1. Overview of the Work

This work contains three parts in two volumes. Volume I consists of Part I *Theory*, Volume II of Part II *Applications* and Part III *Program System*. Part I *Theory* contains Chapters 3 to 8, Part II *Applications* the Chapters II- 2 to II- 4, and Part III *Program System*, the documentation of the entire program system.

Celestial Mechanics, as we understand it today, has a long history. Its impact on the concepts of physics and mathematical analysis and, more recently, on geophysics can hardly be overestimated. Chapter 2 reviews the development of classical Celestial Mechanics, but also the developments related to the motion of artificial satellites.

1.1 Part I: Theory

The Equations of Motion. The title of this work demands that we depart from the equations of motion for the celestial bodies, which may be considered either as point masses or as extended bodies in our developments. In the latter case the bodies may be either rigid or deformable. This initial problem description is suited to address a great variety of problems: the orbital motion in galaxies, globular clusters, planetary systems, binaries, the orbital and rotational motion of planets, and the motion of natural and artificial satellite systems around a planet. Such a general treatment of Celestial Mechanics would be demanding, it could, however, hardly be dealt with in only two volumes.

We focus our treatise on the planetary system (consisting of a limited number of about $N \leq 20$ of point masses), on the orbital and rotational motion of the Earth-Moon-Sun system as an example of an N-body problem with extended bodies, and on the orbital motion of artificial Earth satellites (the attitude of satellites is briefly addressed, as well). With this selection of topics we leave aside many fascinating and important problems in dynamical astronomy, in particular the entire field of galactic dynamics. The latter topic is, e.g., very nicely treated in the standard textbook by Binney and Tremaine [20]. Our selection of topics still is rather ambitious, however.

The equations of motion for three types of problems, namely for the planetary system, for the three-body problem Earth-Moon-Sun and for the motion of artificial satellites are derived in Chapter 3. The method, simple and transparent in principle, is the same for all problem types, for the orbital as well as for the rotational motion: Using classical mechanics, Newton's law of universal gravitation, and the Newton-Euler formalism equating the time-derivatives of the linear momentum of individual bodies (or of mass elements of extended bodies) with the forces acting on the particles, the equations for the orbital and rotational motion are obtained in the inertial system. Depending on the problem type, the equations are then transformed to refer to the primary body for the particular problem type. When the motions of planets, minor planets or comets are studied, the position vectors in the equations of motion are heliocentric, in the other two cases, these position vectors are geocentric.

The three problem types have certain peculiarities: (1) "only" the orbital motion needs to be addressed in the first case, orbital *and* rotational motion in the second case; (2) "only" gravitational forces must be considered in the first two cases, whereas non-gravitational forces have to be dealt with, as well, in the third case; (3) due to the artificial satellites' proximity to the Earth, the gravitational potential of the Earth needs to be modelled very accurately; (4) in the latter application the equations of motion of different satellites are not coupled mathematically, allowing it to deal with each satellite orbit separately.

The developments are in essence based on classical mechanics. The relativistic equations of motion are, however, reproduced, as well. In Chapter 3 the so-called PPN (Parametrized Post-Newtonian) version of the equations of motion for the N-body problem is introduced and discussed, as well. The PPN equations may be viewed as a perturbation of the classical N-body motion. The direct use of the PPN equations (e.g., used for the production of planetary and lunar ephemerides in [107]) in numerical integrations over millions of years prohibitively affects the efficiency of the solution algorithms. This is why approximations of the correct PPN equations are considered, as well (and implemented as options into the computer programs for the planetary and satellite motion).

The Classical Two- and Three-body Problems. The two-body problem must be an integral part of each treatise of Celestial Mechanics, and it is found as an important issue in many textbooks of "ordinary" mechanics. Here, it is dealt with in Chapter 4 together with the three-body problem. That chapter thus deals with the two presumably simplest problems encountered in Celestial Mechanics.

The two-body problem, the motion of two point masses w.r.t. each other, may be solved "analytically", i.e., in terms of a finite set of elementary mathematical functions of time. These analytical solutions are extremely important as *first approximations* of more complicated problems. In Celestial Mechanics the two-body motion often is referred to as the *unperturbed* motion, implying that all other motions may be viewed as perturbed two-body motions. The chapter introduces the one-to-one relationship between the set of position- and velocity-vectors on one hand, and the orbital elements on the other hand. This relationship allows to introduce the concept of *osculating orbital elements*, by assigning one set of six orbital elements to each set of position- and velocity-vectors of a perturbed motion, using the formulae of the two-body problem.

