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## Foreword

This text is an expanded version of informal notes prepared by the first author for a minicourse of eight hours, reviewing the links between hypoelliptic techniques and the spectral theory of Schrödinger type operators. These lectures were given at Rennes for the workshop “Equations cinétiques, hypoellipticité et Laplacien de Witten” organized in February 2003 by the second author. Their content has been substantially completed after the workshop by the two authors with the aim of showing applications to the Fokker-Planck operator in continuation of the work by Hérau-Nier. Among other things it will be shown how the Witten Laplacian occurs as the natural elliptic model for the hypoelliptic drift diffusion operator involved in the kinetic Fokker-Planck equation. While presenting the analysis of these two operators and improving recent results, this book presents a review of known techniques in the following topics : hypoellipticity of polynomial of vector fields and its global counterpart, global Weyl-Hörmander pseudo-differential calculus, spectral theory of non self-adjoint operators, semi-classical analysis of Schrödinger type operators, Witten complexes and Morse inequalities.

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