(x)f(x)]dx,$$

where  $x_i$  and  $x_f$  are the initial and final (upper) values of the instrument scale,  $\varepsilon(x)$  is the relative error of the instrument, and  $f(x)$  is the probability distribution of the indications of the instrument.

This suggestion has the drawback that the probability distribution of the indications of instruments is, in general, unknown. More importantly, however, this weighted-mean characteristic, as any other average characteristic, is completely unsuitable for standardizing the properties of measuring instruments, because uniformity of measuring instruments cannot be achieved in this manner. For example, an instrument that has one or two significant error components, and for which other errors are small, can have the same weighted-mean error as an instrument whose errors are approximately the same.

In addition, when using an instrument whose errors are standardized as weighted means, experimenters cannot estimate the error of a specific result they have obtained, because in this method of standardization, the error of the instrument with a fixed indication can in principle be virtually arbitrarily large.

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Thus, none of the goals of standardization is achieved with this method of standardization of errors of measuring instruments and this method cannot be used.

(4) It has been repeatedly suggested that the additional error caused by the simultaneous action of all influence quantities be standardized. It can be conjectured that some of them will mutually compensate one another so that it will be possible to use the instruments more fully or, vice versa, the error will be larger than in the case when each influence quantity acts separately.

In practice, however, normally not all influence quantities assume their worst (for us) values simultaneously, and it is impossible to take into account only some influence quantities by standardizing in this manner. Instead of lowering the estimate of the measurement error or increasing its accuracy, the measurement error will increase and it will not be estimated as accurately. In addition, the testing equipment would become much more complicated.

In reality, some additional errors can be correlated with one another, and it would be correct to determine these cases and to standardize the correlation of the corresponding errors. However, I have never encountered in practice such cases of standardization of additional errors. There does not appear to be any need to do so. However, this question deserves a detailed study.

(5) In the former USSR, in 1972, a standard that decisively changed the practice of standardization of errors of measuring instruments was adopted. This standard greatly complicates the standardization of errors of measuring instruments and contains wholly unrealistic requirements. They include, for example, the requirement that the mathematical expectation and the rms deviation of the systematic component of the errors of measuring instruments of each type be standardized. These characteristics must be estimated according to the formulas

$$\bar{x} = \frac{\sum_{i=1}^m x_i}{m}, \quad S = \sqrt{\frac{\sum_{i=1}^m (x_i - \bar{x})^2}{m - 1}},$$

where  $m$  is the number of instruments in a batch and  $x_i$  is the systematic error of the  $i$ th instrument.

Estimates of these parameters can be calculated by checking instruments in a batch. Let us assume that they satisfy the standards. Does this mean that the systematic error is sufficiently small for all instruments in the batch? Obviously not. In exactly the same way, nothing can be said about a separate instrument, if these estimates do not satisfy the standards. According to these standards, an instrument cannot be rejected, and it cannot be judged satisfactory in the case of verification. Of course, once estimates have been found for a batch of instruments, then the entire batch of instruments can either be discarded or accepted. But this action is absurd: A bad batch can contain several good instruments, and they should not be discarded. Conversely, it is absurd to pass as satisfactory some unsatisfactory instruments simply because they are contained in the batch that has been found to satisfy the standards. Every instrument is used individually, and the standards must

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make it possible to determine whether the instrument is good or bad. Parameters pertaining to batches do not meet this requirement, and for this reason, they cannot be used as standards for measuring instruments.

I opposed the adoption of this standard, but the standard was adopted (GOST 8.009-72). In 1984, a new edition of this standard was published. Now the characteristics that we studied above are no longer obligatory, and this eases somewhat the situation of instrument manufacturers: They do not have to standardize these characteristics, and this will not be a violation of the standard.

In conclusion, we shall formulate the basic rules for standardization of errors of measuring instruments:

- (i) all properties of a measuring instrument that affect the accuracy of the results of measurements must be standardized;
- (ii) every property that is to be standardized should be standardized separately;
- (iii) methods of standardization must make it possible to check experimentally, and as simply as possible, how well each sample of a measuring instrument corresponds to the established standards; and
- (iv) the standardization must be performed so that measuring instruments can be chosen based on the established standards and so that the measurement error can be estimated.

