2 Radically Quantum: Liberation and Purification from Classical Prejudice

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A hundred years after Max Planck's surprising interpretation of the puzzling features (on the basis of classical physics) in the measured spectral distribution of the black body radiation which initiated quantum mechanics, we return in this book to the question: quo vadis quantum mechanics? Should one really ask the question: where is *quantum mechanics* going? Or should we not rather ask: where are *we* going with quantum mechanics? The former skeptical question, which returns over and over again, reflects the obvious irritation of many people in view of the surprising paradoxes of quantum mechanics, which are indeed irreconcilable with our macroscopically perceived reality. But we may retort: So what? In the historical development of science, we have often experienced the situation in which former theories had to be modified because they proved to be valid only in a certain range of our experience. Regarding quantum physics, despite its uncontested empirical validity in presently known physics, the numerous verifications of newly predicted microscopic effects, and successful application in modern technologies, many still feel quite uneasy about it and urge us to 'improve' this theory or even to replace it altogether by something more familiar.

This frequently stated viewpoint takes us back to Albert Einstein's position in his famous controversy with Niels Bohr about 75 years ago. Although honored with the Nobel Prize for his early contributions to quantum mechanics, Einstein considered these paradoxes unacceptable as a satisfactory answer, in contrast to Niels Bohr and his young student Werner Heisenberg. The latter, after numerous futile attempts to remove these oddities, finally saw themselves cornered and forced to surrender, accepting the physically strange consequences of the otherwise mathematically fully consistent new mechanics. But their defeat led them to one of the greatest scientific discoveries of the modern age: quantum theory turns out to be not simply a paradigm shift in the sense of Thomas Kuhn, but does actually imply a revolutionary change in our view of 'reality', compared to the commonly accepted classical Cartesian–Newtonian reality, by rejecting its ontic character. Rather than being based on interacting objects, 'things' that 'exist', establishing a material reality (in Latin *res* means 'thing') the quantum reality (in German, 'Wirklichkeit' or actionality) is based on an immaterial and irreducible connectedness, a 'potentiality' representing a holistic, inseparable, causally undetermined, genuinely creative world with an infinite-valued multiple logic. This, however, as can be demonstrated, constitutes no irreconcilable clash with our present world view. The commonly employed and experimentally established classical description is effectively recovered for most macroscopic systems and therefore validates the applicability of the classical reality notion in our familiar mesoscopic world.

From this viewpoint, quantum physics definitely provides a more general, more comprehensive and richer theoretical framework than classical physics, which only holds approximately under certain conditions in the limit of many quanta. We may more appropriately call the 'parts' of the 'Wirklichkeit' 'wirks' or 'haps' as minute happenings, or 'acts', as David Finkelstein [1] calls them. Hence the paradoxes of the quantum world only arise if the notions of the classical subsystem are inadequately extended to the superior system. The superior system becomes apparent particularly in microphysics (small numbers of 'wirks' or 'haps') and usually gets averaged out in meso- and macroscopic systems involving many 'wirks'. However, there may also be some surprising consequences in our much larger mesoscopic world if quantum features, by some intrinsic amplification mechanisms, manage to 'surface' and can thereby be observed and measured. In addition, it appears very likely that quantum features do somehow relate to the phenomenon of life and the spiritual dimension of our perception [2]. Because of its basic holistic structure, the quantum world offers a grand 'fulcrum', intimately connecting all fields of human experience, and in particular those which are still considered now as being completely disjoint and even mutually exclusive.

Hence I want to ask people who still have trouble accepting quantum physics the question: what kind of theory do you envisage or seek? What kind of theory would you like to advocate to replace quantum physics as a better description of physical reality? What do you mean by a 'better description' or what do you perceive as particularly 'bad' about the quantum description? What are the aspects you consider irreconcilable with macroscopic reality?

From my viewpoint there seem to be no basic problems with quantum theory as such. This does not mean that we have answers to all questions which can be asked. The problems lie more with interpretation. The question, in fact, has to be asked in a different way and directed to ourselves: how can we come to an arrangement with quantum mechanics so as to feel sufficiently comfortable or even convinced?

I would like to outline my personal perspective and approach to this question. Obviously quantum *mechanics* (QM) is still too limited. However, this is not because of its 'quantum' aspects, but rather because of its reference to 'mechanics', which only served historically as a starting point for the surprising journey into quantum physics (QPh) and still plays an important pedagogical role in this regard. It had to be and has been generalized to a relativistically invariant theory, which automatically requires the consideration of many-particle systems and quantum *field* theory (QFT). There are still a lot of open questions but no real contradictions [see for example the Standard Model (SM) of particle physics]. The mathematical treatment of quantum field theories, however, is still unsatisfactory because, with the present mathematical tools, basic issues can only be superficially addressed. The present limitation to quasilinear systems is too restrictive and actually unjustified.

The formulation of a general, all-embracing quantum theory in terms of a relativistically invariant quantum field theory has met a number of severe difficulties in the past, which we have not been able to resolve, but only effectively tame for a certain type of theory by some formal procedures (renormalization). It seems to me that these difficulties are connected to the fact that we are still sticking too close to the classical analogues: we start from classical field theories which are subsequently 'quantized', evaluate them in a semilinear approximation (perturbation theory), and appropriately cleanse them from irritating 'divergencies' by some subtraction procedures. The whole procedure only succeeds for sufficiently 'soft' interactions. A rather baroque but up-to-now successful example is the present Standard Model. The concept of spacetime-dependent fields, the priority of propagation over interaction, the close correspondence between the constituent fields and observed 'particles' and their symmetry properties are still clear traces of our classical entry and prejudice. A future 'improved' quantum theory, must therefore, in my opinion, be more quantum rather than less, i.e., even more radical in its quantum structure. In particular it should not start with classical features, e.g., presupposing a spacetime continuum as a given general background. This 'classical' space should ultimately arise as a sufficiently valid approximation from a fundamental quantum algebra.

As a concrete example for such a more radical quantum approach, a radically unified nonlinear pure spinor theory will be sketched, the essential features of which were introduced and developed over the period 1953–1983. Viewed from the presently adopted unified quantum field theories, including the Standard Model (SM), or the supersymmetric string theories, it appears outrageously radical, but from our own point of view it should, because of its field concept, be considered only as an intermediate step, although with regard to our intuition and for our guidance, an indispensable step towards a fully-fledged quantum formulation which ultimately employs only algebraic algorithms.

2.1 The Need for a New Approach to Science Indicated by Quantum Physics

2.1.1 General Considerations

Quantum physics is significantly different in structure from classical physics. Classical physics presumes an 'ontic structure' of the world. It starts with the question: What is? What exists? It talks about ‘reality’ consisting primarily of things, shaped material objects which can be distinctly localized in a 3-dimensional space and change location and form in a 1-dimensional directed time. For an explicit description we ask: what are things made of? In an attempt to separate the time-varying features from the invariant matter, we break ‘things’ up into ever smaller pieces, hoping to peel off their changing form from an invariant local core, the formless matter, the atoms. The form or ‘gestalt’ we interpret in terms of changing aggregates comprised of a large multitude of these formless atoms, i.e., localizable ‘pure’ particles of matter (‘pure’ in the sense of lacking any spatial form). The main characteristic of this approach is the priority of timeless pure particles, localized independently from each other at different space points and moving around unaware of each other.

Only in second place, because of additionally introduced interactions between these particles (this actually involves the assumption of other qualities, non-spatial ‘form’ features, like the electric charge of matter particles) do they become ‘aware’ of each other and form a common system, where ‘the whole is more than the sum of its parts’. This, however, can only be described in terms of a joint system of particles if the interaction does not destroy the ‘individuality’ of the particles. The interaction, therefore, has to be sufficiently weak, or in mathematical terms, the equations of dynamics must be quasilinear (differential equations in time with small nonlinear, i.e., sufficiently soft interaction terms) admitting a convergent perturbation expansion. Generalizing the many-particle system to a continuum of particle densities leads to classical fields and partial differential equations describing their dynamics.

In QPh there are no ‘things’, but basically only connectedness. The elements are not ‘pure particles’ or classical fields but simply elements of connectedness which we may call ‘wirks’ or ‘haps’. Numbers and functions of numbers are replaced by operators and operator-valued functions. ‘Wirklichkeit’ (or ‘actionality’) is no longer ‘reality’, but rather ‘potentiality’, a capability of manifesting itself as footprints in a material-energetic reality. The question is not: What is?, but rather: What is going on? What is happening? [1]. The starting point should therefore be an algebra of operators obeying the quantum commutation or anticommutation rules. Continous parameters, including time and space variables, should only show up in connection with particular representations of these algebras. The formulation of quantum field theories (QFT) in terms of dynamic equations or Lagrangians for operator-fields or operator-valued functions of such continuous (spacetime) parameters, will only occur as an effective, restrictedly valid approximation. Interaction will be more fundamental than propagation in the following sense: primarily, there is only interaction, a nonlinearity (an interacting of the field with itself), and only on a second level do we find, as a consequence of this interaction by a kind of a self-organization (constructive feedback), the effective appearance of propagators of ‘particles’. These propagators will be characterized by certain numerical normalization factors and also contain certain masses as softer terms. The 'particles' will be related to the empirically observed particles with the various coupling constants in their interactions with other particles being proportional to the inverse of the numerical propagation normalization factors.

In fact, the appearance of a large number of numerical constants in microphysics, in terms of mass ratios of elementary particles and coupling constants, was one of the main reasons why Werner Heisenberg in his first attempt [3] to formulate a fundamental field theory (in 1950) insisted on starting with a nonlinear theory of the non-renormalizable type. The non-renormalizable character indicates that interaction at small distances dominates propagation. As a consequence, the nonlinearity, a strong local interaction, provides for the possibility that dimensionless numbers can be generated by the dynamics. Heisenberg was familiar with this feature from his very early research work for his dissertation in 1923 on the theory of (classical) turbulent motion of fluids, where such dimensionless numbers (like those of Rayleigh and Reynold) result naturally from the nonlinear Navier–Stokes equations. As another example, the quantum mechanics of a single electron in the H atom produces infinite towers of stationary (or rather, quasi-stationary) states, and as a consequence, a corresponding infinite number of numerical energy ratios. Of course, it is exactly this close connection between relevant non-trivial results and the still continuing inability to treat such nonlinear problems successfully (as being non-renormalizable, i.e., no decoupling at small distances and hence non-applicability of perturbation theory), that has led to the broadly shared opinion that Heisenberg had definitely failed in his quite novel and highly ambitious approach.

The main purpose of the present paper is to retrace the Heisenberg approach to a fundamental quantum theory of reality in terms of the nonlinear spinor field theory of elementary particles, which he started in 1950, more than fifty years ago, and in which I actively participated for 25 years, from 1958 to 1983. Indeed, I am still involved. From my present vantage point, I consider the field theoretical formulation, as I mentioned before, as a kind of intermediary step towards an ultimate comprehensive unified theory, an in-between formulation which offers the possibility to establish approximation schemes, although perhaps only on a rather poor level (e.g., of the Tamm–Dancoff or Bethe–Salpeter type) for actually calculating mass ratios and coupling constants from basic principles. These considerations are not only of historical value but offer, to my mind, the chance to point out again the much more courageous and radical character of the approach suggested by Heisenberg et al. for the formulation of a unified theory of matter than the ones favored at present, with the Standard Model of elementary particles as the most prominent example. Clearly, the Standard Model is very successful in interpreting, or at least being without exception consistent with the extensive range of experimental data now available. Still, in the mind of most particle physicists, the SM could hardly be said to represent anything like a 'final solution' for the fundamental problems of the material world and its dynamics.

To avoid misunderstandings at this point, let me state this more clearly. We should not and we are not aiming at an ultimate 'Weltformel', as our own approach was phrased by newspapers early in 1958. No! Quantum physics, being non-ontic, excludes such a possibility altogether. It is an 'open', largely undetermined theory. In this and many other points I fully agree with what Henry Stapp has said on this issue [4]. But the present Standard Model formulation, in our opinion, beyond this necessary openness and indetermination, has some limitations, in principle. It is more like what one might call an 'engineering model', because, to define the theory uniquely, we have to supply in an ad hoc manner numerous features concerning the character of the dominant fields and also many dimensionless numbers (related to mass ratios and coupling constants) without providing any satisfactory hint of their origin. The 'anthropic principle' – i.e., suggesting that numbers are thrown in arbitrarily by 'God' with the Big Bang at the beginning of the universe but filtered as an 'end of the pipe' condition so to speak by the requirement of coexistence of these specific numbers with the existence of the human being as observer – is hardly a hint but more of a 'poor man's' excuse.

Of course, there is nothing wrong with scientists being modest regarding their actual capability and general claims to offer explantions for any and all phenomena. But it appears unnecessarily fatalistic to use this as a starting point. Anyway, I definitely prefer to compare models, like the SM, with a van der Waals potential model in atomic physics, which allowed rather successful approximations for calculating the spectra of the heavier atoms generated by outer shell electrons by using effective classical van der Waals potentials instead of the electric potential of the atomic nucleus shielded by the electrons of the inner shells. Of course, the real breakthrough in QPh occurred much earlier with the exact calculation of the simple hydrogen atom by Wolfgang Pauli using operator algebra. The trouble repeating this success story for the more general and more complicated problems of relativistic elementary particle dynamics is that there does not seem to be any a nalogue of the simple hydrogen atom, where we could explicitly detect and convincingly isolate and extract the basic principles.

