

Contents

I	Mathematical Preliminaries	1
1	Vector Calculus	3
1.1	The Main Operations of Vector Calculus: <i>div</i> , <i>grad</i> , and Δ	3
1.2	Conservative Vector Fields	5
1.3	Curvilinear Integrals and the Geometric Meaning of the Existence of a Potential	7
1.4	Multiple and Repeated Integrals	8
1.5	The Flow of a Vector Field and the Gauss-Ostrogradsky Theorem	12
1.6	The Circulation of a Vector Field and the Green Formula .	15
1.7	Exercises	17
1.8	Bibliographic Notes	18
2	Partial Differential Equations	19
2.1	The First Order Partial Differential Equations	19
2.1.1	The Complete Integral and the General Integral . .	20
2.1.2	The Singular Integral	21
2.1.3	The Quasilinear Equations and the Method of Characteristics	22
2.1.4	Compatible Systems of the First Order PDEs	24
2.1.5	The Method of Characteristics for a Non-quasilinear First Order PDE	26

2.1.6	Examples	27
2.2	The Second Order Partial Differential Equations	30
2.2.1	Classification of the Linear Second Order Partial Differential Equations	30
2.2.2	Boundary Value Problems for Elliptic Equations	31
2.2.3	Examples	32
2.3	Group Theoretic Analysis of the Systems of Partial Differential Equations	38
2.3.1	One Parameter Lie Groups	38
2.3.2	Invariance of PDEs, Systems of PDEs, and Boundary Problems under Lie Groups	41
2.3.3	Calculating a Lie Group of a PDE	44
2.3.4	Calculating Invariants of the Lie Group	45
2.3.5	Examples	46
2.4	Exercises	47
2.5	Bibliographic Notes	49
3	Theory of Generalized Convexity	51
3.1	Definition and Properties of the Generalized Fenchel Conjugates	52
3.2	Generalized Convexity and Cyclic Monotonicity	55
3.3	Examples	58
3.4	Exercises	59
3.5	Bibliographic Notes	59
4	Calculus of Variations and the Optimal Control	61
4.1	Banach Spaces and Polish Spaces	61
4.2	Hilbert Spaces	65
4.3	Dual Space for a Normed Space and a Hilbert Space	66
4.4	Frechet Derivative of a Mapping between Normed Spaces	69
4.5	Functionals and Gateaux Derivatives	71
4.6	Euler Equation	73
4.7	Optimal Control	74
4.8	Examples	76
4.9	Exercises	78
4.10	Bibliographic Notes	79
5	Miscellaneous Techniques	81
5.1	Distributions and Generalized Solutions for the Partial Differential Equations	81
5.1.1	A Motivating Example	82
5.1.2	The Set of Test Functions and its Dual	83
5.1.3	Examples of Distributions.	84
5.1.4	The Derivative of a Distribution	87

5.1.5	The Product of a Distribution and a Test Function and the Product of Distributions	88
5.1.6	The Resultant of a Distribution and a Dilation Operator	89
5.1.7	Adjoint Linear Differential Operators and Generalized Solutions of the Partial Differential Equations .	91
5.2	Sobolev Spaces and Poincare Theorem	92
5.3	Sweeping Operators and Balayage of Measures	94
5.4	Coercive Functionals	96
5.5	Optimization by Vector Space Methods	96
5.6	Calculus of Variation Problem with Convexity Constraints .	98
5.7	Supermodularity and Monotone Comparative Statics	99
5.8	Hausdorff Metric on Compact Sets of a Metric Space	103
5.9	Generalized Envelope Theorems	107
5.10	Exercises	109
5.11	Bibliographic Notes	110

II Economics of Multi-dimensional Screening 111

6	The Unidimensional Screening Model	115
6.1	Spence-Mirrlees Condition and Implementability	116
6.2	The Concept of the Information Rent	119
6.3	Three Approaches to the Unidimensional Relaxed Problem	119
6.3.1	The Direct Approach	119
6.3.2	The Dual Approach	120
6.3.3	The Hamiltonian Approach	121
6.4	Hamiltonian Approach to the Unidimensional Complete Problem	122
6.5	Type Dependent Participation Constraint	124
6.6	Random Participation Constraint	126
6.7	Examples	127
6.8	Exercises	133
6.9	Bibliographic Notes	134
7	The Multi-dimensional Screening Model	135
7.1	The Genericity of Exclusion	137
7.2	Generalized Convexity and Implementability	141
7.2.1	Is Bunching Robust in the Multi-dimensional Case?	143
7.3	Path Independence of Information Rents	144
7.4	Cost Based Tariffs	145
7.5	Direct Approach and Its Limitations	148
7.6	Dual Approach for $m = n$	151
7.6.1	The Relaxed Problem	152
7.6.2	An Alternative Approach to the Relaxed Problem .	153

7.6.3	The Complete Problem	154
7.6.4	The Geometry of the Participation Region	155
7.6.5	A Sufficient Condition for Bunching	156
7.6.6	The Extension of the Dual Approach for $n > m$	156
7.7	Hamiltonian Approach and the First Order Characterization of the Solution	158
7.7.1	The Economic Meaning of the Lagrange Multipliers	160
7.8	Symmetry Analysis of the First Order Conditions	161
7.9	Some Remarks on the Hamiltonian Approach to the Com- plete Problem	166
7.10	Examples and Economic Applications	167
7.11	Exercises	173
7.12	Bibliographic Notes	173
8	Beyond the Quasilinear Case	175
8.1	The Unidimensional Case	176
8.2	The Multi-dimensional Case	179
8.2.1	Implementability of a Surplus Function	180
8.2.2	Implementability of an Allocation	181
8.3	The First Order Characterization of the Solution of the Re- laxed Problem	184
8.4	Exercises	186
8.5	Bibliographic Notes	187
9	Existence, Uniqueness, and Regularity Properties of the Solution	189
9.1	Existence and Uniqueness of the Solution of the Relaxed Problem	189
9.2	Existence of a Solution for the Complete Problem	191
9.3	Continuity of the Solution	192
9.4	Bibliographic Notes	194
10	Conclusions	195



<http://www.springer.com/978-3-540-23906-2>

Multidimensional Screening

Basov, S.

2005, XIV, 202 p., Softcover

ISBN: 978-3-540-23906-2