In some cases, exceptions must be made to these rules. Such an exception is necessary for strip strain gauges that can be glued on an object only once. For this reason, the strain gauges that are checked can no longer be used for measurements, whereas the gauges that are used for measurements usually cannot be checked or calibrated. In this case, it is necessary to resort to regulation of the properties of a collection of strain gauges, such as, for example, the standard deviation of the sensitivity and mathematical expectation of the sensitivity. The sensitivity of a separate strain gauge, which is essentially not a random quantity, is a random quantity in application to a collection of strain gauges. Once the sensitivity  $x_i$  of every strain gauge chosen at random from a batch (sample) has been determined, it is possible to construct a statistical tolerance interval, i.e., the interval into which the sensitivity of a prescribed fraction  $p$  of the entire collection of strain gauges will fall with a chosen probability  $a$ . As  $a \neq 1$  and  $p \neq 1$ , there is a probability that the sensitivity of any given strain gauge falls outside these tolerance limits. For this reason, the user must take special measures that exclude such a case. In particular, several strain gauges, rather than one, should be used.

## 2.5. Dynamic Characteristics of Measuring Instruments and Their Standardization

The dynamic characteristics of measuring instruments reflect the relation between the change in the output signal and one or another action that produces this change. The most important action is a change in the input signal. In this case, the dynamic characteristic is called the dynamic characteristic for the input signal. Dynamic



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characteristics for one or another influence quantity and for a load (for measuring instruments whose output signal is an electric current or voltage) are also studied.

Complete and partial dynamic characteristics are distinguished [28].

The complete dynamic characteristics determine uniquely the change in time of the output signal caused by a change in the input signal or other action. Examples of such characteristics are a differential equation, transfer function, amplitude–and phase–frequency response, the transient response, and the impulse characteristic. These characteristics are essentially equivalent, but the differential equation is still the source characteristic.

A partial dynamic characteristic is a parameter of the full dynamic characteristic or a functional of it. Examples are the response time of the indications of an instrument and the transmission band of a measuring amplifier.

Measuring instruments can most often be regarded as inertial systems of first or second order. If  $x(t)$  is the signal at the input of a measuring instrument and  $y(t)$  is the corresponding signal at the output, then the relation between them can be expressed with the help of first-order (2.3) or second-order (2.4) differential equations, respectively, which reflect the dynamic properties of the measuring instrument:

$$Ty'(t) + y(t) = Kx(t), \quad (2.3)$$

$$\frac{1}{\omega_0^2}y''(t) + \frac{2\beta}{\omega_0}y'(t) + y(t) = Kx(t). \quad (2.4)$$

The parameters of these equations have specific names:  $T$  is the time constant of a first-order device,  $K$  is the transduction coefficient in the static state,  $\omega_0$  is the angular frequency of free oscillations, and  $\beta$  is the damping ratio.

Equations (2.3) and (2.4) reflect the properties of real devices, and for this reason, they have zero initial conditions: for  $t \leq 0$ ,  $x(t) = 0$  and  $y(t) = 0$ ,  $y'(t) = 0$ , and  $y''(t) = 0$ .

For definiteness, in what follows, we shall study the second-order equation and we shall assume that it describes a moving-coil galvanometer. Then  $\omega_0 = 2\pi f_0$ , where  $f_0$  is the frequency of free oscillations of the moving part of the galvanometer.

To obtain transfer functions from differential equations, it is first necessary to transfer from signals in the time domain to their Laplace transforms, and then to form their ratio. Thus

$$\begin{aligned} \mathcal{L}[x(t)] &= x(s), & \mathcal{L}[y(t)] &= y(s), \\ \mathcal{L}[y'(t)] &= sy(s), & \mathcal{L}[y''(t)] &= s^2y(s), \end{aligned}$$

where  $s$  is the Laplace operator.

For the first-order system, we obtain

$$W(s) = \frac{y(s)}{x(s)} = \frac{K}{1 + sT},$$

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and for the second-order system, we obtain

$$W(s) = \frac{y(s)}{x(s)} = \frac{K}{(1/\omega_0^2)s^2 + (2\beta/\omega_0)s + 1}. \quad (2.5)$$

If in the transfer function the operator  $s$  is replaced by the complex frequency  $j\omega$  ( $s = j\omega$ ), then we obtain the complex frequency response. We shall study the relation between the named characteristics for the example of a second-order system. From (2.4) and (2.5), we obtain

$$W(j\omega) = \frac{K}{(1 - \omega^2/\omega_0^2) + j2\beta\omega/\omega_0}, \quad (2.6)$$

where  $\omega = 2\pi f$  is the running angular frequency.