It may well be, however, that the basic principles are in fact already known: they are the principles of quantum physics. Since physics in the old meaning barely survives, we may just refer to it as the quantum principle. The trouble we meet in quantum physics and the difficulties we still have in understanding many features of it, e.g., in the context of the Standard Model, may very well be connected with our hesitation to take quantum principles really seriously. This is the reason why I have given this contribution the rather challenging title: Radically Quantum.

Before going into more detail, let me briefly indicate some features where the classical prejudice is most visible in the presently favored approach.

2.1.2 Classical Egg Shells in Quantum Physics Today

In a way, present quantum physics still looks like a kind of formal deformation of classical physics. We start from classical considerations with a classical Lagrangian or Hamiltonian or the corresponding classical equations of dynamics. We then 'quantize' the complementary canonical variables according to Heisenberg by replacing the classical variables by the corresponding operators obeying certain commutation rules consistent with the canonical theory (replacing Poisson brackets by commutators or in a Grassmann algebra by anticommutators). We limit calculations to cases which can (at least in principle) be solved explicitly. We extend these models to ones where additional interaction terms are included which can be handled as a small perturbation using the perturbation theory expansions. The corresponding space representation of the solutions leads to the Schrödinger wave equation. They are classical field equations, however, for complex fields of space and time, in contrast to the real classical fields. And there are new ones, without classical analogy, the spinor fields. These fields are again quantized (sometimes misleadingly called second quantization) where spacetime remains a classical background field, like the time parameter in quantum mechanics, on which the quantum field 'lives'.

In the case of interaction, the quantum field formulation is the only consistent way to incorporate relativistic invariance. This is connected with the possibility of pair creation and the severe consequence that the one-particle sector is intimately coupled to the many-particle sectors. Only free field dynamics expressed in term of linear operator-valued field equations can be solved exactly. More complicated field equations containing interaction terms (non-linearities) can only be solved if the interaction is sufficiently 'soft' at small distances (less dominant than the uncertainty fluctuations of the free particle at small distances) and hence can be treated as a small perturbation using the perturbation theory expansion as a valid approximation (super-renormalizable or even to a certain extent renormalizable theories).

Spacetime in these field theories is like an external classical background field. With its metric structure being intrinsically connected to the gravitational field as formulated by Einstein's general theory of relativity, it should be quantized as well to avoid inconsistencies. To quantize by starting from classical general relativity using the canonical procedure leads to a non-renormalizable theory which does not justify perturbation expansion even for an extremely weak coupling of the gravitational field to matter fields. But there are other features which suggest a rather different approach for incorporating gravitation into quantum theory. This will be discussed later.

Altogether the procedure which starts with the phenomenological particles, i.e., the particles which appear if they are far away from each other (asymptotic domain), and represents them by the corresponding local quantum fields, which on the contrary, characterize the behavior at very small distances, appears too luxurious (candidates to be cut off by Occam's razor) for the formulation of a fundamental theory. Such a theory should certainly also be able to generate 'particles', which are not basic entities but rather something like 'bound states' of a much smaller number of building-block fields. The presently highly favored Standard Model is a hybrid model using as basic fields, firstly, fields related to phenomenologically established particles, but also, secondly, other basic fields, like the quarks and the gluons, which do not show up themselves as particles but only play the role of constituent fields of compounds, related to other well-known particles, the strongly interacting and heavy hadrons. String theory starts solely from constituent constructs, strings, but again from the extreme classical end, with generalizations of classical theories similar to general relativity where the emphasis is on the geometric properties and we are guided by some highly attractive mathematical properties. In this context, quantum theory will only be grafted onto the classical formulation afterwards by the usual quantization procedure. From my point of view, string theory is an extremely luxurious starting point which leaves, as I see it, too much arbitrariness for selecting a specially distinguished and plausible form for the fundamental theory.

2.2 Additive Unified Quantum Field Theories

2.2.1 General Remarks

Let me start by describing briefly the presently most favored unified field theory models, which I call the additive unified quantum field theoretical models. The AUQFT models started around 1961 in the wake of the 'radically unified quantum field theoretical models', initiated essentially by Werner Heisenberg in 1950 [5]. The latter theories, RUQFT, will be treated in the next section. In fact, this short presentation with a description of their key consequences will constitute the main purpose of this contribution. Although historically they were devised later, I choose to start with the AUQFT models because they include, in particular, the so-called Standard Model (SM), which appears to be consistent with all the presently known experimental data.

The AUQFT models belong to the category of phenomenologically guided field theoretical models, in the sense that they are closely constructed from well-established subtheories connected with known particles or interactions. They differ, however, in the way this additive patchwork is glued together using general principles, mainly group theoretical considerations, to achieve some kind of unification in order to minimize the number of arbitrary fudge factors, like appropriate masses and coupling constants which are left undetermined by these theories. Like 'engineering models', they are sufficently satisfactory for practical purposes, allowing detailed calculations and predictions for experimental outcomes – and this is indispensable for establishing sound theories – but they are barely acceptable as the final theoretical answer to the very ambitious questions we expect to be answered by a fundamental unified theory of matter, at least if we take the viewpoint of Albert Einstein, as expressed in his autobiographical notes [6]:

> Before I enter upon a critique of mechanics as the foundation of physics, something of a broadly general nature will first have to be said concerning the points of view according to which it is possible to criticize physical theories at all.
>
> The first point of view is obvious: the theory must not contradict empirical facts. However evident this demand may in the first place appear, its application turns out to be quite delicate. For it is often, perhaps always, possible to adhere to a general theoretical foundation by securing the adaption of the theory to the facts by means of artificial additional assumptions. In any case, however, this first point of view is concerned with the confirmation of the theoretical foundation by the available empirical facts.
>
> The second point of view is not concerned with the relation to the material of observation but with the premises of the theory itself, with what may briefly but vaguely be characterized as the 'naturalness' or 'logical simplicity' of the premises (of the basic concepts and of the relations between these which are taken as a basis). This point of view, an exact formulation of which meets with great difficulties, has played an important role in the selection and evaluation of theories since time immemorial. The problem here is not simply one of a kind of enumeration of the logically independent premises [...], but that of a kind of reciprocal weighing of incommensurable qualities. Furthermore, among theories of equally 'simple' foundation that one is to be taken as superior which most sharply delimits the qualities of the system in the abstract (i.e., contains the most definite claims) [...].
>
> The second point of view may briefly be characterized as concerning itself with the 'inner perfection' of the theory, whereas the first point of view refers to the 'external confirmation'. The following I reckon as also belonging to the 'inner perfection' of a theory: We prize a theory more highly if, from the logical standpoint, it is not the result of an arbitrary choice among theories which, among themselves, are of equal value and analogously constructed.

One may argue, of course, whether this point of view should be regarded as universally valid. Einstein himself only applied it 'to such theories whose object is the totality of all physical appearances'. On the basis of a more pragmatic and positivistic attitude where functionality is predominant, many scientists today actually believe that such an expectation is too idealistic in the Platonic sense and should be given up as antiquated. They may tend to favor an anthropic principle according to which an apparent theoretical non-uniqueness of possible world models is dramatically reduced to a very small number of models which require, as a severe limitation, the compatibility of the existence of the universe with the existence of *homo sapiens sapiens* asking all these intricate questions. It is true that we should not be so arrogant as to demand rules about how this universe should be constructed and run, but I find it a rather easy and lazy attitude to discard or externalize such fundamental questions right from the start. There is no question in my mind that there will be no 'world formula' for the universe in the sense, as some people imagine, giving us very precise and unique answers to all our questions. In fact, quantum physics has taught us that the laws of nature are of a much more general form than we ever expected or imagined, leaving enough room for an infinite number of different solutions reflecting the tremendous diversity of structures, forms and processes we observe around us.

To use a more limited but quite illustrative example for this situation, let us look at the large number of light spectra of different atoms and molecules with their complicated sequences of spectral lines and continua, which serve as precise fingerprints for their existence and structure. There are long shelves of books in our libraries with tables of the measured frequencies of the emitted and absorbed light waves in these spectra. The ratio of these frequencies, corresponding to ratios of energies of states, constitute a huge collection of (dimensionless) numbers demanding to be explained by an appropriate theory for atoms and molecules. These problems can be considered to be solved, in principle, by quantum mechanics. But apart from a few very simple cases, in particular the hydrogen atom, or the H_2^+ molecule, and in a rather good approximation for some higher atoms (replacing the atomic nucleus by an effective van der Waals potential for the nucleus shielded by the electrons of the inner shells), this has not actually been carried out explicitly. Nobody is bothered by this because we have full confidence that, with the present theory, this could be demonstrated to a satisfactory degree, if necessary.

The Standard Model of elementary particles could be regarded as a van der Waals approximation to a perhaps much simpler basic theory. But we do not know this, because in particle theory we have the disadvantage that there is no simple example like the hydrogen atom to play around with. But even without the existence of such a simple system where this can be explicitly demonstrated, it may not be unreasonable to believe that there does exist such a simple underlying theoretical structure. I would guess that, even without knowledge of the H atom, scientists would not have suggested an anthropic principle to explain the huge number of spectral lines or the energy levels of the atoms, the differences of which relate to the spectral lines. For the formulation of a theory, it was in this context important not to cling to the large number of phenomenologically apparent spectral lines themselves, but to move conceptually one level below to the electrons, where each is capable of generating an infinite number of eigenstates.

2.2.2 Phenomenologically Guided Field Theoretical Models

Elementary particle theories start from the concept of a classical particle which in a relativistic quantum theory is represented via Heisenberg's interpretation of Bohr's correspondence principle by an operator-valued quantum field $\psi(x)$ depending on a classical (c-number-type) spacetime position x. The position space is important to define a local operator interaction by a multilinear or nonlinear product of local field operators $\cdots \psi_{n+1}(x) \cdot \psi_n(x) \cdots$. The sequential operator products express a time-sequence of the operation (in our mathematical convention, the later $n+1$ is to the left of the earlier n). Hence time is actually represented twice although in a different form: firstly, in the classical fashion as the 0 th component of $x \equiv x^\mu$, and secondly, algebraically, as 'bare time' through an ordering parameter in the operator product. For a free particle, the x-label has only secondary importance and is more conveniently replaced via Fourier transformation by the 4-momentum p, with the restriction $p^2 = m^2$ relating to the rest mass of the particle. The free motion is expressed by the kinetic term in the Lagrange–Hamilton formulation or the propagator in the Feynman S-matrix expansion.

The phenomenological approach starts with phenomenologically known and, in a theoretical description, asymptotically surviving (stable) particles, and associates these with an appropriate tensor-type or spinor-type particle-operator field depending on their intrinsic spin properties (integer: tensorial, half-integer: spinorial). A practical difficulty arises over how to handle the large number of quasi-stable particles which only travel a finite rather than an asymptotically infinite distance before decaying. These decaying particles are usually treated like stable particles in the first approximation. As a consequence of the particle–wave duality, a virtual exchange of particles at smaller distances generates interactions with a Yukawa-type potential $\mathrm{e}^{-r/R}/(r/R)$, with a range R that is inversely proportional to the mass of the virtually exchanged particle. However, there is no one-to-one correlation between particle (asymptotic and on-energy-shell) and interaction (virtual and off-shell). In fact, the concept of interaction is more general than that of particles. This is evident for the massless fields. In particular the very prominent electromagnetic field has a non-particle infinite-range type interaction, the Coulomb interaction, and a similar (but 4-fold) situation holds for the gravitational field.

The phenomenologically guided field theoretical models were quite popular in the early 1960s, before the large number of rather short-lived strong interaction particles (hadrons) were discovered. All of the then-known particles were treated as quasi-stable with electromagnetic and weak interaction, to which a strong interaction was added later on, assumed to be mediated by pions as the relevant force fields. For lack of appropriate tools, all interactions, irrespective of their actual strength, had to be treated in an unsatisfactory manner as 'sufficiently weak impacts' by perturbation theory. This is hardly a viable approach.

2.2.3 Symmetries of the Dynamics and their Effective Up- and Downgrading

The invariance of the Lagrangian and the field equations (derived from the Lagrangian according to the Hamilton principle as a functional extremum of the total action) under certain symmetry transformations are of eminent dynamical importance because they lead, according to the Noether theorems, to conservation laws for certain quantities in physical processes, the Lie generators of the dynamical symmetries. Local fields, in contrast to the asymptotic particle fields, can be viewed as appropriate local parametrizations of the symmetries of the fundamental dynamics.

There are global symmetries characterized by x-independent symmetry transformations G, usually expressed in terms of Lie groups with a certain number of real-valued Lie parameters. Some of these symmetries hold strictly (for Lagrangians which are invariant under these symmetry transformations), leading to exact conservation laws. But there are other symmetry transformations which are phenomenologically only approximately valid. There are also the more general local symmetries or gauge-type symmetries, characterized by symmetries $G(\text{loc})$, with x-dependent Lie parameters. In connection with kinetic terms, they generate certain well-defined (gauge-type) interactions.

