Preface

Variational and boundary integral equation techniques are two of the most useful methods for solving time-dependent problems described by systems of equations of the form

$$\frac{\partial^2 u}{\partial t^2} = Au,$$

where $u = u(x, t)$ is a vector-valued function, x is a point in a domain in $\mathbb{R}^2$ or $\mathbb{R}^3$, and A is a linear elliptic differential operator. To facilitate a better understanding of these two types of methods, below we propose to illustrate their mechanisms in action on a specific mathematical model rather than in a more impersonal abstract setting. For this purpose, we have chosen the hyperbolic system of partial differential equations governing the nonstationary bending of elastic plates with transverse shear deformation. The reason for our choice is twofold. On the one hand, in a certain sense this is a "hybrid" system, consisting of three equations for three unknown functions in only two independent variables, which makes it more unusual—and thereby more interesting to the analyst—than other systems arising in solid mechanics. On the other hand, this particular plate model has received very little attention compared to the so-called classical one, based on Kirchhoff's simplifying hypotheses, although, as acknowledged by practitioners, it represents a substantial refinement of the latter and therefore needs a rigorous discussion of the existence, uniqueness, and continuous dependence of its solution on the data before any construction of numerical approximation algorithms can be contemplated.

The first part of our analysis is conducted by means of a procedure that is close in both nature and detail to a variational method, and which, for this reason, we also call variational. Once the results have been established in the general setting of Sobolev spaces, we carry out the second part of the study by seeking useful, closed-form integral representations of the solutions in terms of dynamic (retarded) plate potentials.

The problems discussed in this book include those with Dirichlet and Neumann boundary conditions (corresponding, in particular, to the clamped-edge and free-edge plate), with elastic (Robin), mixed, and combined displacement-traction (simply supported edge) boundary data, transmission (contact) problems, problems for plates with homogeneous inclusions, plates with cracks, and plates on a generalized elastic foundation. For each of them, the variational version is formulated and its solvability is examined in spaces of distributions; subsequently, the solutions are found in the form of time-dependent single-layer and double-layer potentials with distributional densities that satisfy nonstationary integral equations. The analysis technique consists in using the Laplace transformation to reduce the original problems to boundary value problems depending on the transformation parameter, and on establishing estimates for the solutions of the latter that allow conclusions to be drawn about the existence and properties of the solutions to the given initial-boundary value problems. The transformed problems are solved by means of specially constructed algebras of singular integral operators defined by the boundary values of the transformed potentials.

The distributional setting has the advantage over the classical one in that it enables the method to be applied in less smooth domains—for example, in regions with corners and cuts. Furthermore, Sobolev-type norms are particularly useful in the global error analysis of numerical schemes, but such analysis falls outside the scope of this book and we do not pursue it.

To the authors' knowledge, this is the first time that so many typical initial-boundary value problems have been considered in the same book for a model in conjunction with both variational and boundary integral equation methods. The text provides full details of the proofs and is aimed at researchers interested in the use of applied analysis as a tool for investigating mathematical models in mechanics. The presentation assumes no specialized knowledge beyond a basic understanding of functional analysis and Sobolev spaces.

We want to emphasize that the book does not intend to explain the mechanical background of plate theory. Details of that nature and a fuller discussion of the limitations of the model that we have chosen as our object of study can be found in the article

> J.R. Cho and J.T. Oden, A priori modeling error estimate of hierarchical models for elasticity problems for plate and shell-like structures, *Math. Comput. Modelling* **23** (1996), 117–133.

Ours is a purely mathematical that aims to acquaint the interested reader with two of the most powerful and general techniques of solution for this type of linear problem. We reiterate that the theory of bending of plates with transverse shear deformation has been selected merely as an application vehicle because of its unusual features and lack of previous strict mathematical treatment. The book is a natural complement to our earlier monograph [7], where we investigated the corresponding equilibrium problems.

Some of the results discussed below have already been announced in concise form in the literature (see [4]–[6] and [8]).

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Last but by no means least, we would like to place on record the debt of gratitude that we owe our wives, *sine quibus non,* who have guided us wisely, patiently, and selflessly, by word and by deed, to exciting and challenging new shores.

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