The complex frequency response is often represented for its real and imaginary parts,

$$W(j\omega) = P(\omega) + jQ(\omega).$$

In our case,

$$P(\omega) = \frac{K(1 - (\omega^2/\omega_0^2))}{(1 - (\omega^2/\omega_0^2))^2 + 4\beta^2(\omega^2/\omega_0^2)},$$

$$Q(\omega) = \frac{2\beta(\omega/\omega_0)K}{(1 - (\omega^2/\omega_0^2))^2 + 4\beta^2(\omega^2/\omega_0^2)}.$$

The complex frequency response can also be represented in the form

$$W(j\omega) = A(\omega)e^{j\varphi(\omega)},$$

where  $A(\omega)$  is the amplitude-frequency response and  $\varphi(\omega)$  is the frequency response of phase. In the case at hand

$$A(\omega) = \sqrt{P^2(\omega) + Q^2(\omega)} = \frac{K}{\sqrt{(1 - (\omega^2/\omega_0^2))^2 + 4\beta^2(\omega^2/\omega_0^2)}}, \quad (2.7)$$

$$\varphi(\omega) = \arctan \frac{Q(\omega)}{P(\omega)} = -\arctan \frac{2\beta(\omega/\omega_0)}{1 - (\omega^2/\omega_0^2)}.$$

Equations (2.7) have a well-known graphical interpretation.

The transient response is the function  $h(t)$  representing the output signal produced by a unit step function  $1(t)$  at the input. We recall that the unit step function is a function  $x(t)$  satisfying the following conditions:  $x(t) = 0$  for  $t < 0$  and  $x(t) = 1$  for  $t \geq 0$ . As the input is not periodic,  $h(t)$  is calculated with (2.3) or (2.4). Omitting the simple but, unfortunately, complicated calculations, we arrive at the final

2.5. Dynamic Characteristics of Measuring Instruments 55

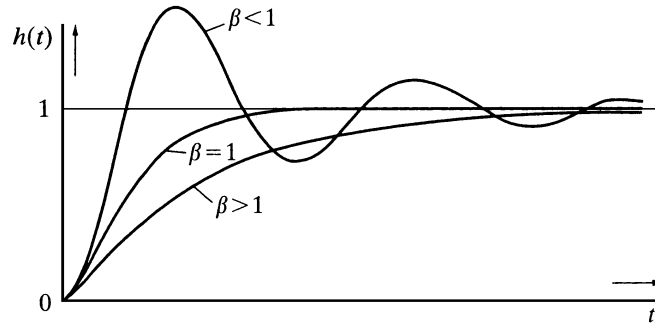


FIGURE 2.4. The transient response of an instrument described by a second-order differential equation;  $\beta$  is the damping ratio.

form of the transient response of the instrument under study:

$$h(t) = \begin{cases} 1 - e^{-\beta\tau} \frac{1}{\sqrt{1-\beta^2}} \sin \left( \tau \sqrt{1-\beta^2} + \arctan \frac{\sqrt{1-\beta^2}}{\beta} \right) & \text{if } \beta < 1, \\ 1 - e^{-\tau}(\tau + 1) & \text{if } \beta = 1, \\ 1 - e^{-\beta\tau} \frac{1}{\sqrt{\beta^2-1}} \sinh \left( \tau \sqrt{\beta^2-1} + \operatorname{arctanh} \frac{\sqrt{\beta^2-1}}{\beta} \right) & \text{if } \beta > 1. \end{cases}$$

Here  $\tau = \omega_0 t$  and the steady-state value of the output signal is taken to be equal to unity, i.e.,  $h(t) = y(t)/K$ . Thanks to this condition, the formulas above and the graphs corresponding to them, presented in Fig. 2.4, are universal in the sense that they do not depend on the specific values of  $\omega_0$  and  $K$ .

