
Contents

1	Introduction	1
1.1	Objectives of Analyzing Multiple Time Series	1
1.2	Some Basics	2
1.3	Vector Autoregressive Processes	4
1.4	Outline of the Following Chapters	5

Part I Finite Order Vector Autoregressive Processes

2	Stable Vector Autoregressive Processes	13
2.1	Basic Assumptions and Properties of VAR Processes	13
2.1.1	Stable VAR(p) Processes	13
2.1.2	The Moving Average Representation of a VAR Process	18
2.1.3	Stationary Processes	24
2.1.4	Computation of Autocovariances and Autocorrelations of Stable VAR Processes	26
2.2	Forecasting	31
2.2.1	The Loss Function	32
2.2.2	Point Forecasts	33
2.2.3	Interval Forecasts and Forecast Regions	39
2.3	Structural Analysis with VAR Models	41
2.3.1	Granger-Causality, Instantaneous Causality, and Multi-Step Causality	41
2.3.2	Impulse Response Analysis	51
2.3.3	Forecast Error Variance Decomposition	63
2.3.4	Remarks on the Interpretation of VAR Models	66
2.4	Exercises	66
3	Estimation of Vector Autoregressive Processes	69
3.1	Introduction	69
3.2	Multivariate Least Squares Estimation	69

3.2.1	The Estimator	70
3.2.2	Asymptotic Properties of the Least Squares Estimator .	73
3.2.3	An Example	77
3.2.4	Small Sample Properties of the LS Estimator	80
3.3	Least Squares Estimation with Mean-Adjusted Data and Yule-Walker Estimation	82
3.3.1	Estimation when the Process Mean Is Known	82
3.3.2	Estimation of the Process Mean	83
3.3.3	Estimation with Unknown Process Mean	85
3.3.4	The Yule-Walker Estimator	85
3.3.5	An Example	87
3.4	Maximum Likelihood Estimation	87
3.4.1	The Likelihood Function	87
3.4.2	The ML Estimators.....	89
3.4.3	Properties of the ML Estimators	90
3.5	Forecasting with Estimated Processes	94
3.5.1	General Assumptions and Results	94
3.5.2	The Approximate MSE Matrix.....	96
3.5.3	An Example	98
3.5.4	A Small Sample Investigation	100
3.6	Testing for Causality.....	102
3.6.1	A Wald Test for Granger-Causality	102
3.6.2	An Example	103
3.6.3	Testing for Instantaneous Causality.....	104
3.6.4	Testing for Multi-Step Causality	106
3.7	The Asymptotic Distributions of Impulse Responses and Forecast Error Variance Decompositions	109
3.7.1	The Main Results	109
3.7.2	Proof of Proposition 3.6	116
3.7.3	An Example	118
3.7.4	Investigating the Distributions of the Impulse Responses by Simulation Techniques	126
3.8	Exercises	130
3.8.1	Algebraic Problems	130
3.8.2	Numerical Problems	132
4	VAR Order Selection and Checking the Model Adequacy ..	135
4.1	Introduction	135
4.2	A Sequence of Tests for Determining the VAR Order.....	136
4.2.1	The Impact of the Fitted VAR Order on the Forecast MSE.....	136
4.2.2	The Likelihood Ratio Test Statistic	138
4.2.3	A Testing Scheme for VAR Order Determination	143
4.2.4	An Example	145
4.3	Criteria for VAR Order Selection	146

