

## Preface

This book grew out of courses given at the University of Wales Swansea to second- and third-year undergraduates. It is designed to provide enough material for a one-year course and splits naturally into a preliminary topology course (Chapters 1–6) and a follow-on course in algebraic topology (Chapters 7–11).

It is often said that topology is a subject which is poorly served for textbooks, and when preparing the lecture courses I found no book that was both accessible to our undergraduates and relevant to current research in the field. This book is an attempt to fill that gap. It is generally accepted that a one-year course on topology is not long enough to take a student to a level where she or he can begin to do research, but I have tried to achieve that as nearly as possible. By omitting some of the more traditional material such as metric spaces, this book takes a student from a discussion of continuity, through a study of some topological properties and constructions, to homotopy and homotopy groups, to simplicial and singular homology and finally to an introduction to fibre bundles with a view towards  $K$ -theory. These are subjects which are essential for research in algebraic topology, and desirable for students pursuing research in any branch of mathematics. In fact, if I may be so bold as to say so, the subjects covered by this book are those areas of topology which *all* mathematics undergraduates should ideally see. In that sense, the material is *essential* topology.

With this range of topics, and the low starting level, the coverage of each subject is, inevitably, not exhaustive. For example, there are many results about connectivity whose proofs could be understood by undergraduates at this level, but which do not appear in this book. Instead, a representative sample of such results is included, together with enough examples that the reader should fully understand the results presented. In an undergraduate course it seems better

to present a brief account of several topics and give a feel for the overall shape of a subject, rather than an in-depth study of a small number of topics.

Some of the deeper results included are presented without proof, so that the student may meet an important theorem in the area even though the proof would lengthen the book unacceptably. In every such case references are given to books which do contain a complete proof.

Given the target audience, the book is designed to require as little prior knowledge as possible. Anyone who has some basic familiarity with functions, such as from a beginning course on calculus, should be able to follow the first four chapters. From Chapter 5 onwards, a little knowledge of algebra is required, in particular equivalence relations for Chapters 5 and 6, some familiarity with groups for Chapters 8 to 11, and with linear algebra and quotient groups for Chapters 9 and 10.

There is a short bibliography included, listing books where students can find details of the proofs which have been omitted. I have not included a list of further reading, as there are many books in topology and algebraic topology that should be intelligible to someone who has read through this book. The choice of which follow-on text to use is a matter of personal taste or, for students embarking on postgraduate study, is something that their supervisor will advise them about.

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