The impulse characteristic  $g(t)$  is found from the transient response in accordance with its definition:

$$g(t) = \frac{dh(t)}{dt}.$$

It should be noted that some types of measuring instruments do not have dynamic characteristics at all: measures of length, weights, vernier calipers, and so on. Some measuring instruments, such as measuring capacitors (measures of capacitance), do not have an independent dynamic characteristic. But when they are connected into an electric circuit, which always has some resistance and sometimes an inductance, the circuit always acquires, together with a capacitance, definite dynamic properties.

Measuring instruments are diverse. Occasionally, to describe adequately their dynamic properties, it is necessary to resort to linear equations of a higher order, nonlinear equations, or equations with distributed parameters. However, complicated equations are used rarely, which is not an accident. After all, measuring instruments are created specially to perform measurements, and their dynamic properties are made to guarantee convenience of use. For example, in designing an automatic plotting instrument, the transient response is made to be short,

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approaching the established level monotonically or oscillating insignificantly. In addition, the instrument scale is made to be linear. But when these requirements are met, the dynamic properties of the instrument can be described by one characteristic corresponding to a linear differential equation of order no higher than second.

A differential equation of high order is most often obtained when synthesizing the dynamic characteristics of an instrument based on the dynamic characteristics of its subunits. Thus, for example, calculating the dynamic characteristic of a galvanometric amplifier with a photoelectric converter that converts the angle of rotation of the moving part of the galvanometer into a voltage (current), we formally obtain an equation of third order: The galvanometer gives two orders, and the photoelectric converter gives one order. Such a description of the properties of the amplifier is necessary at the design stage, because otherwise it is impossible to understand why self-excited oscillations sometimes arise in the system. But when the design is completed and reasonable parameters of the subunits are chosen, it is desirable to simplify the description of the dynamic properties. Thus, the amplifier must be regarded as a black box. Analyzing the relation between the input and output, we find that it is described well by a second-order equation. The same result can also be obtained informally, as done, for example, in [42]. Decomposing the dynamic characteristics of all subunits of the amplifier into first-order characteristics and comparing them, we can see that one can be neglected.

Standardization of the dynamic characteristics of measuring instruments is performed for a specific type of instrument. The problem is solved in two stages. First, an appropriate dynamic characteristic must be chosen, after which the nominal dynamic characteristic and the permissible deviations from it must be established. Thus, for recording instruments and universal measuring transducers, one complete dynamic characteristic must be standardized: Without having the complete dynamic characteristic, a user cannot effectively use these instruments.

For indicating instruments, it is sufficient to standardize the response time. In contrast to the complete characteristics, this characteristic is a partial dynamic characteristic. The dynamic error is another form of a partial dynamic characteristic. Standardization of the limits of a permissible dynamic error is convenient for the measuring instruments employed, but it is justified only when the form of the input signals does not change much.

For measuring instruments described by linear first- and second-order differential equations, the coefficients of all terms in the equations can be standardized. In the simplest cases, the time constant is standardized in the case of a first-order differential equation, and the natural frequency and the damping ratio of the oscillations are standardized in the case of a second-order differential equation.

When imposing requirements on the properties of measuring instruments, it is always necessary to keep in mind how compliance will be checked. For dynamic characteristics, the basic difficulties are connected with creating test signals of predetermined (with sufficient accuracy) form, or with recording the input signal with a dynamically more accurate measuring instrument than the measuring instrument whose dynamic properties are being checked.

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If test signals with adequate accuracy can be created and the dynamic characteristic is found with the help of the corresponding signal, i.e., a transient response as a response of a unit step function signal and frequency response as a response of a sinusoidal test signal, then in principle the obtained experimental data can be processed without any difficulties.

But sometimes the problem must be solved with a test signal that does not correspond to the signal intended for determining the complete dynamic characteristic. For example, one would think that the problem can be solved given the tracing of signals at the input and output of a measuring instrument. In this case, however, special difficulties arise because small errors in recording the test signal and reading the values of the input and output signals often lead to the fact that the dynamic characteristic obtained based on them do not correspond to the dynamic properties of the measuring instrument and are physically meaningless. Such an unexpected effect is explained because the problem at hand is a so-called improperly posed problem. A great deal of attention is currently being devoted to such problems in mathematics, automatics, geophysics, and other disciplines. Improperly posed problems are solved by the methods of regularization, which essentially consist of the fact that the necessary degree of filtering (smoothing) of the obtained solution is determined based on a priori information about the true solution.