4.3.1	Minimizing the Forecast MSE	146
4.3.2	Consistent Order Selection	148
4.3.3	Comparison of Order Selection Criteria	151
4.3.4	Some Small Sample Simulation Results	153
4.4	Checking the Whiteness of the Residuals	157
4.4.1	The Asymptotic Distributions of the Autocovariances and Autocorrelations of a White Noise Process	157
4.4.2	The Asymptotic Distributions of the Residual Autocovariances and Autocorrelations of an Estimated VAR Process	161
4.4.3	Portmanteau Tests	169
4.4.4	Lagrange Multiplier Tests	171
4.5	Testing for Nonnormality	174
4.5.1	Tests for Nonnormality of a Vector White Noise Process	174
4.5.2	Tests for Nonnormality of a VAR Process	177
4.6	Tests for Structural Change	181
4.6.1	Chow Tests	182
4.6.2	Forecast Tests for Structural Change	184
4.7	Exercises	189
4.7.1	Algebraic Problems	189
4.7.2	Numerical Problems	191
5	VAR Processes with Parameter Constraints	193
5.1	Introduction	193
5.2	Linear Constraints	194
5.2.1	The Model and the Constraints	194
5.2.2	LS, GLS, and EGLS Estimation	195
5.2.3	Maximum Likelihood Estimation	200
5.2.4	Constraints for Individual Equations	201
5.2.5	Restrictions for the White Noise Covariance Matrix	202
5.2.6	Forecasting	204
5.2.7	Impulse Response Analysis and Forecast Error Variance Decomposition	205
5.2.8	Specification of Subset VAR Models	206
5.2.9	Model Checking	212
5.2.10	An Example	217
5.3	VAR Processes with Nonlinear Parameter Restrictions	221
5.4	Bayesian Estimation	222
5.4.1	Basic Terms and Notation	222
5.4.2	Normal Priors for the Parameters of a Gaussian VAR Process	223
5.4.3	The Minnesota or Litterman Prior	225
5.4.4	Practical Considerations	227
5.4.5	An Example	227

5.4.6	Classical versus Bayesian Interpretation of $\hat{\alpha}$ in Forecasting and Structural Analysis	228
5.5	Exercises	230
5.5.1	Algebraic Exercises	230
5.5.2	Numerical Problems	231

Part II Cointegrated Processes

6	Vector Error Correction Models	237
6.1	Integrated Processes	238
6.2	VAR Processes with Integrated Variables	243
6.3	Cointegrated Processes, Common Stochastic Trends, and Vector Error Correction Models	244
6.4	Deterministic Terms in Cointegrated Processes	256
6.5	Forecasting Integrated and Cointegrated Variables	258
6.6	Causality Analysis	261
6.7	Impulse Response Analysis	262
6.8	Exercises	265
7	Estimation of Vector Error Correction Models	269
7.1	Estimation of a Simple Special Case VECM	269
7.2	Estimation of General VECMs	286
7.2.1	LS Estimation	287
7.2.2	EGLS Estimation of the Cointegration Parameters	291
7.2.3	ML Estimation	294
7.2.4	Including Deterministic Terms	299
7.2.5	Other Estimation Methods for Cointegrated Systems...	300
7.2.6	An Example	302
7.3	Estimating VECMs with Parameter Restrictions	305
7.3.1	Linear Restrictions for the Cointegration Matrix	305
7.3.2	Linear Restrictions for the Short-Run and Loading Parameters	307
7.3.3	An Example	309
7.4	Bayesian Estimation of Integrated Systems	309
7.4.1	The Model Setup	310
7.4.2	The Minnesota or Litterman Prior	310
7.4.3	An Example	312
7.5	Forecasting Estimated Integrated and Cointegrated Systems ..	315
7.6	Testing for Granger-Causality	316
7.6.1	The Noncausality Restrictions	316
7.6.2	Problems Related to Standard Wald Tests	317
7.6.3	A Wald Test Based on a Lag Augmented VAR	318
7.6.4	An Example	320
7.7	Impulse Response Analysis	321