Osculating and mean elements – the latter defined as averages of osculating elements over certain time intervals – are introduced as fundamental concepts. The computation of ephemerides, one of the important practical problems in Celestial Mechanics, is briefly addressed, as well. The three-body problem already contains many (if not most) of the characteristics and difficulties of the general planetary N-body motion. It was studied by many eminent astronomers and mathematicians (from Euler to Poincaré). The attempts to find “analytical” solutions of the 3-body problem were only moderately successful. They led, however, to the discovery of the problème restreint, one of the most charming mathematical miniatures found in dynamical astronomy. It is treated as a preparation to more general problems.

Variational Equations. The trajectory of a celestial body contains much information. Studies of the development of the osculating orbital elements as a function of time are indeed extremely informative, but yet it is impossible to decide in an objective way whether or not the findings are representative for other trajectories with similar initial characteristics. In order to answer such questions it is mandatory to study the so-called *variational equations*, which may be associated with each individual trajectory. The variational equations are of greatest importance in Celestial Mechanics – in theory as well as in application. They are required for orbit determination and for solving more general parameter estimation problems, in questions concerning the stability of a particular solution, and in error propagation studies. Chapter 5 introduces the variational equations as linear differential equations for the partial derivatives of the position vector(s) of celestial bodies w.r.t. the parameters defining the particular solution of the equations of motion considered. The chapter also provides analytical solutions (in the sense mentioned above) of the variational equations associated with the equations of motion of the two-body problem, and compares their characteristics with the solution characteristics related to perturbed motion.

Theory of Perturbations. Perturbation theory is the central topic of Chapter 6. Each method to solve an initial value problem associated with the equations of motion of a particular orbital motion may be viewed as a *perturbation method*. Usually one expects, however, that perturbation methods make (intelligent) use of the known approximative solutions, i.e., of the solutions of the corresponding two-body motion. The knowledge of an approximative solution may be exploited in many different ways. It is, e.g., possible to set up a differential equation for the difference vector between the actual solution and the known two-body approximation. The best-known of these attempts is the so-called *Encke method*, which is analyzed and considerably expanded when introducing the problems in the chapter. The best possible way to exploit the known analytical solution of the two-body problem consists of the derivation of differential equations for the osculating orbital elements. These equations are derived in an elementary way, without making use of the results of analytical mechanics. Our approach first leads to the perturbation equations in the Gaussian form, which allow it to consider a very broad class of perturbing functions. Only afterwards we derive the so-called Lagrange planetary equations, requiring the perturbing functions to be gradients of a scalar (so-called perturbation) function. The method to derive the equations is very simple and transparent, and the general form of the equations is amazingly simple. The drawback lies in the necessity to calculate the gradients of the orbital elements (w.r.t. Cartesian position- and velocity-components), a task which was performed in the last, technical section of the chapter.

When comparing the mathematical structure of the perturbation equations for different orbital elements (either in the Gaussian or in the Lagrangian form), one finds that all except one are essentially of the same simple mathematical structure. The exception is the equation for the time of pericenter passage T_0 (alternatively for the mean anomaly σ_0 referred to the initial epoch t_0), because the time argument figures outside the trigonometric functions on the right-hand sides of the equations. This is a nuisance independently of whether one solves the equations analytically or numerically. As opposed to the usual method of introducing new, auxiliary functions (as, e.g., the function ρ introduced by Brouwer and Clemence [27]), we derive directly a differential equation for the mean anomaly $\sigma(t)$ which does not show the problems mentioned above. $\sigma(t)$ is of course not an orbital element (an integration constant of the two-body motion), but any other auxiliary functions that might be introduced are not first integrals either.

Numerical Solution of Ordinary Differential Equations. Numerical analysis, in particular the numerical solution of the equations of motion and the associated variational equations, is studied in considerable detail in Chapter 7. In view of the fact that first- and second-order equations as well as definite integrals have to be solved in Celestial Mechanics, the general problem of solving non-linear differential equation systems of order n is studied first. Linear systems and integrals may then be considered as special cases of the general problem.