The cardinal question of particle physics relates, firstly, to the origin of the particular set of symmetries of the field dynamics (the Lagrangian or the corresponding field equations) exhibited phenomenologically, and, secondly, to why some symmetries only hold approximately, indicating perhaps some second step distortion by which originally perfect symmetries are 'broken' in their phenomenological appearance.

The conservation laws of physical processes and the mass spectrum (multiplets) of the particles exhibit a rather complicated structure which at first glance does not hint at a simple fundamental dynamics characterized by some single high symmetry group. From their additive construction, reflecting this in a straightforward combination of the various symmetries of the different interactions (strong, electromagnetic, weak and gravitational), the AUQFT models lead to a 'stutter' structure:

$$G = G_1 \otimes G_2 \otimes G_3 \otimes G_4 \, .$$

In particular the factor groups may be the same group or particular subgroups thereof. Such a situation is well-known from the physics of the heavier atoms with many electrons, which are arranged in different shells because of the Pauli principle related to the anticommutativity of the spinor field operators. The electrons all move in the same rotational $O(3)$-symmetric Coulomb potential of the point-like nucleus (disregarding its spatial extension and its spin). The fundamental symmetry group is just $O(3)$ or the covering group $SU(2)$. [In the special Coulomb case, we even have the higher dynamical symmetry $O(4)$.] If the electrons are considered to be approximately independent of each other, then the basic symmetry group in the case of N electrons seems to be inflated to the N-fold 'stutter' symmetry

$$G = \bigotimes_{1}^{N} G_n \; .$$

Here the symmetry inflation is caused by the assumed independence of the N (electron) subsystems. This is, of course, a rather poor approximation because the electrons are all correlated by the antisymmetrization of the total wave function of the electrons enforcing the Pauli principle, and in addition are affected by their mutual electric Coulomb repulsion and their magnetic dipole interactions connected with their spin properties. In fact, because of the weak spin–spin and spin–orbit interactions, their approximately decoupled spins allow an even higher $2N$-fold 'stutter' symmetry, which by taking the various weak couplings into account step-by-step as perturbations, is then consecutively broken down to the original basic $SU(2)$ characterized by the total angular momentum J.

To demonstrate explicitly the rather sophisticated approximate symmetry structure [7], the lowest energy level structure of the 5-electron boron atom is given in Figs. 2.1 and 2.2 according to the LS and jj coupling schemes. They exhibit different in-between 'broken' higher symmetries related to subshell occupations, subspin arrangements, seniority, etc. The altogether $\binom{8}{3} = 56$ energy levels of the second (less than half-filled) atomic subshell eventually split up into 12 energy levels which only have the degeneracy of the remaining exact $SU(2)$ symmetry group. It is interesting to observe that the fine structure contains split doublets as well as singlets and split triplets.

The investigation of the spectrum of atoms hence gives another interesting hint as to how symmetries can appear to be 'broken'. In particular, there is a well-established method for approximately calculating the lowest energy states of higher atoms. In the so-called optical approximation, only a single 'outermost' electron is taken into account and considered to move in an effective classical van der Waals potential of the spherically symmetric nuclear potential screened by the averaged cloud of inner (all but the outermost) electrons. This effective screened potential will not in general be rotationally symmetric and will thus exhibit orientation properties. The spin orientation degeneracy of the energy states of the singled out (optical) electron will be removed, imitating a breakdown of the basic rotational symmetry.

In the case of a quantum field theory which involves a virtually infinite number of quanta, the above consideration relates to a certain arbitrariness in separating a certain limited subsystem from the total infinite quantum aggregation, referring to the particular system to be explicitly observed and

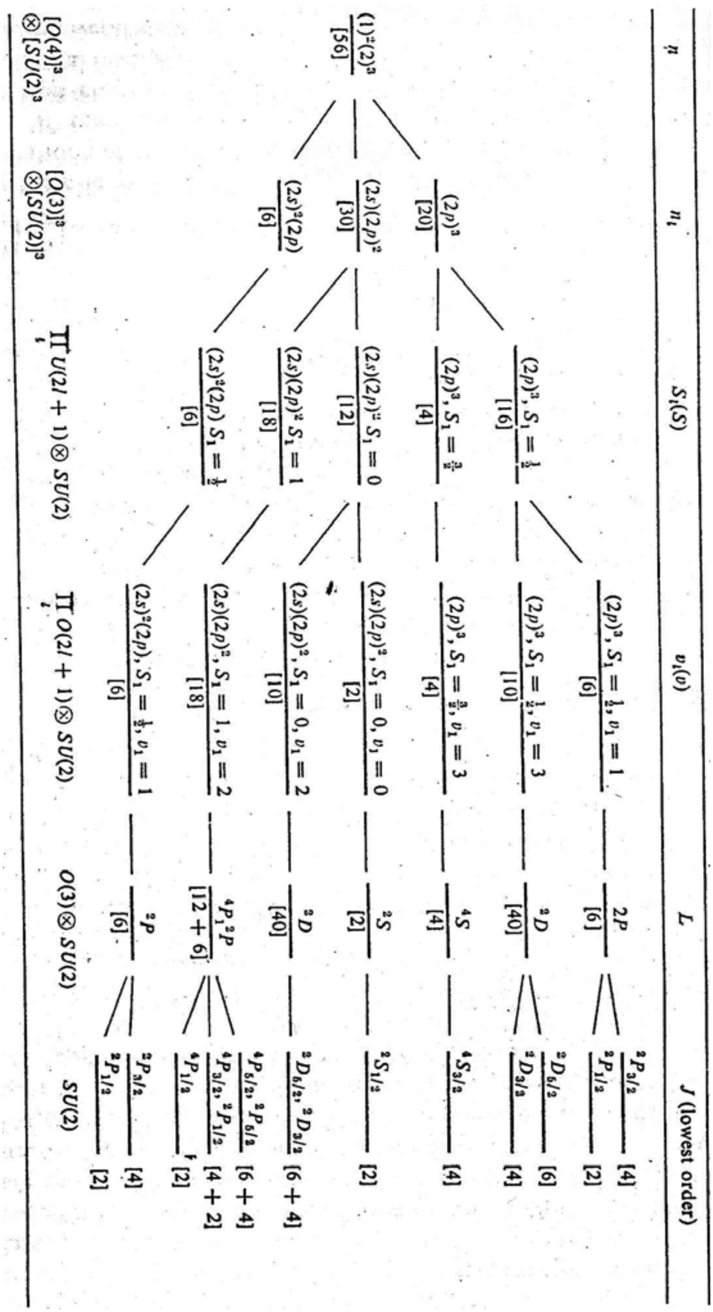

Fig. 2.1. Lowest energy level structure of the 5-electron boron atom according to the LS coupling scheme

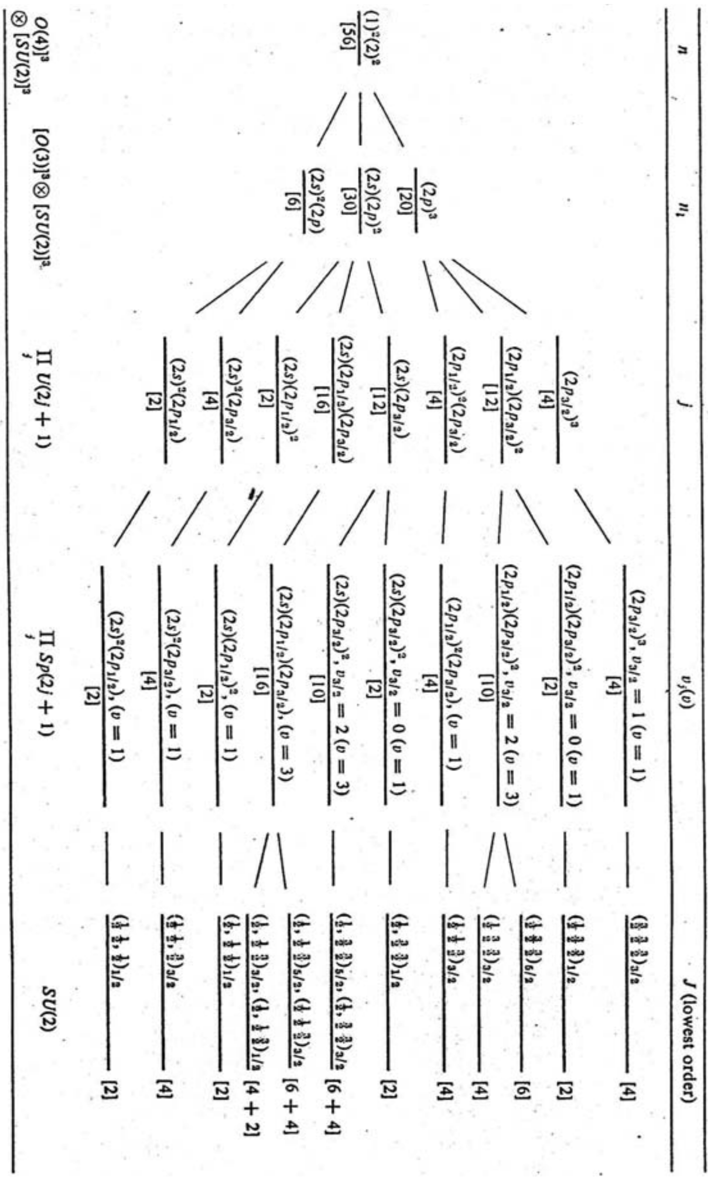

Fig. 2.2. Lowest energy level structure of the 5-electron boron atom according to the jj coupling scheme

analysed (similar to the optical electron of the atom), from the remaining, still infinite quantum system, but treated approximately as a classical 'background state' (similar to the appropriatedly shielded van der Waals potential in our atomic example). In quantum field theory this background state is usually taken as the 'vacuum' or 'ground state', characterizing the 'empty' or 'energetically lowest state', and hence 'stable state'. Mathematically, the ground state defines the representation of the quantum field algebra in the complex-valued infinite-dimensional linear state space, the Hilbert space. Different ground states define inequivalent representations of the algebra and hence disjoint Hilbert spaces. Obviously, the theoretically introduced and commonly used 'vacuum' or 'ground state' is a convenient artifact and serves as an approximation for the real physical background for our observation, which comprises the classical universe with all its galaxies, including the immediate environment of our experiment and in particular the measuring equipment and the observer, all defined in classical terms. Every measurement leading to 'facts', e.g., a blackening of a photographic film, a water droplet track (water condensation strip) of a charged particle in a Wilson cloud chamber, establishes a different inequivalent representation. The singular change of the representation by the measurement is usually interpreted as a discontinuous change, 'a collapse of the wave function' in the observed system.

Of course, simulating the 'background state' by a vacuum only works if the background is invariant under the symmetry groups of the fundamental dynamics, in particular if it is invariant under the 10-parameter spacetime Poincaré group. Due to the extreme weakness of gravitational forces and the neutralization of electric forces by opposite charges, the external influences of distant objects can generally be neglected except for some explicit nearby classical electric or magnetic force fields that are part of our measuring equipment. Such classical force fields can, however, be part of the system of observation, e.g., in the case of a ferromagnet where, due to quantum exchange forces, the electron spins can line up to produce a near-classical magnetic polarization $\boldsymbol{M}$. This state can be used as the reference state, the background state, which in this case will violate the full rotational invariance, breaking the 3-parameter rotational symmetry down to a 1-parameter rotational symmetry around the polarization axis. Different orientations of the polarization $\boldsymbol{M}$ define different Hilbert spaces which are essentially orthogonal to each other, proportional to e^{-N} with N the number of aligned spins or magnetons, and become fully orthogonal (non-equivalent) in the limit $N \to \infty$.

For infinite systems or field models, the asymmetric ground state cannot even be rotated to an infinitesimally different orientation by (angular momentum) operators defined in that Hilbert space. Only sub-infinitesimal rotations $\sim \lambda/L$ correspond to this space, relating to rotations of finite pieces of length of order λ of the ferromagnet L. They can be interpreted as wavepackets of size λ of Bloch spin waves $\theta_\pm(x)$ with energy $E = \hbar c/\lambda \to 0$ with wavelength $\lambda \to \infty$, relating to massless boson modes in a relativistic formulation. As a consequence of the asymmetric background state, the whole tower of excited states will also exhibit this asymmetry.

The idea of interpreting the appearance of broken symmetries as a strictly valid fundamental symmetry but distorted by an asymmetric ground state, was discussed by Heisenberg from the very beginning of his unified theory in connection with isospin symmetry, broken by an isospin-polarized ground state in analogy with the ferromagnet or antiferromagnet. In connection with a unified theory of elementary particles, it was suggested in more general terms in the well-known but unpublished preprint of Heisenberg and Pauli [8], and in a more detailed paper by Dürr, Heisenberg, Mitter, Schlieder, and Yamazaki [9], i.e., three and four years, respectively, earlier than the papers by Goldstone [10] and Goldstone, Salam, Weinberg [11].