7.8	Exercises	323
7.8.1	Algebraic Exercises	323
7.8.2	Numerical Exercises	324
8	Specification of VECMs	325
8.1	Lag Order Selection	325
8.2	Testing for the Rank of Cointegration	327
8.2.1	A VECM without Deterministic Terms	328
8.2.2	A Nonzero Mean Term	330
8.2.3	A Linear Trend	331
8.2.4	A Linear Trend in the Variables and Not in the Cointegration Relations	331
8.2.5	Summary of Results and Other Deterministic Terms ..	332
8.2.6	An Example	335
8.2.7	Prior Adjustment for Deterministic Terms	337
8.2.8	Choice of Deterministic Terms	341
8.2.9	Other Approaches to Testing for the Cointegrating Rank	342
8.3	Subset VECMs	343
8.4	Model Diagnostics	345
8.4.1	Checking for Residual Autocorrelation	345
8.4.2	Testing for Nonnormality	348
8.4.3	Tests for Structural Change	348
8.5	Exercises	351
8.5.1	Algebraic Exercises	351
8.5.2	Numerical Exercises	352

Part III Structural and Conditional Models

9	Structural VARs and VECMs	357
9.1	Structural Vector Autoregressions	358
9.1.1	The A-Model	358
9.1.2	The B-Model	362
9.1.3	The AB-Model	364
9.1.4	Long-Run Restrictions à la Blanchard-Quah	367
9.2	Structural Vector Error Correction Models	368
9.3	Estimation of Structural Parameters	372
9.3.1	Estimating SVAR Models	372
9.3.2	Estimating Structural VECMs	376
9.4	Impulse Response Analysis and Forecast Error Variance Decomposition	377
9.5	Further Issues	383
9.6	Exercises	384
9.6.1	Algebraic Problems	384
9.6.2	Numerical Problems	385

10 Systems of Dynamic Simultaneous Equations	387
10.1 Background	387
10.2 Systems with Unmodelled Variables	388
10.2.1 Types of Variables	388
10.2.2 Structural Form, Reduced Form, Final Form	390
10.2.3 Models with Rational Expectations	393
10.2.4 Cointegrated Variables	394
10.3 Estimation	395
10.3.1 Stationary Variables	396
10.3.2 Estimation of Models with $I(1)$ Variables	398
10.4 Remarks on Model Specification and Model Checking	400
10.5 Forecasting	401
10.5.1 Unconditional and Conditional Forecasts	401
10.5.2 Forecasting Estimated Dynamic SEMs	405
10.6 Multiplier Analysis	406
10.7 Optimal Control	408
10.8 Concluding Remarks on Dynamic SEMs	411
10.9 Exercises	412

Part IV Infinite Order Vector Autoregressive Processes

11 Vector Autoregressive Moving Average Processes	419
11.1 Introduction	419
11.2 Finite Order Moving Average Processes	420
11.3 VARMA Processes	423
11.3.1 The Pure MA and Pure VAR Representations of a VARMA Process	423
11.3.2 A VAR(1) Representation of a VARMA Process	426
11.4 The Autocovariances and Autocorrelations of a VARMA(p, q) Process	429
11.5 Forecasting VARMA Processes	432
11.6 Transforming and Aggregating VARMA Processes	434
11.6.1 Linear Transformations of VARMA Processes	435
11.6.2 Aggregation of VARMA Processes	440
11.7 Interpretation of VARMA Models	442
11.7.1 Granger-Causality	442
11.7.2 Impulse Response Analysis	444
11.8 Exercises	444
12 Estimation of VARMA Models	447
12.1 The Identification Problem	447
12.1.1 Nonuniqueness of VARMA Representations	447
12.1.2 Final Equations Form and Echelon Form	452
12.1.3 Illustrations	455