It is not sufficient to consider only initial value problems in Celestial Mechanics. The so-called *local boundary value problem* (where the boundary epochs are close together in time) is of particular interest. It is, e.g., used in orbit determination problems. Euler's original analysis (see Figure 7.1) is the foundation for all modern algorithms. It meets all requirements an algorithm should offer, except one: Euler's method is prohibitively inefficient. We show that the (not so well known) collocation methods may be viewed as the logical generalization of Euler's method. *Cum grano salis* one might say that the Euler method and the collocation method are identical except for the order of the approximation. Euler's method corresponds to a local Taylor series approximation of order n (where n is the order of the differential equation system); the order $q \geq n$ of the collocation method may be defined by the program user. Collocation methods may be easily adapted to automatically control the local errors of the integration, allowing it to determine efficiently not only orbits of small, but also large eccentricities.

Local error control is one issue, the accumulation of the local errors, due to two different sources, is another one. The accumulation of errors is of course studied for a machine-environment (as opposed to hand calculation). Apart from that our treatment is closely related to the method described in Brouwer's brilliant analysis [26]. Based on this theory a rule of thumb is provided for selecting the proper (constant) stepsize for producing planetary ephemerides (assuming low eccentricity orbits). This approximate treatment of the accumulation error is not applicable to very long integration spans or to problems involving strong perturbations (as, e.g., in the case of resonances). The correct theory of error accumulation therefore must to be based on the variational equations as well.

Orbit Determination and Parameter Estimation. Orbit determination and more general parameter estimation procedures are the topic of Chapter 8. The decision to conclude Part I with the chapter on orbit determination is justified by the fact that the problem reveals many interesting theoretical aspects related to parameter estimation theory. The determination of orbits may, however, also be viewed as one of the important practical tasks in Celestial Mechanics. The chapter may thus also be viewed as a transition chapter to the application part.

Orbits of celestial bodies may only be determined if they were repeatedly observed. For generations of astronomers the expression "observation" was synonymous for "direction observation" (usually an astrometric position), defining (in essence) the unit vector from the observer to the observed object at the epoch of the observation (a precise definition is provided). Except for the fact that today usually CCD (Charge Coupled Device) observations, and no longer photographic or even visual observations, are made, not much has changed in this view of things, when minor planets or comets are concerned. Orbit determination based on astrometric positions is also an important issue when dealing with artificial satellites and/or space debris. This is why the classical orbit determination problem based on astrometric positions applied to minor planets, comets, and artificial objects orbiting the Earth is addressed first.

It is important to distinguish between *first orbit determination* and *orbit improvement*. In the former case there is no a priori information about the orbital characteristics available. In the latter case, such information is available, and this allows it to linearize the problem and to solve it with standard procedures of applied mathematics. *Cum grano salis* one might say that first orbit determination is an *art*, whereas orbit improvement is *mathematical routine*.

Let us first comment the artistic task: If the force field is assumed to be known (in most cases one even uses the two-body approximation) the problem is reduced to determining six parameters, the (osculating) orbital elements, using the observations. First orbit determination can only succeed, if the number of unknowns can be reduced to only one or two parameters. The principle is explained in the case of determining a circular orbit using two astrometric positions, where the (originally) six-dimensional problem is reduced to one of dimension one.

In the general case, it is possible to reduce the problem to a two-dimensional problem, the topocentric distances corresponding to two astrometric positions being the remaining unknowns. The method is based on the numerical solution of a local boundary value problem as discussed in Chapter 7. The new method presented here is very robust, allowing it to investigate also delicate cases as, e.g., multiple solutions.

Classical orbit determination must be illustrated with standard and difficult examples. Program ORBDET, serving this purpose, allows it to determine first orbits of objects in the planetary system (minor planets, comets, NEO (Near Earth Objects), etc.), and of satellites or space debris. First orbits may be determined with a variety of methods in ORBDET, including the determination of circular and parabolic orbits. Except for the case of the circular orbit the basic method is the new method mentioned above. Sample observations of minor planets, comets, and artificial satellites are provided. Each orbit determination is concluded by an orbit improvement step, where the more important perturbations of the particular problem may be taken into account.

Most of the orbit determination procedures in use today are based on ideas due to Gauss, Laplace, and others. The historical reminiscences are discussed, but not considered for implementation. Often, the original recipes have been simply translated into a computer code and applied – from our point of view a totally unacceptable procedure. Our method outlined in the main text and implemented in program ORBDET is based on Gauss's brilliant insight that the formulation of the orbit determination problem as a boundary value problem (instead of an initial value problem) immediately reduces the number of parameters from six to two, and on the numerical solution of the associated (local) boundary value problem.