2.2.4 Standard Model

Within the AUQFT approach the Standard Model is essentially a phenomenologically guided unified quantum field theory with, however, basic modifications regarding the strong interactions and their corresponding particles (hadrons). These modifications were necessary to deal effectively with the fast growing number of quasi-stable hadrons (short finite half-life), which made it unreasonable to represent them all by genuine local fields. In fact, many decayed in a very short time into longer-living hadrons and eventually stable hadrons, and it appeared more appropriate to consider them as quasi-bound states or resonance states of their decay products. This suggested an approach closer to the treatment of atoms and molecules, introducing a smaller set of constituent fields. However, group theoretical considerations [12] suggested a new type of building block for hadrons: the quarks and the gluons. They were no longer simply connected to phenomenologically known particles or quasi-particles but were merely 'local interaction field constructs' representing certain basic symmetries similar to, but not identical to the constituent fields already proposed by Heisenberg in 1950 and promoted by the radically unified field theories, to be outlined in the next section. In this sense the Standard Model is a hybrid model of the phenomenologically guided field model augmented by the constituent-type-field concept.

The basic symmetry group G_{SM} of the Standard Model is a rather baroque 'stutter' symmetry, reflecting its basic 'additive' phenomenological genesis:

$$G_{\mathrm{SM}} = \underset{F'}{U(1)}\otimes \underset{Y}{U(1,\mathrm{loc})}\otimes \underset{f}{SU(2,\mathrm{loc})}\otimes \underset{c}{SU(3,\mathrm{loc})}\otimes \underset{\text{family}}{G'(?)}\otimes \underset{\text{gravity}}{G''(?)}\ ,$$

with F', Y, f, c the fermion number, hypercharge, flavor (e.g., isospin for the first family) and color properties. The majority of the groups are local groups generating gauge interactions. The 3-fold family diversity is still not satisfactorily incorporated and gravitation is left out.

Excluding gravitation and possible field additions needed to resolve the 'family' puzzle, there are altogether 142 constituent fields in the Standard Model, comprising 90 fermion fields and 52 boson fields. The fermion fields consist of three equivalent families of 15 Weyl spinor fields and their antiparticles each accounting for four 3-colored hadrons (a left-handed flavor doublet and two right-handed flavor singlets) and three colorless leptons (a left-handed flavor doublet and a right-handed flavor singlet). The generalized flavor refers to the up–down (isospin), charm–strange, and top–bottom qualites, respectively, for the three families. The $52 = 4 \times (8+4) + 2 \times 2$ boson fields consist of an 'isospin' singlet, 8-colored, vector gauge field (gluon) and an 'isospin' doublet, uncolored, scalar Higgs field and its antiparticles. Higgs fields are required to generate the breaking of the isospin symmetry, or more generally the flavor symmetries.

In trying to unify this theory further to a Grand Unified Theory (GUT), there have been many attempts to get rid of the 'stutter' structure of the symmetry by embedding it into a higher symmetry (without taking into account family triplication, etc.). There have been various proposals [13], e.g.,

$$G_{\text{grand}} = \underset{F}{U(1)} \otimes \underset{fc}{SU(5, \text{loc})} \ .$$

The 15 spinor fields can then be grouped into a 10-plet and 5-plet (although mixed in helicity and fermion number) of $SU(5, \text{loc})$. The interaction is carried by 24 vector gauge fields, and $24 + 5$ Higgs fields would have to be introduced to break the grand symmetry into flavor and color subsymmetries. Obviously there will be a lot of superfluous and unaccounted-for fields.

It should be mentioned that the Standard Model and the speculative extended models require a particular mechanism to contain all 'colored' fields in such a way that they never show up asymptotically. This may be a consequence of the necessarily nonlinear interaction of non-Abelian gauge fields preventing color charges from ever being pulled apart. But the mathematical treatment is still quite unsatisfactory because it goes beyond a perturbation theory approach. Also to break symmetries in a mathematically transparent way, Lorentz-scalar Higgs fields have to be added in an ad hoc manner to enforce the existence of the desired asymmetric ground state, not to mention the large numbers of mass terms which cannot be deduced from the theory.

This may suffice to indicate that the present status of a 'comprehensive quantum theory of matter' is far from being satisfactory. The open questions may not have a definite and rigorous answer, e.g., as in the case of the anthropic principle. But it is not illegitimate to criticize the particular approach, in particular to ask: why should reality be such as to allow reductionist solutions, which depend on approximate linearizations? Because, after all, quantum theory strongly emphasizes the fundamental dominance of connectedness over separateness. It appears that we should turn the question around. Rather than ask why existing 'separated' things come together to form various complicated compounds, we should make the opposite inquiry: why does it happen that the intrinsic tight connectedness has the tendency to favor processes of differentiation and 'emancipation' – arrangements loosening connectedness and weakening interactions by appropriate mutual compensation of forces in certain 'regions' (such as happens as a consequence of repulsive forces between opposite charges in electrodynamics), or by the 'repulsive' exchange forces enforcing the Pauli exclusion of states, or quite differently, by destructive interference – to create quasi-empty 'in-betweens', and simulate a world consisting approximately of (sufficiently weakly interacting) separate as-if-parts?

2.3 Radically Unified Field Theoretical Models

2.3.1 Historical Remarks

The occurrence of very different types and strengths of interactions is a very interesting feature of matter dynamics and there were early efforts in science to find some theoretical reasons for their particular existence and possibilities for a common origin. Being very familar in our daily life with the phenomenon of 'weight', an attraction of all matter by the earth, it was an impressive 'unification of interactions' by Isaac Newton to discover that the forces of celestial motion – described by Johannes Kepler for the planetary system as planets being attracted by the central sun – were caused by the same gravitational force which made the apple drop to the ground. The electric and magnetic interactions were combined to an electromagnetic interaction, also responsible for the chemical forces which made solid matter appear as spatially extended objects (*res extensa* of Descartes) and included later, as shown by Faraday and Maxwell, the phenomenon of electromagnetic waves. The Maxwell equations allowed an extremely compact 'unified' theoretical description in terms of an electromagnetic field generated by and interacting with electric charges and currents. The field concept could be similarly extended to gravitational interaction. Albert Einstein in his general theory of relativity [14] identified the gravitational field with metric properties of the spacetime continuum (curved pseudo-Euclidean geometry). Einstein [15] and Hermann Weyl [16] attempted later to generate a comprehensive 'unified theory' by appropriately incorporating electrodynamics into a generalized geometric structure, without much success.

Quantum physics, however, put an end to this very promising approach to a classical unified theory. Firstly, gravitation, the heart of the geometric approach, proved rather recalcitrant as regards quantization, in contrast to the electromagnetic field, but, secondly, two new interactions were discovered, the weak interaction connected with radioactive decay, and more importantly, the strong interactions relevant for atomic nuclei. With the discovery of the strongly interacting π-meson, it appeared that strong interactions could be handled in a rather similar way to the electromagnetic one, with light quanta being replaced by pions. This favored the additive approach to a quantum field theory, as described in the last section. But due to stubborn inconsistencies at small distances (divergency problems) in the marriage of quantization and local interactions enforced by special relativity [17], the quantum field theoretical approach caused great frustration and called for a basically novel approach.

As a consequence, the majority of quantum physicists turned away from local quantum field theories altogether and looked for an appropriate description in terms of a scattering-matrix theory. This starts from the experimentally accessible asymptotically large distance region, where, in the case of finite-range interactions, only free particles occur. The S-matrix theory was developed by Heisenberg [18] in the mid-1940s as a kind of phenomenological model for an elementary particle theory starting solely from 'observable quantities'. This approach proved very powerful by the observation that the necessary condition of relativistic causality, represented in field theories by the causal condition of 'locality', could be incorporated there in terms of certain analyticity requirements for the scattering matrix.

Concerning the analyticity properties of the S-matrix theory, although they are mathematically quite attractive and in many ways productive as a substitute for the physically relevant local causality, Heisenberg felt that this new tool was intuitively not very illuminating with regard to the dismal behaviour at short distances. To avoid the obstinate divergence difficulties at small distances, he therefore returned in the early 1950s to the quantum field approach with the idea of postulating a fundamental length [3] indicating a spacetime range within which quantum theory in the presently used form would not be valid but had to be modified. This had two important consequences. Firstly, the local constituent fields would not be identical to the canonical fields appearing asymptotically on the periphery connected with the observable particles. Secondly, the coupling constant characterizing the interaction, a nonlinear term, would be proportional to a positive power of the fundamental length, hence rendering the theory non-renormalizable in the common terminology, i.e., not reducing to a free and hence canonical field theory in the limit of short distances. However, the latter troublesome consequence provided an interesting opportunity to start with a much smaller number of constituent fields compared to the number of asymptotic particle-like fields representing so-called bound states of the constituent fields. This radically new approach opened the way to a 'radically unified field theory'. The price was non-renormalizability. Did this really represent an insurmountable barrier? Or was it on the contrary the key to a new type of solution?

I emphasize this point in order to demonstrate that these RUQFTs based on non-particle-like constituent fields with possibly unusual (non-canonical) properties, developed in the early 1950s by Heisenberg [19, 20] were, and still are actually more revolutionary in their basic approach than the AUQFT constructions, including the presently accepted Standard Model. In fact, the non-canonical character (unusual transformation under dilatations) required an indefinite metric in the state space (Nevanlinna space instead of a Hilbert space) [19, 20] familiar in the Gupta–Bleuler formulation of quantum electrodynamics and gravitation [21, 22].

We should realize that, at the time of the conception of the RUQFT, although only the hadronic charged π-mesons had been discovered [23], Heisenberg, experiencing the many-particle production in cosmic radiation showers [24, 26, 27] clearly anticipated the later development of particle physics into a physics with a very large number of strongly interacting particles. It was only 14 years later with the work of Murray Gell-Mann [12] and others, as a reaction to the ever increasing number of hadrons and after playing around with an $SU(3)$ and an 8-fold approach to resolve the problem of *flavor* multiplicity [28], that such non-particle-type constituent fields were introduced into the AUQFT in the form of non-observable 'color' fields, the quarks and gluons, changing the AUQFT into hybrid theories, as described in the last section. Only the hadrons, in contrast to the non-hadrons, are treated in the constituent field fashion. However, they still adhere to their canonical behavior at the unobservable short distances, perhaps an unnecessary luxury because this is only required for asymptotic fields to enable a unitary S-matrix for the in- and out-states.

The further development of Heisenberg's more fundamental RUQFT approach was hindered and later totally ignored because of the phenomenologically highly successful Standard Model which, however, aimed primarily at a half-way manageable theory – convergent-renormalizable and hence accessible at small distances by perturbation theory – rather than seeking a more profound understanding of the phenomenologically exhibited dynamics and its strange symmetry pattern. As was expected, at least from the RUQFT point of view, a more profound insight into the basic dynamics would prove rather difficult if one simply tried now to improve on the present Standard Model. A more radical approach seems to be unavoidable.

But how radical should it be? New investigations along the lines of RUQFT since the early 1980s seem to indicate that the necessary changes to the familiar QFT may not be as severe as to require a dramatic modification of the basic quantum principles (e.g., the introduction of a fundamental length ℓ indicating a spacetime region where the quantum principle is grossly violated). Hence my goal in the present paper is not to compete with the SM in its results, but rather to take a different path, which allows us to reach what we believe is the heart of the problem: to establish from basic (mainly group-theoretical) considerations a fundamental highly nonlinear quantum field theory with a radically simple and unique structure which, under various more complicated conditions than hitherto assumed, permits approximate linearizations and can effectively lead to a 'baroque' phenomenological field theory of the same type as the SM. In addition, this very simple and compact fundamental quantum theory may offer a glimpse of an even more profound level: a purely algebraic description at the foundation which avoids the spacetime-dependent fields altogether at the outset, but prepares their effective appearance, relating them to operators parameterized by the orbits of the fundamental Lie group.

2.3.2 Heisenberg–Pauli Nonlinear Spinor Theory

Observations and studies of the multiple-particle high-energy showers of cosmic rays convinced Heisenberg [17, 24–26] that this could not be solely the result of many consecutive few-particle showers but must be indicative of many-particle production at very small distances connected with a rather strong interaction there. Hence it was rather unlikely that a fundamental particle theory would be convergent-renormalizable or even super-renormalizable, reflecting a free theory behavior. The divergencies in non-convergent-renormalizable and non-renormalizable theories on the other hand should not be considered a basic deficiency but simply a reflection of the fact that the canonical commutation rules commonly applied for quantization are intimately connected with the classical solutions of the free particle wave equation with a $\delta(\boldsymbol{x})$-function point source. For non-renormalizable theories the commutation relations are affected by the short-range interactions and hence have to be changed accordingly [5, 39].

This led Heisenberg to suggest a quantum field theory based solely on a single, massless anticommuting (4-component) Dirac spinor field $\psi(x)$ as the basic constituent field, coupled non-linearly to itself:

$$\mathrm{i}\gamma^{\mu}\partial_{\mu}\psi(x) + \ell^2 \Gamma\psi(\bar{\psi}\Gamma\psi) = 0 \, , \tag{2.1}$$

rather analogously to the anharmonic oscillator in quantum mechanics. In this case of a canonical field (mass or inverse length dimension $\dim\psi = 3/2$), the coupling constant will have the dimension of a length-square ℓ^2 (or mass dim $= -2$) which indicates the dominance of the interaction term over the kinetic term, identifying the theory as non-renormalizable.