Improperly posed problems in dynamics in application to measurement engineering are reviewed in [28].

A separate problem, which is important for some fields of measurement, is the determination of the dynamic properties of measuring instruments directly when the instruments are being used. An especially important question here is the question of the effect of random noise on the accuracy with which the dynamic characteristics are determined.

This section, then, has been a brief review of the basic aspects of the problem of standardizing and determining the dynamic properties of measuring instruments.

## 2.6. Statistical Analysis of the Errors of Measuring Instruments Based on Data Provided by Calibration Laboratories

A general characteristic of the errors of the entire population of measuring instruments of a specific type could be their distribution function. I made an attempt to find such functions for several types of measuring instruments. The results of these investigations, which were performed together with T.L. Yakovleva, were published in [43] and [53].

The errors of measuring instruments are determined by calibration, and a decision was made to use the data provided by calibration laboratories. Because it is impossible to obtain the errors of all instruments of a given type that are in use, the use of a sampling method is unavoidable.

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TABLE 2.2. The example of main statistical characteristics of errors for six types of measuring instruments.

Type of measuring instrument	Year of calibration	Point of check	Sample size	Moment		Coefficient	
				First	Second central	Skewness	Excess
Э59 ammeter	1974	80 Divisions	160	0.163	0.0074	-0.40	0.56
	1976		160	0.180	0.042	-1.33	4.27
Э59 voltmeter	1974	150 Divisions	120	0.050	0.063	-0.47	-0.29
	1976		108	0.055	0.065	-0.18	0.15
Д 566 wattmeter	1974	150 Divisions	86	0.088	0.024	-0.50	-0.54
	1976		83	0.062	0.021	0.05	0.81
TH-7 thermometer	1975	100 °C	92	-0.658	0.198	0.14	-0.14
	1976		140	-0.454	0.128	0.45	1.57
Standard spring manometer	1973	9.81 kPa	250	0.158	0.012	0.55	0.54
	1976		250	0.128	0.012	0.59	-0.13
P331 resistance measure	1970	100 Ω	400	$0.33 \times 10^{-3}$	$1.6 \times 10^{-2}$	0.82	1.08
	1975		400	$0.1 \times 10^{-3}$	$1.2 \times 10^{-2}$	0.44	2.02

To establish a property of an entire group (general collection) based on a sample, the samples must be representative. Sample homogeneity is a necessary indicator of representativeness. In the case of two samples, to be sure that the samples are homogeneous, it is necessary to check the hypothesis  $H_0 : F_1 = F_2$ , where  $F_1$  and  $F_2$  are distribution functions corresponding, respectively, to the first and second samples.

The results of the check, as is well known, depend not only on the error of the measuring instrument being calibrated but also on the error of the standard. For this reason, measuring instruments that are checked with not less than a fivefold margin of accuracy were selected for analysis.

In addition, to ensure that the samples are independent, they were formed either based on data provided by calibration laboratories in different regions of the former USSR or, if data from a single laboratory were used, the data were separated by a significant time interval. The sample size was maintained approximately constant.

We shall discuss [43] first. Table 2.2 gives the basic statistical characteristics of the samples for six types of different instruments. Two samples, obtained at different times, are presented for each of them. For brevity, the data referring to only one numerical scale marker are presented. The arithmetic mean of the values obtained by continuously approaching the marker checked from both sides was taken as the value of the error. The first initial and second central moments are given in the same units in which the value of the point of checking is presented, i.e., in fractions of a scale graduation, in degrees Celsius, and so on. (in the corresponding power). Errors exceeding twice the limit of permissible error were eliminated from the analysis.

The test was made with the help of the Wilcoxon–Mann–Whitney and Siegel–Tukey criteria with a significance level  $q = 0.05$ . The technique of applying these criteria is described in Chapter 4.

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TABLE 2.3. The results of testing the hypothesis of homogeneity for samples of six types of measuring instruments.