Not only angles, but also distances, distance differences, and other aspects of a satellite orbit may be observed. This naturally leads to much more general orbit determination problems in satellite geodesy. Usually, one may assume moreover that good approximations of the true orbits are known – meaning that "only" standard methods (based on linearizing a non-linear parameter estimation problem) are required to improve the orbits.

An observation of a celestial body does not only contain information concerning the position (and/or velocity) vector of the observed object, but also about the observer's position and motion. This aspect is widely exploited in satellite geodesy. Some of the general parameter estimation schemes and of the results achieved are briefly mentioned in Chapter 8, as well.

The chapter is concluded with two modern examples of "pure" orbit determination problems. One is related to SLR (Satellite Laser Ranging), the other to the determination of LEO (Low Earth Orbiter) orbits, where the LEO is equipped with a GPS receiver. The latter orbit determination problem is based on the LEO positions (and possibly position differences) as determined from the data of the spaceborne GPS receiver. Program SATORB may be used to determine these orbits. This latter application is attracting more and more attention because more and more LEOs are equipped with GPS receivers.

1.2 Part II: Applications

Rotation of Earth and Moon. Chapter II- 2 deals with all aspects of the three-body problem Earth-Moon-Sun. All developments and analyses are based on the corresponding equations of motion developed in Chapter 3; the illustrations, on the other hand, are based almost exclusively on the computer program ERDROT (see section 1.3).

In order to fully appreciate the general characteristics of Earth (and lunar) rotation, it is necessary to understand the orbital motion of the Moon in the first place. This is why the orbital motion of the Moon is analyzed before discussing the rotation of Earth and Moon.

The main properties of the rotation of Earth and Moon are reviewed afterwards under the assumption that both celestial bodies are rigid. Whereas the characteristics of Earth rotation are well known, the rotational properties of the Moon are usually only vaguely known outside a very limited group of specialists. Despite the fact that the structure of the equations is the same in both cases, there are noteworthy differences, some of which are discussed in this chapter. The analysis pattern is the same for the two bodies: The motions of the rotation axis in the body-fixed system and in the inertial system are established by computer simulations (where it is possible to selectively "turn off" the torques exerted by the respective perturbing bodies); the simulation results are then explained by approximate analytical solutions of the equations of motion. The simulations and the approximate analytic solutions are compared to the real motion of the Earth's and Moon's rotation poles. Many, but not all aspects are explained by the rigid-body approximation.

This insight logically leads to the discussion of the rotation of a non-rigid Earth. This discussion immediately leads in turn to very recent, current and possible future research topics. Initially, the "proofs" for the non-rigidity of the Earth are provided. This summary is based mainly on the Earth rotation series available from the IERS and from space geodetic analysis centers. Many aspects of Earth rotation may be explained by assuming the Earth to consist of a solid elastic body, which is slightly deformed by "external" forces. Only three of these forces need to be considered: (1) the centrifugal force due to the rotation of the Earth about its figure axis, (2) the differential centrifugal force due to the rotation of the Earth about an axis slightly differing from this figure axis, and (3) the tidal forces exerted by Sun and Moon (and planets). The resulting, time-dependent deformations of the Earth are small, which is why in a good approximation they may be derived from Hooke's law of elasticity. The elastic Earth model brings us one step closer to the actual rotation of the Earth: The difference between the Chandler and the Euler period as well as the observed bi-monthly and monthly LOD (Length of Day) variations can be explained now.

The elastic Earth model does not yet explain all features of the observed Earth rotation series. There are, e.g., strong annual and semi-annual variations in the real LOD series, which may *not* be attributed to the deformations of the solid Earth. Peculiar features also exist in the polar motion series. They are observed with space geodetic techniques because the observatories are attached to the solid Earth and therefore describe the rotation of this body (and not of the body formed by the solid Earth, the atmosphere and the oceans). Fortunately, meteorologists and oceanographers are capable of deriving the angular momentum of the atmosphere from their measurements: by comparing the series of AAM (Atmospheric Angular Momentum) emerging from the meteorological global pressure, temperature, and wind fields with the corresponding angular momentum time series of the solid Earth emerging from space geodesy, the "unexplained" features in the space geodetic observation series of Earth rotation are nowadays interpreted by the exchange of angular momentum between solid Earth, atmosphere and oceans – implying that the sum of the angular momenta of the solid Earth and of atmosphere and oceans is nearly constant.