Pauli [29] noticed in 1957 that the neutrino equation

$$i\gamma^{\mu}\partial_{\mu}\psi = 0 \tag{2.2}$$

is not only invariant under the $U(1)$ phase transformation, but also with regard to the extended 3-parameter group, the Pauli transformations

$$\psi \to a\psi + b\gamma_5\tilde{\psi} \, , \qquad |a|^2 + |b|^2 = 1 \, , \qquad \tilde{\psi} \equiv C^{-1}\bar{\psi}^{\mathrm{t}} \, , \tag{2.3}$$

which later on was shown by Gürsey [30] to be isomorphic to the $SU(2)$ isospin transformations. Together with the $U(1)$ Touschek transformation [31]

$$\psi \longrightarrow \mathrm{e}^{\mathrm{i}\alpha\gamma_5}\psi \, , \tag{2.4}$$

related to a fermion number F, this results in the group

$$G = U(2) = \underset{F}{U(1)} \otimes \underset{I}{SU(2)} \tag{2.5}$$

as internal invariance group of the massless fermion equation. The surprising feature, however, was that this enlarged invariance group G_ν could be maintained by adding a nonlinear pseudovector self-interaction

$$\mathrm{i}\gamma^\mu \partial_\mu \psi + \ell^2 \gamma_5 \gamma^\mu \psi (\bar{\psi} \gamma_5 \gamma^\mu \psi) = 0 \; , \tag{2.6}$$

leading to the Heisenberg–Pauli spinor equation in 1958 [8]. This result seemed very interesting, firstly because the quartic interaction term could be fixed uniquely, and secondly, because it automatically provided the isospin group without the usual ad hoc doubling of the number of components of the spinor field. Shortly afterwards this surprise was somewhat dampened by the present author [32], with the discovery that the Heisenberg–Pauli equation can be simply interpreted as arising from an equation for a 2-component Weyl spinor field χ and the usual doubling of components, written as $\boldsymbol{\chi}$, to include isospin:

$$\mathrm{i}\sigma^\mu \partial_\mu \boldsymbol{\chi} + \ell^2 \sigma^\mu \boldsymbol{\chi} (\boldsymbol{\chi}^* \sigma_\mu \boldsymbol{\chi}) = 0 \; , \tag{2.7}$$

where the uniqueness of the quartic term prevails after doubling (in the σ-notation an isospin unit matrix is suppressed). Of course, The internal invariance group (2.5) does not suffice to embrace the empirical classification, which at that time (1958) required at least two charge-type numbers and two different fermion numbers. The electric charge was not identical with the third component of the isospin I_3 but

$$Q = I_3 + Y \tag{2.8}$$

contains in addition a hypercharge Y, which the theory (2.7) does not provide. Similarly, the fermion number has to be augmented by the introduction of an additional fermion-type number Λ such that baryon and lepton number can be distinguished:

$$B = F + \Lambda \; , \qquad L = F - \Lambda \; . \tag{2.9}$$

The difference of the non-provided quantum numbers, i.e.,

$$S = Y - \Lambda = Y - \frac{1}{2}(B - L) \; , \tag{2.10}$$

corresponds empirically to the property ‘strangeness’, the only member of the other flavor families known at the time. The corresponding larger internal invariance group would then have the extended form

$$\begin{array}{c} G' = U(1) \otimes U(1) \otimes U(1) \otimes SU(2) \; , \\ \quad Y \qquad\;\; \Lambda \qquad\;\; F \qquad\quad I \end{array} \tag{2.11}$$

which, if basically incorporated into a spinor equation, would require a quadrupling of the spinor components to 16. Instead, at that time, the invariance group suggested was

$$\begin{array}{c} G'' = U(1) \otimes U(1) \otimes SU(3) \; , \\ \quad \Lambda \qquad\;\; F \qquad \text{flavor} \end{array} \tag{2.12}$$

involving 12 components with fermion number doubling and extending the isospin $SU(2)$ group to a flavor $SU(3)$, not to be confused with the present color $SU(3)$.

It was actually this question, how best to accommodate the additional quantum numbers – we stuck to $SU(2)$ flavor, whilst others favored a more pragmatic approach by simply adding a sufficient number of new fields – that led to the departure of the development that finally became the well-known Standard Model. The main goal of RUQFT was different. From the point of view of the radical approach, an increase in the number of basic constituent fields was not an admissible procedure because it automatically destroyed the uniqueness of the field equation, provided that no higher internal symmetries like $SU(3)$ and higher were enforced. It is noteworthy that even in the SM, the $SU(3)$ flavor group did not survive, although only by transferring the group theoretical deficiencies by postulating the existence of three 'families', the origin of which has remained a mystery.

The new mechanism for obtaining 'additional' symmetry groups was viewed in our RUQFT approach as abandoning the requirement for the 'ground state' to be simply the 'vacuum', i.e., a representation of 'empty' or a state of pure 'nothingness', which, by definition, must have the property of being unique and invariant under the full symmetry group of the field dynamics. In fact, it was argued that the ground state should be considered as an effective 'background state' approximating everything 'outside' what is actually observed. In particular, the observed 'broken' isospin invariance reflected by the non-degenerate isospin multiplets seemed to suggest an asymmetric 'iso-ferromagnetic' or 'anti-ferromagnetic' type of polarisation of the ground state, which arose, as in superconductivity, from a Bose–Einstein condensation of isospinor–spinor pairs. Such an isospin-asymmetric ground state has two consequences:

- The occurrence of zero-mass modes similar to the Bloch spin waves of the ferro- and antiferromagnet, but here in the form of isospin-flip modes, or in modern terminology, Goldstone modes connected with the asymmetry of the ground state.
- The possibility of uncommon 'dressings' of the original bare constituent fields and their 'bound states' forming the dressed particles to be identified with the observed particles that may differ not only in their mass from

their bare modes but also by properties carried by the ground state, i.e., isospin properties.

In the original RUQFT papers [9], it was suggested that isospin-Bloch waves (zero-mass charged bosons) were eventually responsible for the appearance of the massless photons assuming an anti-isoferromagnetic background structure and neutral pairs of flip-up/flip-down modes to form photons [33]. This did not prove to be very successful.

Surprisingly, the second aspect of the unusual dressings has never been taken up in the quantum field theoretical models which dominate today, including the SM. This feature allows the introduction of 'isospin-frozen' fields, i.e., isospinor fields where the non-aligned part of the isospin is shielded by an isospin-wave dressing. They are nonlinear representations of the $SU(2)$ [34–37]. These dressings were used to give meaning to the 'spurion' introduced in 1956 by Wentzel [38] to generate strange baryons (Λ and Σ) from nucleons. In this case the hypercharge is simply the remaining I_3 charge of a frozen isopinor. The linear and nonlinear representations of $SU(2)$ cannot transform into each other, and hence define different classes of particles which are independently conserved. In addition, the dressings, as in the polaron case, will not necessarily be local but may lead to a finite spatial extension of the dressed particle (a soliton involving an infinite number of Bloch-wave bosons), in contrast to the local constituent fields. This opens a completely new way of looking at hadrons and also offers an opportunity to distinguish the different flavor families.

The close connection between asymmetric ground states and the existence of zero-mass spinless modes, although in special cases well known before, was demonstrated in 1961 for the non-relativistic case by Goldstone [10] and shortly later generalized to the relativistic case by Goldstone, Salam and Weinberg [11]. The Goldstone particles can be imagined as localized infinitesimal transformations that change the oriented ground state into an off-direction infinitesimally over a finite region, which, if infinitely extended (not localized and hence zero momentum), should not change the energy because of the formal energy degeneracy of different oriented states. The Goldstone modes are thus operator-valued Lie parameters of the broken symmetry transformations.

The asymmetry of the ground state $|\Omega\rangle$ can be expressed by the asymmetry condition of the ground state expectation value

$$\langle\Omega|\tilde{\boldsymbol{\chi}}^*\boldsymbol{\tau}^i\boldsymbol{\chi}|\Omega\rangle = \text{const.} \times \delta_3^i \tag{2.13}$$

of the isovector Higgs field $\tilde{\boldsymbol{\chi}}^*\boldsymbol{\tau}\boldsymbol{\chi}$, a scalar pair of the constituent $\boldsymbol{\chi}$ fields, in fact, the only possible local Lorentz-invariant isovector field (aside from its Hermitian conjugate) constructible from the $\boldsymbol{\chi}$. This allows the representation

$$\tilde{\boldsymbol{\chi}}^*(x)\boldsymbol{\tau}_i\boldsymbol{\chi}(x) = \exp\left[\mathrm{i}\varphi_0(x) + \mathrm{i}\varphi_3(x)\right]\left[\varphi_1(x)\times + \varphi_2(x)\times\right]\tilde{\boldsymbol{\chi}}^*(x)\boldsymbol{\tau}_3\boldsymbol{\chi}(x)\ , \tag{2.14}$$

with $\varphi_{1,2}(x)$ the two Goldstone fields of the isospin transformations, and $\varphi_0(x)$ an additional Goldstone field connected with the additional asymmetry of the ground state as exhibited by the non-Hermitian condition (2.13) with regard to the F-phase transformation. The latter implies the possibility for a freezing of the fermion number F by appropriate dressings and offers the opportunity to generate the new quantum number Λ introduced ad hoc in (2.9).

I will not go into a detailed discussion of the calculation of the various 'bound' states of the constituent fields, which was extensively described and treated in earlier papers [9, 39–41]. To succeed in producing finite results for masses and coupling constants, an effective change (averaging over an essential singularity) of the quantization rule was necessary, involving a cutoff of the divergent 2-point function at small distances $|x| \leq \ell$ (universal length) and applying the New–Tamm–Dancoff approximation method for calculating the masses of the nucleons and mesons. The simplest mesons consisted of the isoscalar and isovector Lorentz pseudoscalars and Lorentz-vector mesons generated from the S-bound state of the urfield/anti-urfield pairs $\boldsymbol{\chi}^*\boldsymbol{\chi}$, $\boldsymbol{\chi}^*\boldsymbol{\tau}\boldsymbol{\chi}$, $\boldsymbol{\chi}^*\sigma_\mu\boldsymbol{\chi}$, $\boldsymbol{\chi}^*\sigma_\mu\boldsymbol{\tau}\boldsymbol{\chi}$ phenomenologically connected to the η, π, ω and ρ and this at a time (1958) when only the π mesons were known. The mass of the η meson was correctly predicted relative to the π mass. A calculation of the Sommerfeld fine-structure constant was also attempted [42].

Numerical results could, however, only be obtained by using the rather crude cutoff procedure which had the serious consequence that the metric of the quantum-mechanical Hilbert space was no longer positive definite (Nevanlinna space). Therefore states with negative norm (ghost states) in principle could not be avoided, leading to a non-unitary S-matrix and violation of the probability interpretation of the wave functions. Details and many references can be found in Heisenberg's book [47].

Many interesting attempts were made using various dressing mechanisms to construct effective fields and interactions which could simulate the fields of the SM, mainly concentrating on the electromagnetic–weak interactions, but also including the difficult task of obtaining some understanding of the strong interaction and its apparent color $SU(3)$ quality [43–46].

I will restrict myself here to considerations concerning the symmetry group structure. The main weakness of the Heisenberg–Pauli spinor theory seemed to be connected to overshooting the effective regularization of the interaction to the extent that it was treated in practical calculations like a super-renormalizable theory where a coupling constant of mass dimension rather than of length dimension provided the ad hoc assumed basic length scale. This caused difficulties in generating gauge-invariant interactions, in particular electrodynamics from 'bound state' considerations. However, gauge-invariant interactions proved to be of decisive importance in the SM. On the other hand, gauge invariance (i.e., invariance under x-dependent symmetry transformations) is directly linked to the equal importance of the kinetic and the interaction term, indicating the absence of an explicit length parameter (universal length) at the outset and hence requiring an invariance under scale transformations. This requirement leads to an even more radical formulation of the RUQFT, which will be sketched in the next section.

2.3.3 Gauge-Invariant and General Spacetime-Invariant Nonlinear Spinor Theory

To establish a gauge-invariant pure nonlinear spinor theory in the context of the RUQFT approach requires that gauge fields should not be added ad hoc but rather generated from the constituent spinor field, the urfield. This necessarily means that no spacetime derivatives, and hence no kinetic term for the urfield can occur in the basic Lagrangian or the corresponding field equation. The Lagrangian should only contain an 'interaction' term, a purely nonlinear expression of the constituent fields. It should be appreciated at this point that, because of the anticommuting property (Grassmann algebra) of the operator fields $\boldsymbol{\chi}$ and $\boldsymbol{\chi}^*$, the number of their local products is extremely limited. The Pauli exclusion principle limits their local clustering, so to speak, and restricts them to just one mode at each x-point. Because of the 4 components of the field $\boldsymbol{\chi}$ and also 4 for their Hermitian conjugates $\boldsymbol{\chi}^*$, the maximum local product will be an octonic expression, which also exhibits the maximum symmetry. This suggests [48, 49] starting with the maximally symmetrical action

$$S = \int \mathrm{d}^4 x \sqrt{-g} L(x) \;, \tag{2.15}$$

with the ultralocal Lagrangian (density)

$$\sqrt{-g} L(x) = -\frac{1}{N} : \boldsymbol{\chi\chi\chi\chi\chi}^*\boldsymbol{\chi}^*\boldsymbol{\chi}^*\boldsymbol{\chi}^* : (x) = -\frac{1}{24N} : \det(\boldsymbol{\chi\chi}^*) : (x) \;, \tag{2.16}$$

constructed from the 4-component, anticommuting, non-Hermitian isospinor–spinor field $\boldsymbol{\chi}(x) \neq \boldsymbol{\chi}^*(x)$ ('urfield'):

$$\boldsymbol{\chi} \equiv \boldsymbol{\chi}_\alpha(x) \;, \qquad \alpha = 1, 2, 3, 4 \;, \tag{2.17}$$

where N is an appropriate normalization factor. Such a self-interaction term was already considered in 1977 by Heinrich Saller [50].

