

## Preface

Charles François Sturm, through his papers published in the 1830's, is considered to be the founder of Sturm-Liouville theory. He was born in Geneva in September 1803. To commemorate the 200th anniversary of his birth, an international colloquium in recognition of Sturm's major contributions to science took place at the University of Geneva, Switzerland, following a proposal by Andreas Hinz. The colloquium was held from 15 to 19 September 2003 and attended by more than 60 participants from 16 countries. It was organized by Werner Amrein of the Department of Theoretical Physics and Jean-Claude Pont, leader of the History of Science group of the University of Geneva. The meeting was divided into two parts. In the first part, historians of science discussed the many contributions of Charles Sturm to mathematics and physics, including his pedagogical work. The second part of the colloquium was then devoted to Sturm-Liouville theory. The impact and development of this theory, from the death of Sturm to the present day, was the subject of a series of general presentations by leading experts in the field, and the colloquium concluded with a workshop covering recent research in this highly active area.

This drawing together of historical presentations with seminars on current mathematical research left participants in no doubt of the degree to which Sturm's original ideas are continuing to have an impact on the mathematics of our own times. The format of the conference provided many opportunities for exchange of ideas and collaboration and might serve as a model for other multidisciplinary meetings.

The organizers had decided not to publish proceedings of the meeting in the usual form (a complete list of scientific talks is appended, however). Instead it was planned to prepare, in conjunction with the colloquium, a volume containing a complete collection of Sturm's published articles and a volume presenting the various aspects of Sturm-Liouville theory at a rather general level, accessible to the non-specialist. Thus Jean-Claude Pont will edit a volume<sup>1</sup> containing the collected works of Sturm accompanied by a biographical review as well as abundant historical and technical comments provided by the contributors to the first part of the meeting.

The present volume is a collection of twelve refereed articles relating to the second part of the colloquium. It contains, in somewhat extended form, the survey lectures on Sturm-Liouville theory given by the invited speakers; these are the first

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<sup>1</sup> *The Collected Works of Charles François Sturm*, J.-C. Pont, editor (in preparation).

six papers of the book. To complement this range of topics, the editors invited a few participants in the colloquium to provide a review or other contribution in an area related to their presentation and which should cover some important aspects of current interest. The volume ends with a comprehensive catalogue of Sturm-Liouville differential equations. At the conclusion of the Introduction is a brief description of the articles in the book, placing them in the context of the developing theory of Sturm-Liouville differential equations. We hope that these articles, besides being a tribute to Charles François Sturm, will be a useful resource for researchers, graduate students and others looking for an overview of the field.

We have refrained from presenting details of Sturm's life and his other scientific work in this volume. As regards Sturm-Liouville theory, some aspects of Sturm's original approach are presented in the contributions to the present book, and a more detailed discussion will be given in the article by Jesper Lützen and Angelo Mingarelli in the companion volume. Of course, the more recent literature concerned with this theory and its applications is strikingly vast (on the day of writing, MathSciNet yields 1835 entries having the term "Sturm-Liouville" in their title); it is therefore unavoidable that there may be certain aspects of the theory which are not sufficiently covered here.

The articles in this volume can be read essentially independently. The authors have included cross-references to other contributions. In order to respect the style and habits of the authors, the editors did not ask them to use a uniform standard for notations and conventions of terminology. For example, the reader should take note that, according to author, inner products may be anti-linear in the first or in the second argument, and deficiency indices are either single natural numbers or pairs of numbers. Moreover, there are some differences in terminology as regards spectral theory.

The colloquium would not have been possible without support from numerous individuals and organizations. Financial contributions were received from various divisions of the University of Geneva (Commission administrative du Rectorat, Faculté des Lettres, Faculté des Sciences, Histoire et Philosophie des Sciences, Section de Physique), from the History of Science Museum and the City of Geneva, the Société Académique de Genève, the Société de Physique et d'Histoire Naturelle de Genève, the Swiss Academy of Sciences and the Swiss National Science Foundation. To all these sponsors we express our sincere gratitude. We also thank the various persons who volunteered to take care of numerous organizational tasks in relation with the colloquium, in particular Francine Gennai-Nicole who undertook most of the secretarial work, Jan Lacki and Andreas Malaspinas for technical support, Danièle Chevalier, Laurent Freland, Serge Richard and Rafael Tiedra de Aldecoa for attending to the needs of the speakers and other participants. Special thanks are due to Jean-Claude Pont for his enthusiastic collaboration over a period of more than three years in the entire project, as well as to all the speakers of the meeting for their stimulating contributions.

As regards the present volume, we are grateful to our authors for all the efforts they have put into the project, as well as to our referees for generously

giving of their time. We thank Norrie Everitt, Hubert Kalf, Karl Michael Schmidt, Charles Stuart and Peter Wittwer who freely gave their scientific advice, Serge Richard who undertook the immense task of preparing manuscripts for the publishers, and Christian Clason for further technical help. We are much indebted to Thomas Hempfling from Birkhäuser Verlag for continuing support in a fruitful and rewarding partnership.

The cover of this book displays, in Liouville's handwriting, the original formulation by Sturm and Liouville, in the manuscript of their joint 1837 paper, of the regular second-order boundary value problem on a finite interval. The paper, which is discussed here by W.N. Everitt on pages 47–50, was presented to the Paris Académie des sciences on 8 May 1837 and published in *Comptes rendus de l'Académie des sciences*, Vol. IV (1837), 675–677, as well as in *Journal de Mathématiques Pures et Appliquées*, Vol. 2 (1837), 220–223. The original manuscript, with the title “Analyse d'un Mémoire sur le développement des fonctions en séries, dont les différents termes sont assujettis à satisfaire à une même équation différentielle linéaire contenant un paramètre variable”, is preserved in the archives of the Académie des sciences to whom we are much indebted for kind permission to reproduce an extract.

Geneva, September 2004

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Past and Present

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