12.2	The Gaussian Likelihood Function	459
12.2.1	The Likelihood Function of an MA(1) Process	459
12.2.2	The MA(q) Case	461
12.2.3	The VARMA(1, 1) Case	463
12.2.4	The General VARMA(p, q) Case	464
12.3	Computation of the ML Estimates	467
12.3.1	The Normal Equations	468
12.3.2	Optimization Algorithms	470
12.3.3	The Information Matrix	473
12.3.4	Preliminary Estimation	474
12.3.5	An Illustration	477
12.4	Asymptotic Properties of the ML Estimators	479
12.4.1	Theoretical Results	479
12.4.2	A Real Data Example	486
12.5	Forecasting Estimated VARMA Processes	487
12.6	Estimated Impulse Responses	490
12.7	Exercises	491
13	Specification and Checking the Adequacy of VARMA Models	493
13.1	Introduction	493
13.2	Specification of the Final Equations Form	494
13.2.1	A Specification Procedure	494
13.2.2	An Example	497
13.3	Specification of Echelon Forms	498
13.3.1	A Procedure for Small Systems	499
13.3.2	A Full Search Procedure Based on Linear Least Squares Computations	501
13.3.3	Hannan-Kavalieris Procedure	503
13.3.4	Poskitt's Procedure	505
13.4	Remarks on Other Specification Strategies for VARMA Models	507
13.5	Model Checking	508
13.5.1	LM Tests	508
13.5.2	Residual Autocorrelations and Portmanteau Tests	510
13.5.3	Prediction Tests for Structural Change	511
13.6	Critique of VARMA Model Fitting	511
13.7	Exercises	512
14	Cointegrated VARMA Processes	515
14.1	Introduction	515
14.2	The VARMA Framework for $I(1)$ Variables	516
14.2.1	Levels VARMA Models	516
14.2.2	The Reverse Echelon Form	518
14.2.3	The Error Correction Echelon Form	519
14.3	Estimation	521

14.3.1	Estimation of ARMA_{RE} Models	521
14.3.2	Estimation of EC-ARMA_{RE} Models	522
14.4	Specification of EC-ARMA_{RE} Models	523
14.4.1	Specification of Kronecker Indices	523
14.4.2	Specification of the Cointegrating Rank	525
14.5	Forecasting Cointegrated VARMA Processes	526
14.6	An Example	526
14.7	Exercises	528
14.7.1	Algebraic Exercises	528
14.7.2	Numerical Exercises	529
15	Fitting Finite Order VAR Models to Infinite Order Processes	531
15.1	Background	531
15.2	Multivariate Least Squares Estimation	532
15.3	Forecasting	536
15.3.1	Theoretical Results	536
15.3.2	An Example	538
15.4	Impulse Response Analysis and Forecast Error Variance Decompositions	540
15.4.1	Asymptotic Theory	540
15.4.2	An Example	543
15.5	Cointegrated Infinite Order VARs	545
15.5.1	The Model Setup	546
15.5.2	Estimation	549
15.5.3	Testing for the Cointegrating Rank	551
15.6	Exercises	552
<hr/>		
Part V Time Series Topics		
<hr/>		
16	Multivariate ARCH and GARCH Models	557
16.1	Background	557
16.2	Univariate GARCH Models	559
16.2.1	Definitions	559
16.2.2	Forecasting	561
16.3	Multivariate GARCH Models	562
16.3.1	Multivariate ARCH	563
16.3.2	MGARCH	564
16.3.3	Other Multivariate ARCH and GARCH Models	567
16.4	Estimation	569
16.4.1	Theory	569
16.4.2	An Example	571
16.5	Checking MGARCH Models	576
16.5.1	ARCH-LM and ARCH-Portmanteau Tests	576