This Lagrangian is 'ultralocal' in a more extended sense than used earlier by Klauder [51], that it lives solely on separate spacetime points and hence, prima facie, cannot generate interaction but only self-action, i.e., no dialogues but merely monologues. This, however, only holds in a classical interpretation of the fields. In the case of quantum fields, the products of the quantum fields become singular as a consequence of the quantization condition. These singularities have to be subtracted from the local product by a regularization procedure, a Wick product prescription, which is indicated by the (still undefined) double-dots : : . This implies an infinitesimal nonlocality which can generate effective derivative terms for various local operators constructed from $\boldsymbol{\chi}$ and $\boldsymbol{\chi}^*$ and therefore corresponding effective kinetic terms in the Lagrangian and propagators. In particular, in the case of spontaneous symmetry breaking, there should appear Goldstone modes with 'soft' kinetic terms which disappear in the limit of small distances.

To secure scale-invariance of the action S, the urfield must be given the mass (or inverse length) dimension

$$\dim \boldsymbol{\chi}(x) = \dim \boldsymbol{\chi}^*(x) = \frac{1}{2} \ , \tag{2.18}$$

i.e., transform under scale transformations as

$$\boldsymbol{\chi} \longrightarrow \mathrm{e}^{\eta/2}\boldsymbol{\chi} \ , \tag{2.19}$$

which is subcanonical as compared to the canonical dimension $\dim \psi = 3/2$ of physical spinor fields ψ corresponding to $\psi^*\psi$ being a 3-space density.

To specify the Wick finite-part product : :, we postulate for the point-split $\boldsymbol{\chi}(x_+)\boldsymbol{\chi}^*(x_-)$ product the formal Laurent expansion in terms of the (timelike) split-vector ξ (with $x_\pm = x \pm \xi/2$) and in accordance with the dimension assignment (2.18):

$$\boldsymbol{\chi}_\alpha(x_+)\boldsymbol{\chi}^*_{\dot\beta}(x_-) = \frac{\mathrm{i}}{2\pi}\bar{N}\frac{\xi_\mu}{\xi^2}h^\mu_{\ \alpha\dot\beta}(x) + V_{\alpha\dot\beta}(x,\xi) \ , \tag{2.20}$$

with the first term explicitly exhibiting the singular part $\sim \xi^{-1}$. This can also be expressed by the condition

$$h^\mu_{\ \alpha\dot\beta}(x) = \frac{1}{\bar{N}}\lim_{\xi\to 0}\xi^\mu\boldsymbol{\chi}_\alpha(x_+)\boldsymbol{\chi}^*_{\dot\beta}(x_-) \neq 0 \ . \tag{2.21}$$

Hence the finite, ξ-independent Lagrangian (2.16) can now be cast into the more transparent form

$$\begin{aligned}\sqrt{-g}L(x) &= -\frac{1}{N}\lim_{\xi\to 0} X(x_+)X^*(x_-)\\ &= -\frac{1}{24\times 8N}\lim_{\xi\to 0}\partial^2_\xi\partial^2_\xi\xi^2\xi^2[X(x_+)X^*(x_-)] \ .\end{aligned} \tag{2.22}$$

The action (2.15) is formally invariant under the huge 47-parameter or respectively, the x-dependent parameter function ('local' or gauge) symmetry group

$$G_{\max} = \underset{\text{hybrid}}{D(1,\mathrm{loc})} \otimes \underset{\text{external}}{SL(4,R,\mathrm{loc})} \otimes U(1,\mathrm{loc}) \otimes \underset{\text{internal}}{SL(4,C,\mathrm{loc})} \ , \tag{2.23}$$

which contains:

- the external volume-conserving 15-parameter special linear transformations in the $d = 4$-dimensional spacetime manifold corresponding, in the x-dependent form, to the volume-conserving transformations of general relativity;
- the internal 1-parameter phase transformation for a fermion number F and the 30-parameter linear complex transformations of the $2n = 4$ spinor components (n = number of Weyl fields), ultimately interpreted as isospinor–spinor components;
- the 1-parameter hybrid internal–external dilations acting on the coordinate differentials as well as on the urfields according to the specification (2.19) under the condition $d = 2n$.

The Lagrangian even exhibits other symmetries. We can establish an invariance under spacetime translations and the general 'local' conformal transformations [50, 52, 53] and also an $N = 2$ supersymmetry [54].

The invariance of the action under the corresponding extended x-dependent transformations is, of course, a consequence of the formal complete separateness of all spacetime points. It is only because of the 'softening' of the 'ultra-locality' by the : : regularization prescription connected with the quantum character of the urfields without destroying this local invariance that the local invariance with regard to the internal groups now acquires the non-trivial meaning of a 'gauge' group. This implies that all derivatives which are effectively generated from the 'softening' of the nonlinear term now appear automatically as 'covariant' derivatives involving compensating vector gauge fields, which must be constructs from the urfields. In particular, we immediately realize [55] that the finite-part isovector–vector field

$$\boldsymbol{A}_\mu(x) = \frac{1}{2} : \boldsymbol{\chi}^* \sigma_\mu \boldsymbol{\tau} \boldsymbol{\chi} : (x) \ , \tag{2.24}$$

under x-dependent isospin transformations

$$\boldsymbol{\chi}(x) \longrightarrow \mathrm{e}^{\boldsymbol{\tau}\cdot\boldsymbol{\varphi}(x)/2} \boldsymbol{\chi}(x) = \left[1 + \frac{1}{2} \boldsymbol{\tau} \cdot \boldsymbol{\varphi}(x) + \cdots \right] \boldsymbol{\chi}(x) \ , \tag{2.25}$$

automatically shows the inhomogeneous behavior

$$\boldsymbol{A}_\mu(x) \to \mathrm{e}^{\boldsymbol{\varphi}(x)\times} \boldsymbol{A}_\mu(x) + \partial_\mu \boldsymbol{\varphi}(x) = \boldsymbol{A}_\mu(x) + \boldsymbol{\varphi}(x) \times \boldsymbol{A}_\mu(x) + \partial_\mu \boldsymbol{\varphi}(x) + \cdots \ , \tag{2.26}$$

which always occurs in the combination of a covariant derivative

$$D_\mu \equiv \partial_\mu + \frac{\mathrm{i}}{2} \boldsymbol{\tau} \cdot \boldsymbol{A}_\mu \ . \tag{2.27}$$

However, the huge symmetry group (2.23) is not realized for the quantum field theory because of the smaller symmetry of the quantization condition of the fields under the following requirement for the anticommutator (suppressing a unit isospin matrix in the notation):

$$\begin{aligned}\lim_{\xi\to 0}\{\boldsymbol{\chi}_\alpha(x_+)\boldsymbol{\chi}^*_{\dot\beta}(x_-)\} &= -\mathrm{Im}\frac{\bar N}{2\pi}\frac{\xi_\mu}{\xi^2-\epsilon(\xi^0)}h^\mu_{\alpha\dot\beta}(x)\\ &= \bar N h^\mu_{\alpha\dot\beta}(x)\xi_\mu\epsilon(\xi^0)\delta(\xi^2)\\ &= -\bar N|\xi_0|\delta(\xi_0^2) = \text{const.}\end{aligned} \tag{2.28}$$

This quantization condition can also be phrased as a condition for the ground state, viz.,

$$\langle\Omega \mid h^\mu_{\alpha\dot\beta}(x) \mid \Omega\rangle = \delta^\mu_m\bar\sigma^m_{\alpha\dot\beta}\ . \tag{2.29}$$

This states that $h(x)^\mu_{\alpha\dot\beta}$ is essentially the mixed tetrad tensor related to the metric tensor in the usual way, and is required to contain the Minkowski flat space metric as exhibited by the most singular part of (2.28).

The quantization condition (2.28) induces a tremendous breaking of the maximum group (2.23) down to the 11-parameter stability group of the (assumed) translationally invariant ground state:

$$G_\Omega = \underset{\text{hybrid}}{D(1)} \otimes \underset{\text{hybrid Lorentz}}{SO(3,1)} \otimes \underset{\text{internal}}{U(1)\otimes SU(2)}\ , \tag{2.30}$$

giving rise to $47-11=36$ Goldstone modes connected with the 36-parameter coset

$$\frac{G}{G_\Omega} = \frac{SL(4,R)}{SO(3,1)}\otimes\frac{SL(4,C)}{SL(2,C)}\otimes SU(2)\ . \tag{2.31}$$

In contrast to the usual examples of a condensate caused by a Higgs boson field, no mass scale is established in our case on this first level of symmetry-breaking. It is rather the Planck constant $\hbar$ (not explicitly written) that is responsible. The connection between spacetime properties and spin degrees of freedom, investigated by many [56–58], is thus enforced by quantization; or the other way around, quantization provides the foundation for such a parameter space.

It is actually interesting to observe that the symmetry breakdown condition for the mixed tensor $h^\mu_{\alpha\dot\beta}$ enforces a fixed 'spin–orbit' coupling, which is the 4-dimensional analogue of a $J=2$ (S-triplet/P-wave). It imprints a Minkowski metric η_{ik}, natural to the spin algebra ($\sigma_i\bar\sigma_k$), on the spacetime manifold. The 9 Goldstone excitations of this condensate characterize local deviations from the flat Minkowski metric and relate to the 9 (volume conserving) degrees of freedom of the usual 10 components of the gravitational field or the metric tensor with fixed determinant. Hence the ground state has some similarity with the B-phase of supercooled ^{3}He [59], in which a $^2P_2$ configuration with total spin $J=2$ occurs in such a way that, in the corresponding spontaneous breakdown of the orbital and spin symmetry (SBSOS), only the relative orientation of orbital and spin direction gets fixed (pseudo-isotropy). The excitations of the relative orbital-spin (Goldstone) vibrating modes, the spin-nematic waves of the condensate, correspond to the gravitational degrees of freedom in our case.

The 21 Goldstone modes connected with the broken internal symmetries related to the coset $SL(4,C)/SU(2)$ will not really show up because they are absorbed by the corresponding vector gauge fields (Anderson–Higgs mechanism) [60,61] which are also generated, and will eventually give rise to massive vector particles.

In addition to this 'hard' symmetry breaking, we can imagine additional 'soft' breakings taking into account additional terms of the operators in a ξ-expansion $\sim m^2\xi\xi$, $m^4\xi\xi\xi\xi$, etc. In analogy with the usual symmetry-breaking mechanism, e.g., in the case of the phase transformation symmetry $U(1)$ or the isospin symmetry $SU(2)$ as indicated in (2.13), they would then produce corresponding kinetic terms. It should be noticed in this context that the scalar bilinear forms $:\boldsymbol{\chi}(x)\boldsymbol{\chi}(x):$ and $:\boldsymbol{\chi}^*(x)\boldsymbol{\chi}^*(x):$ do not actually require $:\,:$ regularization because the anticommutator of like fields vanishes.

In a similar way additional terms $\sim M^2\xi\xi$ on the right-hand side of (2.20) in $V_{\alpha\dot\beta}(x,\xi)$ would give rise to a general covariant kinetic term for the gravitational field $\sim M^2$ with the important consequence that, in comparison with the interaction term, a gravitational coupling constant $\sim 1/M^2 \sim \ell^2_{\text{Planck}}$ would naturally occur. This means that M should be identified with the huge gravitational mass

$$M = \ell^{-1}_{\text{Planck}} = 1.22\times 10^{19}\ \text{GeV}\ . \tag{2.32}$$

The explicit evaluation of the Lagrangian (2.22) produces the various derivative forms connected to kinetic terms and derivative couplings. They are rather numerous and are given elsewhere [49]. The leading term is a third-derivative expression,

$$L(x) = \frac{2\pi^2}{\bar N} : \boldsymbol{\chi}^* \frac{\mathrm{i}}{2}^3 \sigma \overleftrightarrow{\partial} \overleftrightarrow{\partial^2} \boldsymbol{\chi} : (x) + \cdots\ , \tag{2.33}$$

with the normalization factor $N = \bar N^4/96\pi^2$, where we have suppressed the gauge fields which augment the derivatives to the covariant expressions.

The corresponding Green's function or propagator depicts a 'double pole' in momentum space instead of the common single pole for a particle:

$$G(p) \sim \bar N \frac{\bar\sigma\cdot p}{(p^2)^2}\ , \tag{2.34}$$

which correctly reflects the anticommutator rules for a fermion of subcanonical dimension 1/2. The double pole describes a 'dipole ghost', treated extensively by Heisenberg in 1957 [20] and refers to unphysical states of zero norm and therefore zero probability.