Even after having modelled the Earth as a solid elastic body, partly covered by oceans and surrounded by the atmosphere, it is not yet possible to explain all features of the monitored Earth rotation. Decadal and secular motions in the observed Earth rotation series still await explanation. The explanation of these effects requires even more complex, multi-layer Earth-models, as, e.g., illustrated by Figure II- 2.55. The development of these complex Earth models is out of the scope of an introductory text. Fortunately, most of their features can already be seen in the simplest generalization, usually referred to as the Poincaré Earth model, consisting of a rigid mantle and a fluid core (see Figure II- 2.56). It is in particular possible to explain the terms FCN (Free Core Nutation) and NDFW (Nearly-Diurnal Free Wobble). The mathematical deliberations associated with the Poincaré model indicate the degree of complexity associated with the more advanced Earth models. It is expected that such models will be capable of interpreting the as yet unexplained features in the Earth rotation series – provided that Earth rotation is continuously monitored over very long time spans (centuries).

Artificial Earth Satellites. Chapter II- 3 deals with the orbital motion of artificial Earth satellites. Most illustrations of this chapter stem from program SATORB, which allows it (among others) to generate series of osculating and/or mean elements associated with particular satellite trajectories.

The perturbations of the orbits due to the oblate Earth, more precisely the perturbations due to the term C_{20} of the harmonic expansion of Earth's potential, are discussed first. The pattern of perturbations at first sight seems rather similar to the perturbations due to a third body: No long-period or secular perturbations in the semi-major axis and in the eccentricity, secular perturbations in the right ascension of the ascending node Ω and in the argument ω of perigee. There are, however, remarkable peculiarities of a certain practical relevance. The secular rates of the elements Ω (right ascension of ascending node) and ω (argument of perigee) are functions of the satellite's inclination i w.r.t. the Earth's equatorial plane. The perturbation patterns allow it to establish either sun-synchronous orbital planes or orbits with perigees residing in pre-defined latitudes.

The orbital characteristics are established by simulation techniques (using program SATORB), then explained with first-order general perturbation methods (based on simplified perturbative forces). Higher-order perturbations due to the C_{20}-term and the influence of the higher-order terms of the Earth's potential (which are about three orders of magnitude smaller than C_{20}) are studied subsequently. The attenuating influence of the Earth's oblateness term C_{20} on the perturbations due to the higher-order terms C_{ik} is discussed as well.

If a satellite's revolution period is commensurable with the sidereal revolution period of the Earth, some of the higher-order terms of Earth's potential may produce resonant perturbations, the amplitudes of which may become orders of magnitude larger than ordinary higher-order perturbations. Resonant perturbations are typically of very long periods (years to decades), and the amplitudes may dominate even those caused by the oblateness. Two types of resonances are discussed in more detail, the (1:1)-resonance of geostationary satellites and the (2:1)-resonance of GPS-satellites. In both cases the practical implications are considerable. In the case of GPS-satellites the problem is introduced by a heuristic study, due to my colleague Dr. Urs Hugentobler, which allows it to understand the key aspects of the problem without mathematical developments.

The rest of the chapter is devoted to the discussion of non-gravitational forces, in particular of drag and of solar radiation pressure. As usual in our treatment, the perturbation characteristics are first illustrated by computer simulations, then understood by first-order perturbation methods. Atmospheric drag causes a secular reduction of the semi-major axis (leading eventually to the decay of the satellite orbit) and a secular decrease of the eccentricity (rendering the decaying orbit more and more circular). Solar radiation pressure is (almost) a conservative force (the aspect is addressed explicitly), which (almost) excludes secular perturbations in the semi-major axis. Strong and long-period perturbations occur in the eccentricity, where the period is defined by the periodically changing position of the Sun w.r.t. the satellite's orbital plane.

The essential forces (and the corresponding perturbations) acting on (suffered by) high- and low-orbiting satellites conclude the chapter.

Evolution of the Planetary System. The application part concludes with Chapter II- 4 pretentiously entitled *evolution of the planetary system.* Three major issues are considered: (a) the orbital development of the outer system from Jupiter to Pluto over a time period of two million years (the past million years and the next million years – what makes sure that the illustrations in this chapter will not be outdated in the near future, (b) the orbital development of the complete system (with the exception of the "dwarfs" Mercury and Pluto), where only the development of the inner system from Venus to Mars is considered in detail, and (c) the orbital development of minor planets (mainly of those in the classical asteroid belt between Mars and Jupiter).