16.5.2	LM and Portmanteau Tests for Remaining ARCH	577
16.5.3	Other Diagnostic Tests	578
16.5.4	An Example	578
16.6	Interpreting GARCH Models	579
16.6.1	Causality in Variance	579
16.6.2	Conditional Moment Profiles and Generalized Impulse Responses	580
16.7	Problems and Extensions	582
16.8	Exercises	584
17	Periodic VAR Processes and Intervention Models	585
17.1	Introduction	585
17.2	The VAR(p) Model with Time Varying Coefficients	587
17.2.1	General Properties	587
17.2.2	ML Estimation	589
17.3	Periodic Processes	591
17.3.1	A VAR Representation with Time Invariant Coefficients	592
17.3.2	ML Estimation and Testing for Time Varying Coefficients	595
17.3.3	An Example	602
17.3.4	Bibliographical Notes and Extensions	604
17.4	Intervention Models	604
17.4.1	Interventions in the Intercept Model	605
17.4.2	A Discrete Change in the Mean	606
17.4.3	An Illustrative Example	608
17.4.4	Extensions and References	609
17.5	Exercises	609
18	State Space Models	611
18.1	Background	611
18.2	State Space Models	613
18.2.1	The Model Setup	613
18.2.2	More General State Space Models	624
18.3	The Kalman Filter	625
18.3.1	The Kalman Filter Recursions	626
18.3.2	Proof of the Kalman Filter Recursions	630
18.4	Maximum Likelihood Estimation of State Space Models	631
18.4.1	The Log-Likelihood Function	632
18.4.2	The Identification Problem	633
18.4.3	Maximization of the Log-Likelihood Function	634
18.4.4	Asymptotic Properties of the ML Estimator	636
18.5	A Real Data Example	637
18.6	Exercises	641

Appendix

A	Vectors and Matrices	645
A.1	Basic Definitions	645
A.2	Basic Matrix Operations	646
A.3	The Determinant	647
A.4	The Inverse, the Adjoint, and Generalized Inverses	649
A.4.1	Inverse and Adjoint of a Square Matrix	649
A.4.2	Generalized Inverses	650
A.5	The Rank	651
A.6	Eigenvalues and -vectors – Characteristic Values and Vectors	652
A.7	The Trace	653
A.8	Some Special Matrices and Vectors	653
A.8.1	Idempotent and Nilpotent Matrices	653
A.8.2	Orthogonal Matrices and Vectors and Orthogonal Complements	654
A.8.3	Definite Matrices and Quadratic Forms	655
A.9	Decomposition and Diagonalization of Matrices	656
A.9.1	The Jordan Canonical Form	656
A.9.2	Decomposition of Symmetric Matrices	658
A.9.3	The Choleski Decomposition of a Positive Definite Matrix	658
A.10	Partitioned Matrices	659
A.11	The Kronecker Product	660
A.12	The vec and vech Operators and Related Matrices	661
A.12.1	The Operators	661
A.12.2	Elimination, Duplication, and Commutation Matrices	662
A.13	Vector and Matrix Differentiation	664
A.14	Optimization of Vector Functions	671
A.15	Problems	675
B	Multivariate Normal and Related Distributions	677
B.1	Multivariate Normal Distributions	677
B.2	Related Distributions	678
C	Stochastic Convergence and Asymptotic Distributions	681
C.1	Concepts of Stochastic Convergence	681
C.2	Order in Probability	684
C.3	Infinite Sums of Random Variables	685
C.4	Laws of Large Numbers and Central Limit Theorems	689
C.5	Standard Asymptotic Properties of Estimators and Test Statistics	692
C.6	Maximum Likelihood Estimation	693
C.7	Likelihood Ratio, Lagrange Multiplier, and Wald Tests	694

C.8	Unit Root Asymptotics.....	698
C.8.1	Univariate Processes	698
C.8.2	Multivariate Processes	703
D	Evaluating Properties of Estimators and Test Statistics by Simulation and Resampling Techniques	707
D.1	Simulating a Multiple Time Series with VAR Generation Process	707
D.2	Evaluating Distributions of Functions of Multiple Time Series by Simulation	708
D.3	Resampling Methods.....	709
	References	713
	Index of Notation	733
	Author Index	741
	Subject Index	747

<http://www.springer.com/978-3-540-40172-8>

New Introduction to Multiple Time Series Analysis

Lütkepohl, H.

2005, XXI, 764 p., Hardcover

ISBN: 978-3-540-40172-8