If a softer mass term $\sim M^2$ appears in the expansion of $V[x,\xi]$ in (2.20) then this double pole may be pulled apart to positive and negative norm contributions:

$$G(p) \sim \bar{N} \frac{\bar{\sigma} \cdot p}{p^2(p^2 \pm M^2)} = \pm \bar{N} \frac{\bar{\sigma} \cdot p}{M^2} \left(\frac{1}{p^2} - \frac{1}{p^2 \pm M^2} \right) , \tag{2.35}$$

which looks like a regular neutrino propagator with a momentum cutoff at the mass M, e.g., like the huge gravitational mass (2.32) mentioned above. If the minus sign occurs, the second term will correspond to a negative norm (ghost) fermion of mass M (if $\bar{N} = -1$ is chosen).

I will not go into further details here. It can be demonstrated [49] that essentially all the effective local fields of the Standard Model can be generated from the urfield. This is easily imagined if we consider different groupings of the octonic interaction:

$$\begin{array}{ll} \text{Interaction term} & \boldsymbol{\chi}^*\boldsymbol{\chi}^*\boldsymbol{\chi}^*\boldsymbol{\chi}^*\boldsymbol{\chi}\boldsymbol{\chi}\boldsymbol{\chi}\boldsymbol{\chi} \\ \text{Higgs analog} & \boldsymbol{\Phi}^*\boldsymbol{\Phi}^*\boldsymbol{\Phi}\boldsymbol{\Phi} \sim (\boldsymbol{\chi}^*\boldsymbol{\chi}^*)(\boldsymbol{\chi}^*\boldsymbol{\chi}^*)(\boldsymbol{\chi}\boldsymbol{\chi})(\boldsymbol{\chi}\boldsymbol{\chi}) \\ \text{Fermion–gauge} & \boldsymbol{\psi}^*\boldsymbol{A}\boldsymbol{\psi} \sim (\boldsymbol{\chi}^*\boldsymbol{\chi}^*\boldsymbol{\chi}^*)(\boldsymbol{\chi}^*\boldsymbol{\chi})(\boldsymbol{\chi}\boldsymbol{\chi}\boldsymbol{\chi}) \\ \text{Higgs–gauge} & \boldsymbol{\Phi}^*\boldsymbol{A}\boldsymbol{A}\boldsymbol{\Phi} \sim (\boldsymbol{\chi}^*\boldsymbol{\chi}^*)(\boldsymbol{\chi}^*\boldsymbol{\chi})(\boldsymbol{\chi}^*\boldsymbol{\chi})(\boldsymbol{\chi}\boldsymbol{\chi}) \end{array} \tag{2.36}$$

and the possible kinetic terms

$$\begin{array}{llll} \text{Fermions} & \boldsymbol{\chi}^*\partial\partial\partial\boldsymbol{\chi} & \boldsymbol{\psi}^*\partial\boldsymbol{\psi} & M^2\boldsymbol{\chi}^*\partial\boldsymbol{\chi} \\ \text{Higgs} & & \partial\boldsymbol{\Phi}^*\partial\boldsymbol{\Phi} & m^2\boldsymbol{\Phi}^*\boldsymbol{\Phi} \\ \text{Gauge} & \partial\boldsymbol{A}\partial\boldsymbol{A} & \boldsymbol{A}\boldsymbol{A}\partial\boldsymbol{A} \;\; \boldsymbol{A}\boldsymbol{A}\boldsymbol{A}\boldsymbol{A} & \\ \text{Goldstone} & m^2\partial\varphi\partial\varphi & M^2\partial h\partial h & \end{array} \tag{2.37}$$

with the effective fields

$$\begin{array}{llll} \text{Canonical fermions} & \dim = 3/2 & \text{left}: \boldsymbol{\psi} = \boldsymbol{\chi}\boldsymbol{\chi}\boldsymbol{\chi} & \text{right}: \boldsymbol{\psi}' = \boldsymbol{\chi}^*\boldsymbol{\chi}\boldsymbol{\chi} \\ \text{Canonical bosons} & \dim = 1 & \boldsymbol{\Phi} = \boldsymbol{\chi}^*\boldsymbol{\chi} & \boldsymbol{A} = \boldsymbol{\chi}^*\boldsymbol{\chi} \\ \text{Goldstone fields} & \dim = 0 & \varphi_\pm \quad v & g_{\mu\nu} \quad h^\mu_{\alpha\dot{\beta}} \end{array} \tag{2.38}$$

An important feature are the Goldstone dressings arising from the combined symmetry breakdown of the isospin group and the fermion number phase transformation. The Goldstone degrees of freedom can be formally accentuated by writing the urfield as

$$\begin{aligned} \boldsymbol{\chi}_\alpha(x) &= \exp\left\{\frac{\mathrm{i}}{2}[\varphi_+(x)\tau^- + \varphi_-(x)\tau^+]\right\} \exp\left\{\frac{\mathrm{i}}{2}[\varphi_0(x) + \varphi_3\tau^3(x)]\right\} \underline{\boldsymbol{\chi}}(x) \\ &= \boldsymbol{s}^3_\alpha(x)v_3(x)\underline{\boldsymbol{\chi}}(x) \, . \end{aligned} \tag{2.39}$$

Here $\boldsymbol{s}^3_\alpha(\varphi_\pm)$ is a transmutator $I_{1,2,3} \to I_3 = Y$, freezing the isospin into the third direction, the hypercharge Y (now like an isoscalar) with only local deviations as expressed by the Goldstone modes $\varphi_\pm(x)$. Furthermore, $v_3(\varphi_0)$ is a transmutator tying the I_3 to the fermion number to enforce $F - I_3 = 0$.

Formally $s(x)$ acts like a local isospinor with an additional frozen charge $Y = 1/2$. It has the properties of Wentzel's spurion [34,38]. $v(x)$ is a local isocalar field with $F = -Y = 1/2$. As a consequence there will be numerous additional effective local fields resulting from different dressings of the constituent field constructs. The electric charge Q of the (asymptotically emerging) dressed particles will only result from their isospin degree of freedom:

$$Q = (I_3)_{\text{total}} = (I_3)_{\text{field}} + (I_3)_{\text{dressing}} \,, \tag{2.40}$$

and the same holds for the relevant fermion number

$$F = F_{\text{total}} = (F)_{\text{field}} + (F)_{\text{dressing}} \,. \tag{2.41}$$

Different dressings, involving an infinite number of Goldstone modes, should represent a highly effective barrier for transitions and hence should give rise to a large number of fermion-number-type conservation laws. As mentioned earlier, this may not only be the key to obtaining independent conservation for baryon and lepton numbers, but it may also offer a hint for the existence of the different flavor families. The left-handed leptons of canonical dim $= 3/2$, the neutrino–electron doublet, may be straightforwardly constructed as a dressed triple field

$$\boldsymbol{\psi}_{\mathrm{L}} \sim v\boldsymbol{\chi\chi\chi} \,, \qquad F = 2 \,, \qquad Y = -\frac{1}{2} \,, \tag{2.42}$$

and a right-handed isoscalar field arises by involving an anti-field

$$\psi'_{\mathrm{L}} \sim v^3 \boldsymbol{s\chi\chi\chi}^* \,, \qquad F = 2 \,, \qquad Y = -\frac{1}{2} \,, \tag{2.43}$$

offering with (2.42) the possibility to establish a massive electron. The right-handed nucleons, the proton–neutron doublet, on the other hand, may have the structure

$$\boldsymbol{\psi}_N \sim v^* \boldsymbol{\chi\chi\chi} \,, \qquad F = 1 \,, \qquad Y = +\frac{1}{2} \,. \tag{2.44}$$

They may be regarded as a composition of three quarks $q \sim (v^*)^{1/3}\boldsymbol{\chi}$, a very artificial construction. It would rather suggest dealing instead with a 4-component system with three fermions, each of only half a charge being embedded into a fourth partner, a smeared out ($F = -1/2$, $Q = +1/2$) Goldstone halo, or an extended soliton-type bag. This may all sound strange, but there is at this stage no indicator for establishing an $SU(3)$ color symmetry. Hence, this still represents a serious weakness in the present theory. Many attempts were made to remedy the situation but none proved satisfactory, in particular regarding the apparent high validity of $SU(3)$ color invariance demonstrated by experiments. Nevertheless, I believe we should not give up searching for an adequate solution of the color aspect within our framework.

To my mind, the dressing mechanism is very important and has shown many interesting features [43–46] which have hardly drawn much attention up to now and hence have not been looked at in more detail by others. A comparison with atomic physics, for example, as indicated by the boron example (Figs. 2.1 and 2.2), shows very clearly how in many-fermion systems the energy term system (analogous to the particle spectrum) exhibits approximately valid new symmetries, which are very remote from the symmetries of the underlying dynamics of the constituents. In particular $SU(n)$ symmetries, with n the number of the coexisting electrons in a shell or subshell, play an important role because of their antisymmetrization (permutation group).

The importance of the dressing aspect related to the phenomenon of spontaneous symmetry breakdown and its formulation by Bose–Einstein condensation with the occurrence of zero-mass Goldstone modes and hence the possibility of small energy deformations has recently drawn a lot of attention [63] connected with new quantum technologies, and specifically regarding quantum computers, and it has even reached the popular press [62]. Indeed, this may be another indication that, beyond the appearance of the gapless excitations in a Bose–Einstein condensate in terms of the massless Goldstone modes, there will be, in general, a high spatial deformation instability or sensibility connected with the possibility of low-energy Goldstone clusters of finite extensions or, if interactions are appropriately taken into account, even of localized 'bound states' of such clusters. These may serve as traps for spinorial field configurations leading to soliton-type bags for fermions (dressings) as required to explain the spatial extension of hadrons without employing the usually assumed gluon fields. Such traps effectively suppress transitions between fermion configurations which otherwise appear to be allowed on the basis of non-gauge-type fermion number conservation. Hence the still mysterious high asymmetry of the universe regarding baryon number B and lepton number L, may not actually be related to a PC-violation but rather to an incorrect assignment of B and L to the basic (fermion number) $U(1)$ symmetry.

It is quite cumbersome to pull out the physically interesting aspects explicitly from the constituent field dynamics represented by the Lagrangian, as explicitly demonstrated in [48, 49], and in particular to calculate numerical mass ratios and coupling constants explicitly. We should not be surprised that this 'dynamical map' [64, 65] from local to asymptotic expressions or bare to dressed fields will be and must necessarily be very complicated. It has been demonstrated in simpler cases how this can be done in a very rough way, but better methods should certainly be developed. Phenomenologically more accessible formulations may be obtained by introducing effective local fields through appropriate constraints using the Lagrange multiplier approach.

Although the suggested formulation of our basic theory [66–68] may still be very far from being called a robust theory or even a theory at all, let me finish with some general remarks about how we might go even further in our attempts to be radically quantum. This will be a highly speculative journey but may encourage new thinking and serve to open new vistas.

Our Lagrangian has still some ad hoc basic features which, from our radical point of view, require further probing. One is the introduction of a 4-dimensional spacetime as a parameter background for our description; the other is the reason for using a 4-component non-Hermitian spinor field instead of the simpler 2-component field, or even something else. There is no question that a spinor field has tremendous advantages over other fields to serve as a constituent field. This is obvious (half-spin, anticommutativity). The spacetime continuum we have chosen as background with an approximately Minkowskian pseudo-Euclidean metric structure gives the 'time' dimension a different role than the three 'space' dimensions. But this background is simply a continuous 4-fold label (similar to the index α of the spinor) for the 'octonic spinor field cluster' which defines the Lagrangian, or better, the Lagrangian density or the basic 'building block'. It is just an enumeration of 'more of the same' and, in fact, an infinite number of these clusters. The action is just the total sum or, if really considered continuous, an integral over all these 'octonic stars'. The 'time' plays an important role, because we use it to give meaning to an order in the definition of the Wick product. The Wick product prescription is based on a time-ordering definition for the fields regarded as operators connecting the time sequence with the consecutive multiplication from the left, defining a time arrow (the 'earlier' is always to the right of the 'later'). The operator character of the spinor is essential and is reflected in the anticommutativity of its products. We can characterize this Grassmann property by step operators $a, a^\dagger$,

$$a = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} , \qquad a^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} , \tag{2.45}$$

satisfying the anticommutation rules

$$\{a, a\} = \{a^\dagger, a^\dagger\} = 0 , \qquad \{a, a^\dagger\} = 1 . \tag{2.46}$$

These step operators describe possible changes in a 2-state system, e.g., of 'nonexistence' and 'existence'. Then with the time-arrow interpretation it gets the meaning

$$\begin{aligned} a^\dagger &= (\text{not to be}) \longrightarrow (\text{to be}) = \text{creation operator} , \\ a &= (\text{not to be}) \longleftarrow (\text{to be}) = \text{annihilation operator} . \end{aligned} \tag{2.47}$$

I consider this relationship to be rather profound in the sense that the operation obtains the meaning of a directed process in a parameter sequence which we call time.

The 4-component constituent spinor $\boldsymbol{\chi}_\alpha$ (on one x-point) now relates to four independent pairs of such step operators $a, a^\dagger$. The question now arises: why do we need a doubled Weyl spinor $\boldsymbol{\chi}$ and not simply χ? What is the reason for the isospin?