Type of measuring instrument	Year of calibration	Point of check	Result of testing the hypothesis based on the criterion of	
			Wilcoxon–Mann–Whitney	Siegel–Tukey
Э59 ammeter	1974	30 Divisions	+	–
	1976	60	0	–
		80	0	–
		100	+	+
Э59 voltmeter	1974	70 Divisions	–	0
	1976	150	+	+
Д 566 wattmeter	1974	70 Divisions	+	+
	1976	150	+	+
TH-7 thermometer	1975	100 °C	0	–
	1976	150 °C	–	+
		200 °C	+	+
Standard spring manometer	1973	9.81 kPa	+	+
	1976			
P331 resistance measure	1970	10 kΩ	0	–
	1975	100 Ω	0	–
		10 Ω	0	–

The results of the analysis are presented in Table 2.3. Rejection of the hypothesis is indicated by a minus sign, and acceptance is indicated by a plus sign. The symbol 0 means that a test based on the given criterion was not made.

The Wilcoxon–Mann–Whitney and Siegel–Tukey criteria are substantially different: The former is based on comparing averages, and the latter is based on comparing variances. For this reason, it is not surprising that cases when the hypothesis  $H_0$  is rejected according to one criterion but accepted according to the other are encountered. The hypothesis of sample homogeneity must be rejected if even one of the criterion rejects it. Both samples were found to be homogeneous only for the Д 566 wattmeters and standard manometers. For other measuring instruments, the compared samples were often found to be nonhomogeneous. It is interesting that on one scale marker, they can be homogeneous, on another, they are inhomogeneous (Э59 voltmeters and ammeters). TH-7 thermometers had homogeneous samples in one range of measurement and nonhomogeneous in a different range. The calculations were repeated for significance levels of 0.01 and 0.1, but on the whole, the results were the same in both cases.

The results obtained show that samples of measuring instruments are frequently nonhomogeneous with respect to errors. For this reason, they cannot be used to determine the distribution function of the errors of the corresponding instruments.

The experiment described was formulated to check the temporal stability of the distribution functions of the errors, but because in the samples compared, the numbers of the instruments were not recorded and they, of course, changed, the

**Au:** This sentence is awkward and unclear. Please reword.

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result obtained has a different but no less important meaning: It indicates that they also are nonhomogeneous.

This result is indicated also by the results of [53], in which samples obtained based on data provided for 959 ammeters by four calibration laboratories in different regions of the former USSR were compared. The number of samples was equal to 150–160 everywhere. The errors were recorded at the numerical markers 30, 60, 80, and 100 graduations. The samples were assigned the numbers 1, 2, 3, and 4, and the hypotheses  $H_0 : F_1 = F_2, F_2 = F_3, F_3 = F_4$ , and  $F_4 = F_2$  were checked. The combinations of samples were arbitrary. The hypothesis testing was based on the Wilcoxon–Mann–Whitney criterion with  $q = 0.05$ . The analysis showed that we can accept the hypothesis  $H_0 : F_1 = F_2$  only, and only at the marker 100. In all other cases, the hypothesis had to be rejected.

Thus, the sample method does not permit finding the distribution function of the errors of measuring instruments. There are evidently two reasons for this result. The first reason is that the stock of instruments of each type is not constant. On the one hand, new instruments that have just been manufactured are added to it. On the other hand, in the verification, some instruments are rejected, some instruments are replaced, and others are discarded. The ratio of the numbers of old and new instruments is constantly changing. The second reason is that the errors of the instruments unavoidably change with time. Moreover, many instruments are used under different conditions, and the conditions of use affect differently the rate at which the instrumental errors change.

The temporal instability of measuring instruments raises the question of whether the errors of measuring instruments are in general sufficiently stable so that a collection of measuring instruments can be described by some distribution function. At a fixed moment in time, each type of instruments without doubt can be described by distribution function of errors. The other problem is how to find this distribution function. The simple sampling method, as we saw above, is not suitable. But even if the distribution function can be found by some complicated method, after some time, it would have to be redetermined, because the errors, and the composition of the stock of measuring instruments, change. Therefore it must be concluded that the distribution of errors of measuring instruments cannot be found based on the experimental data.

The results presented above were obtained in the former USSR, and instruments manufactured in the former USSR were studied. However, there are no grounds for expecting that instruments manufactured in other countries will have different statistical properties.





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