The illustrations have three sources, namely (a) computer simulations with program PLASYS, allowing it to numerically integrate any selection of planets of our planetary system with the inclusion of one body of negligible mass (e.g., a minor planet or a comet) with a user-defined set of initial orbital elements (definition in Chapter 2), (b) orbital elements obtained through the MPC (Minor Planet Center) in Cambridge, Mass., and (c) spectral analyses of the series of orbital elements (and functions thereof) performed by our program FOURIER.

By far the greatest part of the (mechanical) energy and the angular momentum of our planetary system is contained in the outer system. Jupiter and Saturn are the most massive planets in this subsystem. Computer simulations over relatively short time-spans (of 2000 years) and over the full span of two million years clearly show that even when including the entire outer system the development of the orbital elements of the two giant planets is dominated by the exchange of energy and angular momentum between them. The simulations and the associated spectra reveal much more information.

Venus and Earth are the two dominating masses of the inner system. They exchange energy and angular momentum (documented by the coupling between certain orbital elements) very much like Jupiter and Saturn in the outer system. They are strongly perturbed by the planets of the outer system (by Jupiter in particular). An analysis of the long-term development of the Earth's orbital elements (over half a million years) shows virtually "no long-period structure" for the semi-major axis, whereas the eccentricity varies between $e \approx 0$ and $e \approx 0.5$ (exactly like the orbital eccentricity of Venus).

Such variations might have an impact on the Earth's climate (annual variation of the "solar constant", potential asymmetry between summer- and winter-half-year). The eccentricity is, by the way, approaching a minimum around the year 35'000 A.D., which does not "promise" too much climate-relevant "action" in the near future – at least not from the astronomical point of view. The idea that the Earth's dramatic climatic changes in the past (ice-ages and warm periods) might at least in part be explained by the Earth's orbital motion is due to Milankovitch. Whether or not this correlation is significant cannot be firmly decided (at least not in this book). The long-term changes of the orbital characteristics (of the eccentricity, but also of the inclination of the Earth's orbital plane w.r.t. the so-called invariable plane) are, however, real, noteworthy and of respectable sizes.

Osculating orbital elements of more than $100'000$ minor planets are available through the MPC. This data set is inspected to gain some insight into the motion of these celestial objects at present. The classical belt of minor planets is located between Mars and Jupiter. Many objects belonging to the so-called Kuiper-belt are already known, today. Nevertheless, the emphasis in Chapter II- 4 is put on the classical belt of asteroids and on the explanation of (some aspects of) its structure. The histogram II- 4.43 of semi-major axes (or of the associated revolution periods) indicates that the Kirkwood gaps must (somehow) be explained by the commensurabilities of the revolution periods of the minor planets and of Jupiter. After the discussion of the observational basis, the analysis of the orbital motion of minor planets is performed in two steps:

- The development of the orbital elements of a "normal" planet is studied. This study includes the interpretation of the (amazingly clean) spectra of the minor planet's mean orbital elements. These results lead to the definition of the (well known) so-called proper elements. It is argued that today the definition of these proper elements should in principle be based on numerical analyses, rather than on analytical theories as, e.g., developed by Brouwer and Clemence [27]. A few numerical experiments indicate, however, that the results from the two approaches agree quite well.

- Minor planets in resonant motion with Jupiter are studied thereafter. The Hilda group ((3:2)-resonance) and the (3:1)-resonance are considered in particular. The Ljapunov characteristic exponent is defined as an excellent tool to identify chaotic motion. A very simple and practical method for its establishment (based on the solution of one variational equation associated with the minor planet's orbit) is provided in program PLASYS. The tools of numerical integration of the minor planet's orbit together with one or more variational equations associated with it, allow it to study and to illustrate the development of the orbital elements of minor planets in resonance zones. It is fascinating to see that the revolutionary numerical experiments performed by Jack Wisdom, in the 1980s, using the most advanced computer hardware available at that time, nowadays may be performed with standard PC (Personal Computer) equipment.