In our formulation, the isospin definitely played a secondary role in comparison with the spin, which is closely connected to the spacetime metric. But the isospin doubling nonetheless proved essential in defining an octonic product. Maximally, a single Weyl spinor allows only a quartic product as used in the Heisenberg–Pauli equation (2.6). But this is not actually the case because $\boldsymbol{\chi}_\alpha$ is non-Hermitian and hence $\boldsymbol{\chi}^*_\beta$ can be connected with other pairs $b, b^\dagger$. This can be readily seen if we restrict ourselves to a single Weyl spinor χ_α, which as usual allows the introduction of a_1, a_2, b_1, b_2 and $a_1^\dagger, a_1^\dagger, b_1^\dagger, b_2^\dagger$ with indices 1, 2 referring to the spin up/down components of the Weyl spinor (helicity). This does indeed admit a non-vanishing octonic product:

$$a_1^\dagger a_2^\dagger b_1^\dagger b_2^\dagger a_1 a_2 b_1 b_2 \ . \tag{2.48}$$

Why does this work? We have dissociated the usual connection between the fermion number F and helicity. Perhaps the old Pauli definition of the 4-component spinor as a Majorana–Dirac spinor may after all be more appropriate, with the important addition now, that the Majorana condition $\psi = \gamma_5 \psi^{\mathrm{C}}$ no longer holds for the operators. This would imply that the basic 8-parameter symmetry group

$$GL(2,C) = D(1)(\eta_0) \otimes \frac{SL(2,C)}{SU(2)}(\eta) \otimes U(1)(\alpha_0) \otimes SU(2)(\alpha) \tag{2.49}$$

could perhaps suffice.

The orbits of the non-compact dilatation group and the coset depending on the parameters (η_0, η) applied to a positive or negative timelike vector fill the forward and backward light cones, respectively. For fixed η_0, we obtain the 3-dimensional space on a hyperbola. For $\eta_0 \to -\infty$, this shrinks back to the light cone. Only in this limit are the positive and negative light cones connected at the origin. This suggests identifying the ad hoc introduced spacetime continuum on which the octonic spinor stars are spread out with the 4-parameter space of the non-compact part of the $GL(2,C)$ group. The octonic spinor star consists of four annihilation beams entering from the backward light cone, and four creation beams emerging into the forward light cone, with both 4-beams not quite touching at the origin because of the regularization of the self-action.

It is tempting to connect the evolution of the cosmos with a steady increase in the dilatation parameter η_0, starting at minus infinity at the origin and sweeping step by step over the whole future cone, enlarging the number of spinor stars along the 3-space hyerbola, which will look more and more like our familiar infinite spacetime continuum. The Planck length ℓ_{Planck} may perhaps be connected with the ‘minimum time’ of the creation and annihilation processes inherent in the basic step operators, i.e., if interpreted according to

$$a \cdot a^\dagger + a^\dagger \cdot a \Longrightarrow \frac{1}{\sqrt{\ell}} \left[a\left(+\frac{\ell}{2}\right) a^\dagger \left(-\frac{\ell}{2}\right) + a^\dagger \left(+\frac{\ell}{2}\right) a \left(-\frac{\ell}{2}\right) \right] \ , \tag{2.50}$$

where the factor in front prepares for dim$\boldsymbol{\chi} = 1/2$ under dilatations. However, because of the scale invariance of the dynamics, the Planck length will only show up at longer distances in the form of logarithmic mass terms, or in softer terms connected with spontaneous symmetry breakdowns.

In a way, the cosmos would resemble a huge, continuously growing, parallel closely-linked computer system with software based on the 8-fold general linear transformations in a 2-dimensional complex space $GL(2, C)$ instead of the (0,1)-bit of our present computers. However, because of the basically creative elements and the infinitely open logic, the quantum cosmos, in stark contrast to our fully determined computer, would be essentially open to the future, and hence would correspond more closely to what in our limited meso-world we experience as being 'fully alive'.

References

1. D.R. Finkelstein: *Quantum Relativity*, Springer, Berlin, Heidelberg (1996)
2. H.P. Dürr: What is Life? Scientific Approaches and Philosophical Positions. In: *Series on the Foundations of Natural Science and Technology*, ed. by H.P. Dürr, F.A. Popp, W. Schommers, World Scientific, New Jersey, London, **4**, 145 (2002)
3. W. Heisenberg: Akad. Wiss. Göttingen **I**, 50 (1950)
4. H. Stapp: *Your Place in the Mindful Universe*, Workshop: *Quo Vadis Quantum Mechanics?*, Center of Frontier Science, Temple University, September (2002)
5. W. Heisenberg: Z. Naturforschung **5a**, 251 (1950)
6. P.A. Schilpp: *Albert Einstein: Philosopher-Scientist*, Tudor Publishing Company, New York (1951) pp. 21–23
7. H.P. Dürr: *Properties of Matter under Unusual Conditions*, in honor of E. Teller's 70th birthday, ed. by H. Mark, S. Feshbach, Wiley & Sons, New York (1969) p. 301; Lect. Notes Math., Springer, Berlin, Heidelberg, New York, **79**, 259 (1977)
8. W. Heisenberg, W. Pauli: *On the Isospin Group in the Theory of Elementary Particles*, preprint Jan. 1958, MPT für Physik, Göttingen (unpublished)
9. H.-P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, K. Yamazaki: Z. Naturforschung **14a**, 441 (1959)
10. J. Goldstone: Nuovo Cim. **19**, 154 (1961)
11. J. Goldstone, J.A. Salam, S. Weinberg: Phys. Rev. **127**, 965 (1962)
12. M. Gell-Mann: Phys. Lett. **8**, 214 (1964); G. Zweig: CERN Reports Nr. 8182/TH 401/TH 412 (1964)
13. P. Langacker: Phys. Rep. **72**, 185 (1981); H. Saller: *Vereinheitlichte Feldtheorie der Elementarteilchen*, Lecture Notes in Physics, Springer, Berlin (1985)
14. A. Einstein: Preuss. Akad. Wiss. Berlin **78**, 799, 831, 844 (1915)
15. A. Einstein: Preuss. Akad. Wiss. Berlin Sitzungsberichte, 541 (1931), 130 (1932)
16. H. Weyl: Preuss. Akad. Wiss. Berlin **26**, 465 (1918)
17. W. Heisenberg: Ann. Physik **32**, 20 (1938); **110**, 251 (1938)
18. W. Heisenberg: Z. f. Physik **120**, 513, 673 (1943); **123**, 93 (1944)
19. T.D. Lee: Phys. Rev. **95**, 1329 (1954); G. Kållen, W. Pauli: Mat. Phys. Medd. **30**, 7 (1955); S. Schlieder: Z. Naturforschung **15a**, 448, 460, 555 (1960)

20. W. Heisenberg: Nucl. Physics **4**, 532 (1957); W. Karowski: **23A**, 126 (1974)
21. S.N. Gupta: Proc. Phys. Soc. A **63**, 681 (1950); K. Bleuler: Helv. Phys. Acta **23**, 567 (1950)
22. H.P. Dürr, E. Rudolph: Nuovo Cim. **65A**, 423 (1970)
23. C.M. Lattes, C.P.S. Occhialini, C.F. Powell: Nature **160**, 453, 486 (1947)
24. W. Heisenberg: Z. f. Physik **101**, 533 (1936); **113**, 61 (1939); Rev. Mod. Physics **11**, 241 (1939)
25. W. Heisenberg: Z. f. Physik **126**, 571 (1949)
26. W. Heisenberg: Nuovo Cim. **6**, 493 (1949); Z. f. Physik **126**, 569 (1949); **133**, 65 (1952); Naturwiss. **39**, 69 (1952)
27. W. Heisenberg: Z. f. Physik **133**, 65 (1952)
28. M. Gell-Mann: Phys. Rev. **125**, 1067 (1962); J. Rosner: Phys. Reports C **11**, 189 (1974)
29. W. Pauli: Nuovo Cim. **6**, 204 (1957)
30. F. Gürsey: Nuovo Cim. **7**, 411 (1958)
31. B. Touschek: Nuovo Cim. **5**, 1281 (1957)
32. H.P. Dürr: Z. Naturforschung **16a**, 327 (1961)
33. W. Heisenberg, H. Wagner, K. Yamazaki: Nuovo Cim. **59A**, 377 (1969); W. Heisenberg, K. Yamazaki: Nuovo Cim. **11B**, 125 (1972)
34. H.P. Dürr, W. Heisenberg: Z. f. Naturforschung **16a**, 726 (1961); Nuovo Cim. **37**, 1446, 1487 (1965); H.P. Dürr, J. Géhéniau: Nouvo Cim. **28**, 132 (1963)
35. H. Saller: Nuovo Cim. **30A**, 541 (1975); **34A**, 99 (1976); H.P. Dürr, H. Saller: Nuovo Cim. **39A**, 31 (1977); **41A**, 677 (1977); **48A**, 505, 561 (1978); **60**, 79 (1980)
36. H.P. Dürr, H. Saller: Phys. Lett. B **84**, 336 (1979); Nuovo Cim. **53A**, 469 (1979); **65A**, 147 (1981)
37. H.P. Dürr, H. Saller: Nuovo Cim. **60A**, 79 (1980)
38. G. Wentzel: Phys. Rev. **101**, 1214 (1956)
39. W. Heisenberg: Nachr. Akad. Wiss. Göttingen **IIA**, 111 (1953); Z. Naturforschung **9a**, 292 (1954)
40. W. Heisenberg, F. Kortel, H. Mitter: Zeits. f. Naturforschung **10a**, 425 (1955); R. Ascoli, W. Heisenberg: Zeits. f. Naturforschung **12a**, 177 (1957)
41. H.P. Dürr, H. Saller: Zeits. f. Naturforschung **22a**, 1819 (1967)
42. H.P. Dürr, W. Heisenberg, H. Yamamoto, K. Yamazaki: Nuovo Cim. **38**, 1220 (1965)
43. H.P. Dürr, H. Saller: Phys. Lett. B **84**, 336 (1979); Nuovo Cim. **53A**, 469 (1979); **63A**, 147 (1981)
44. H.P. Dürr, H. Saller: Phys. Rev. D **22**, 1176 (1980)
45. H. Saller: Nuovo Cim. **64A**, 141 (1981)
46. H. Saller: Nuovo Cim. **77A**, 437 (1983); **82A**, 299 (1984)
47. W. Heisenberg: *Introduction to the Unified Theory of Elementary Particles*, Wiley & Sons, London (1966)
48. H.P. Dürr: Nuovo Cim. **62A**, 69 (1981); **65A**, 195 (1981)
49. H.P. Dürr: Nuovo Cim. **73A**, 165 (1983)
50. H. Saller: Nuovo Cim. **42A**, 189 (1977)
51. J. Klauder: Commun. Math. Phys. **18**, 307 (1970)
52. H.P. Dürr: Nuovo Cim. **4A**, 187 (1971); Gen. Rel. and Grav. **4**, 29 (1973); Nova Acta Leop. **212**, 81 (1974)
53. H.P. Dürr, P. du T. van der Merwe: Nuovo Cim **23A**, 1 (1974); H.P. Dürr: Nuovo Cim. **27A**, 305 (1975); H.P. Dürr, C.C. Chiang: **28A**, 89 (1975)

54. H.P. Dürr: Physics Lett. B **115**, 33 (1982); B **122**, 368 (1983)
55. H.P. Dürr, N.J. Winter: Nuovo Cim. **70A**, 467 (1970); **7A**, 461 (1972)
56. C.F. von Weizsäcker: *Die Einheit der Natur* (Hanser, Munich); *Quantum Theory of Time and Space* (Hanser, München) **1**, 213 (1975); **2**, 86 (1977); **3**, 7 (1979)
57. D. Finkelstein: *Quantum Theory and the Structure of Time and Space* (Hanser, Munich) **2**, 46 (1977)
58. H.P. Dürr: Nova Acta Leop. **44**, 37 (1976)
59. A.J. Leggett: Phys. Rev. Lett. **29**, 1227 (1972); J. Physics C **6**, 1387 (1973); M. Lin, C. Cross: Phys. Rev. Lett. **4**, 250 (1978)
60. P.W. Anderson: Phys. Rev. **130**, 439 (1963)
61. P.W. Higgs: Phys. Lett. **12**, 132 (1964); Phys. Rev. **145**, 1156 (1966)
62. J. Carey: Business Week, 15 March 2004, 103 (2004); ref. to W. Phillips (Gaithersburg), I. Bloch (Mainz)
63. F. Dalfovo, S. Giorgini, L.P. Pitaevskii, S. Stringari: Rev. Mod. Phys. **71**, 463 (1999); L. Pitaevskii, H. Stringari: *Bose-Einstein Condensation*, Oxford University Press (2003)
64. H. Umezawa: Nuovo Cim. **38**, 1415 (1965); **40**, 450 (1965)
65. H.P. Dürr: *Quantum Field Theory*, ed. by F. Mancini, North-Holland, Amsterdam (1986) p. 39
66. H.P. Dürr: Bull. Soc. Math. Belg. **31**, 17 (1979)
67. H.P. Dürr: *Lecture Notes in Physics. Unified Theories of Elementary Particles* (Springer, Berlin), ed. by P. Breitenlohner, H.P. Dürr **160**, 36 (1982)
68. H.P. Dürr: Nova Acta Leop. **NF55**, No. 248, 93 (1982)

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