
Preface

The book is devoted to the mathematical description of probabilistic conditional independence structures. The topic of conditional independence, which falls within both the scope of statistics and of artificial intelligence, has been at the center of my research activity for many years – since the late 1980s. I have been primarily influenced by researchers working in the area of graphical models but I gradually realized that the concept of conditional independence is not necessarily bound to the idea of graphical description and may have a broader impact. This observation led me to an attempt to develop a non-graphical method for describing probabilistic conditional independence structures which, in my view, overcomes an inherent limitation of graphical approaches. The method of structural imsets described in this book can be viewed as an algebraic approach to the description of conditional independence structures although it remains within the framework of discrete mathematics.

The basic idea of this approach was already presented in the middle of the 1990s in a series of papers [137]. However, I was not satisfied with the original presentation of the approach for several reasons. First, the series of papers only dealt with the discrete case, which is a kind of imperfection from the point of view of statistics. Second, the main message was dimmed by unnecessary mathematical peculiarities and important ideas were perhaps not pinpointed clearly. Third, the motivation was not explained in detail. I also think that the original series of papers was difficult for researchers in the area of artificial intelligence to read because “practical” implementation aspects of the presented approach were suppressed there. Another point is that the pictorial representation of considered mathematical objects, to which researchers interested in graphical models are accustomed, was omitted.

Within the next six years, further mathematical results were achieved which amended, supplemented and gave more precision to the original idea. I have also deliberated about suitable terminology and the way to present the method of structural imsets which would be acceptable to statisticians and researchers in the area of artificial intelligence, as well as exact from the mathematical point of view. I wrote it up in my DrSc thesis [146], which became

the basis of this monograph. After finishing the thesis, I realized the potential future practical application of the method to learning graphical models and decided to emphasize this by writing an additional chapter.

Thus, the aim of this monograph is to present the method of structural imsets in its full (present) extent: the motivation; the mathematical foundations, which I tried to present in a didactic form; indication of the area of application; and open problems. The motivation is explained in the first chapter. The second chapter recalls basic concepts in the area of probabilistic conditional independence structures. The third chapter is an overview of classic graphical methods for describing conditional independence structures. The core of the method of structural imsets is presented in the next four chapters. The eighth chapter shows application of the method to learning graphical models. Open problems are gathered in the ninth chapter and necessary elementary mathematical notions are provided in the Appendix for the reader's convenience. Then the List of Notation follows. As there are many cross-references to elementary units of the text, like Lemmas, Remarks etc., they are listed with page numbers afterwards. The text is concluded by the References and the Index.

The book is intended for

- mathematicians who may be attracted by this particular application of mathematics in the area of artificial intelligence and statistics;
- researchers in statistics and informatics who may become interested in deeper understanding of the mathematical basis of the theory of (graphical) models of conditional independence structures;
- advanced PhD students in the fields of mathematics, probability, statistics, informatics and computer science who may find inspiration in the book and perhaps make some progress either by solving open problems or by applying the presented theory in practice.

In particular, I have in mind those PhD students who are thinking about an academic career. They are advised to read the book starting with the Appendix and to utilize the lists at the end of the book.

Many people deserve my thanks for help with this piece of work. In particular, I would like to thank Marie Kolářová for typing the text of the monograph in \LaTeX . As concerns expert help I am indebted to my colleagues (and former co-authors) Fero Matúš and Phil Dawid for their remarks (even for some critical ones made by Fero), various pieces of advice and pointers to the literature and discussion which helped me clarify the view on the topic of the book. I have also profited from cooperation with other colleagues: some results presented here were achieved with the help of computer programs written by Pavel Boček, Remco Bouckaert, Tomáš Kočka, Martin Volf and Jiří Vomlel. Moreover, I am indebted to my colleagues Radim Jiroušek, Otakar Kríž and Jiřina Vejnarová for their encouragement in writing my DrSc thesis, which was quite important for me. The cooperation with all of my colleagues mentioned above involved joint theoretical research as well. A preliminary version of the

book was read by my PhD student Petr Šimeček, who gave me several useful comments and recommendations including an important example. I also made minor changes in response to comments given by Tomáš Kroupa and Helen Armstrong, who read some parts of the manuscript. As concerns the technical help I would like to thank Václav Kellar for making special L^AT_EX fonts for me and to Jarmila Pánková for helping me to prepare several pages with special pictures. I am likewise grateful to Cheri Dohnal and Antonín Otáhal for correcting my (errors in) English. I was very pleased by the positive attitude of Stephanie Harding, who is the Mathematics and Statistics Editor at Springer London; the cooperation with her was smooth and effective. She found suitable reviewers for the book and they gave me further useful comments, which helped me to improve the quality of the book.

I am also indebted to other colleagues all over the world whose papers, theses and books inspired me somehow in connection with this monograph. In particular, I would like to mention my PhD supervisor, Albert Perez. However, many other colleagues influenced me in addition to those who were already mentioned above. I will name some of them here: Steen Andersson, Luis de Campos, Max Chickering, Robert Cowell, David Cox, Morten Frydenberg, Dan Geiger, Tomáš Havránek, Jan Koster, Ivan Kramosil, Steffen Lauritzen, Franco Malvestuto, Michel Mouchart, Chris Meek, Azaria Paz, Judea Pearl, Michael Perlman, Jean-Marie Rolin, Thomas Richardson, Jim Smith, Glenn Shafer, Prakash Shenoy, David Spiegelhalter, Peter Spirtes, Wolfgang Spohn, Nanny Wermuth, Joe Whittaker, Raymond Yeung and Zhen Zhang. Of course, the above list is not exhaustive; I apologize to anyone whose name may have been omitted.

Let me emphasize that I profited from meeting several colleagues who gave me inspiration during the seminar, “Conditional Independence Structures”, which was held from September 27 to October 17, 1999 in the Fields Institute for Research in Mathematical Sciences, University of Toronto, Canada, and during several events organized within the framework of the ESF program, “Highly Structured Stochastic Systems” in the years 1997–2000. In particular, I wish to thank Helène Massam and Steffen Lauritzen, who gave me a chance to participate actively in these wonderful events. For example, I remember the stimulating atmosphere of the HSSS research kitchen “Learning conditional independence models”, held in Třešť, Czech Republic, in October 2000.

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