1.3 Part III: Program System

The program system, all the procedures, and all the data files necessary to install and to use it on PC-platforms or workstations equipped with a WINDOWS operating system are contained on the CDs accompanying both volumes of this work. The system consists of eight programs, which will be briefly characterized below. Detailed program and output descriptions are available in Part III, consisting of Chapters II- 5 to II- 11.

The program system is operated with the help of a menu-system. Figure 1.1 shows a typical panel – actually the panel after having activated the program system *Celestial Mechanics* and then the program PLASYS. The top line of each panel contains the buttons with the program names and the help-key offer real-time information when composing a problem.

The names of input- and output-files may be defined or altered in these panels and input options may be set or changed. By selecting ≪ Next Panel ≫ (bottom line), the next option/input panel of the same program are activated. If all options and file definitions are meeting the user's requirements, the program is activated by selecting ≪ Save and Run ≫ . For CPU (Central Processing Unit) intensive programs, the program informs the user about the remaining estimated CPU-requirements (in %).

The most recent general program output (containing statistical information concerning the corresponding program run and other characteristics) may be inspected by pressing the button ≪ Last Output ≫ . With the exception of LEOKIN all programs allow it to visualize some of the more specific output files using a specially developed graphical tool compatible with the menu-system. The output files may of course also be plotted by the program user with any graphical tool he is acquainted with. All the figures of this book

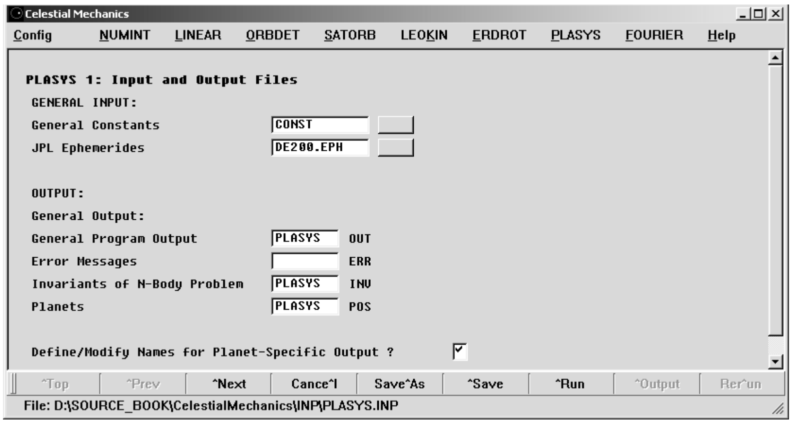

Fig. 1.1. Primary menu for program system *Celestial Mechanics, PLASYS*

illustrating computer output were, e.g., produced with the so-called "gnu"-graphics package. The gnu-version used here is also contained on the CD. The programs included in the package "Celestial Mechanics" are (in the sequence of the top line of Figure 1.1):

1. **NUMINT** is used in the first place to demonstrate or test the mutual benefits and/or deficiencies of different methods for numerical integration. Only two kinds of problems may be addressed, however: either the motion of a minor planet in the gravitational field of Sun and Jupiter (where the orbits of the latter two bodies are assumed to be circular) or the motion of a satellite in the field of an oblate Earth (only the terms C_{00} and C_{20} of the Earth's potential are assumed to be different from zero).

   The mass of Jupiter or the term C_{20} may be set to zero (in the respective program options), in which case a pure two-body problem is solved.

   When the orbit of a "minor planet" is integrated, this actually corresponds to a particular solution of the problème restreint. In this program mode it is also possible to generate the well known surfaces of zero velocity (Hill surfaces), as they are shown in Chapter 4.
2. **LINEAR** is a test program to demonstrate the power of collocation methods to solve linear initial- or boundary-value problems. The program user may select only a limited number of problems. He may test the impact of defining the collocation epochs in three different ways (equidistant, in the roots of the Legendre and the Chebyshev polynomials, respectively).
3. **SATORB** may either be used as a tool to generate satellite ephemerides (in which case the program user has to specify the initial osculating

elements), or as an orbit determination tool using *either* astrometric positions of satellites or space debris as observations *or* positions (and possibly position differences) as pseudo-observations. In the latter case SATORB is an ideal instrument to determine a purely dynamical or a reduced-dynamics orbit of a LEO. It may also be used to analyze the GPS and GLONASS ephemerides routinely produced by the IGS (International GPS Service).

The orbit model can be defined by the user, who may, e.g.,

- select the degree and the order for the development of the Earth's gravity potential,
- decide whether or not to include relativistic corrections,
- decide whether or not to include the direct gravitational perturbations due to the Moon and the Sun,
- define the models for drag and radiation pressure, and
- decide whether or not to include the perturbations due to the solid Earth and ocean tides.

Unnecessary to point out that this program was extensively used to illustrate Chapter II- 3.

When using the program for orbit determination the parameter space (naturally) contains the initial osculating elements, a user-defined selection of dynamical parameters, and possibly so-called pseudo-stochastic pulses (see Chapter 8).

Programs ORBDET and SATORB were used to illustrate the algorithms presented in Chapter 8.

4. **LEOKIN** may be used to generate a file with positions and position differences of a LEO equipped with a spaceborne GPS-receiver. This output file is subsequently used by program SATORB for LEO orbit determination. Apart from the observations in the standard RINEX (Receiver Independent Exchange Format), the program needs to know the orbit and clock information stemming from the IGS.
5. **ORBDET** allows it to determine the (first) orbits of minor planets, comets, artificial Earth satellites, and space debris from a series of astrometric positions. No initial knowledge of the orbit is required, but at least two observations must lie rather close together in time (time interval between the two observations should be significantly shorter than the revolution period of the object considered).

   The most important perturbations (planetary perturbations in the case of minor planets and comets, gravitational perturbations due to Moon, Sun, and oblateness of the Earth (term C_{20}) in the case of satellite motion) are included in the final step of the orbit determination. ORBDET is the only interactive program of the entire package.

The program writes the final estimate of the initial orbital element into a file, which may in turn be used subsequently to define the approximate initial orbit, when the same observations are used for orbit determination in program SATORB.

6. **ERDROT** offers four principal options:
   - It may be used to study Earth rotation, assuming that the geocentric orbits of Moon and Sun are known. Optionally, the torques exerted by Moon and Sun may be set to zero.
   - It may be used to study the rotation of the Moon, assuming that the geocentric orbits of Moon and Sun are known. Optionally, the torques exerted by Earth and Sun may be set to zero.
   - The N-body problem Sun, Earth, Moon, plus a selectable list of (other) planets may be studied and solved.
   - The program may be used to study the correlation between the angular momenta of the solid Earth (as produced by the IGS or its institutions) and the atmospheric angular momenta as distributed by the IERS (International Earth Rotation and Reference Systems Service).

   This program is extensively used in Chapter II- 2.
7. **PLASYS** numerically integrates (a subset of) our planetary system starting either from initial state vectors taken over from the JPL (Jet Propulsion Laboratory) DE200 (Development Ephemeris 200), or using the approximation found in [72]. A minor planet with user-defined initial osculating elements may be included in the integration, as well. In this case it is also possible to integrate up to six variational equations simultaneously with the primary equations pertaining to the minor planet. Program PLASYS is extensively used in Chapter II- 4.
8. **FOURIER** is used to spectrally analyze data provided in tabular form in an input file. The program is named in honour of Jean Baptiste Joseph Fourier (1768–1830), the pioneer of harmonic analysis. In our treatment Fourier analysis is considered as a mathematical tool, which should be generally known. Should this assumption not be (entirely) true, the readers are invited to read the theory provided in Chapter II- 11, where Fourier analysis is developed starting from the method of least squares. As a matter of fact it is possible to analyze a data set using
   - *either* the *least squares* technique – in which case the spacing between subsequent data points may be arbitrary,
   - *or* the *classical Fourier analysis*, which is orders of magnitude more efficient than least squares (but requires equal spacing between observations), and where *all* data points are used,
   - *or* FFT (Fast Fourier Transformation), which is in turn orders of magnitude more efficient than the classical Fourier technique, but where

usually the number of data points should be a power of 2 (otherwise a loss of data may occur).

In the FFT-mode the program user is invited to define the decomposition level (maximum power of 2 for the decomposition), which affects the efficiency, but minimizes (controls) loss of data. The general program output contains the information concerning the data loss.

The program may very well be used to demonstrate the efficiency ratio of the three techniques, which should produce identical results. FOURIER is a pure service program.

The computer programs of Part III are used throughout the two volumes of our work. It is considered a minimum set ("starter's kit") of programs that should be available to students entering into the field of Astrodynamics, in particular into one of the applications treated in Part II of this work. The programs NUMINT, LINEAR, and PLASYS are also excellent tools to study the methods of numerical integration.

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